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An Investigation of the Air Chamber of Horn Type Loudspeakers

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The air chamber is treated as a boundary value problem which results in the solution of the wave equation for the general case in which the horn throat enters the air chamber in any circumferentially symmetrical manner. The following specific cases are analyzed: (1) the case in which the horn throat enters the air chamber by means of a single orifice, (2) the horn throat enters the air chamber by means of a single annulus of radius r and width w, and (3) the horn throat enters the air chamber in "m" annuluses of radii $r_1 \cdots r_m$ and widths $w_1 \cdots w_m$. The analysis reveals that the radial perturbation caused by the horn throat excites higher order modes. At the resonant frequencies of these modes the horn throat pressure becomes zero and the loudspeaker does not radiate. By suitable choice of annulus radii and widths the first "m" modes may be suppressed and the corresponding nulls in the output pressure eliminated.

IN the classical analysis of the air chamber of horn type loudspeakers (Fig. 1) the assumption is made that all of the dimensions are small compared with wavelength.[1–3] In many applications of horn type loudspeakers this assumption is justified, but in the case of horn type tweeters the wavelength becomes a small fraction of the diameter of the air chamber. In order to develop a theory which will accurately predict the high frequency performance of the loudspeaker, the air chamber must be treated as a boundary value problem.

[1] W. P. Mason, *Electromechanical Transducers and Wave Filters* (D. Van Nostrand Company, Inc., New York, 1942), pp. 225–230.
[2] H. F. Olson, *Elements of Acoustical Engineering* (D. Van Nostrand Company, Inc., New York, 1940), pp. 190–191.
[3] B. H. Smith and W. T Selsted, Audio Eng. 34, 16 (1950).

Since the thickness of the air chamber is negligible compared with the radius of curvature of the diaphragm, wave propagation within the air chamber will be negligibly different from that in a similar circular cylindrical cavity. The radius of this cavity is equal to the distance along the diaphragm from the center to the outer edge of the air chamber.

THE LOUDSPEAKER AIR CHAMBER AS A BOUNDARY VALUE PROBLEM

It will be assumed that the horn throat enters the air chamber without circumferential variations. For the present the axial component of velocity over the front of the air chamber where the horn throat enters will not be defined; instead it will be kept general, and called

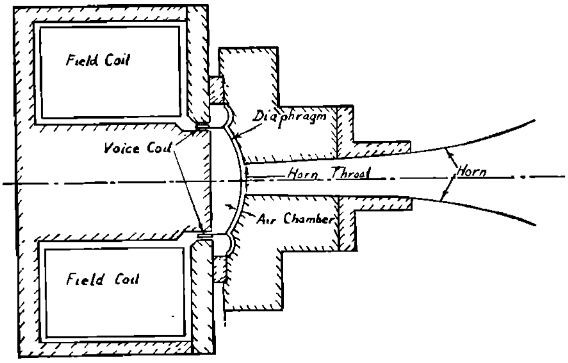


FIG. 1. Typical horn loudspeaker. The cavity between the diaphragm and horn throat is called the air chamber. It has the same cross-sectional area as the diaphragm and is of the order of 0.020 of an inch thick.

$f(r)$. Along the diaphragm the axial component of velocity will be assumed sinusoidal and of amplitude u_0. Along the outer boundary of the air chamber it will be assumed that the radial component of velocity is zero. The immediate problem is to choose a solution of the wave equation which will meet these boundary conditions. Mathematically the problem may be stated as follows:

$$\nabla^2\Phi = k^2\Phi \quad k=\omega/c, \tag{1}$$
$$u_z = u_0, \quad z=0, \quad 0\leqslant r\leqslant a,$$
$$u_z = f(r), \quad z=l, \quad 0\leqslant r\leqslant a,$$
$$u_z = 0, \quad 0\leqslant z\leqslant l, \quad r=a,$$
$$\bar{u} = -\nabla\Phi, \quad p = j\omega\rho\Phi.$$

The solution is

$$\Phi = -\frac{u_0}{k}(\sin kz + \cot kl \cos kz) + \sum_{n=0}^{\infty} \frac{2J_0(k_n r)\cos\gamma_n z}{\gamma_n a^2 J_0^2(p_n')\sin\gamma_n l}\int_0^a f(r)J_0(k_n r)rdr, \tag{2}$$

$$p \simeq j\rho\frac{c}{kl}\left(-u_0 + \frac{2}{a^2}\int_0^a f(r)rdr + \sum_{n=1}^{\infty}\frac{2J_0(k_n r)}{[1-(f_n/f)^2]a^2J_0^2(p_n')} \times \int_0^a f(r)J_0(k_n r)rdr\right), \tag{3}$$

$$f_n = p_n'c/2\pi a. \tag{4}$$

Equation (3) is a general expression for the pressure in any air chamber without circumferential variations. It is apparent that if $f(r)$ is a constant there is neither radial pressure variation nor radial velocity variation. If $f(r)$ is not a constant, the third term in Eq. (3) produces a radial pressure variation, the magnitude of which is a function of frequency. Each of the terms in the series is a higher order mode, and the degree of excitation of this mode depends upon the ratio of the frequency to the resonant frequency of the mode.

First, let us consider the simplest type of air chamber and then proceed to successively more complex types.

THE CASE IN WHICH THE HORN THROAT ENTERS THE AIR CHAMBER BY MEANS OF A CENTER HOLE

The quantity $f(r)$ must be defined, and then Eq. (3) is applied. For the moment we will neglect viscosity and assume that the particle velocity is constant over the horn throat. (See Fig. 2.)

Thus, $f(r)$ may be defined as follows:

$$f(r) = u_\tau = p_\tau/\rho c, \quad 0\leqslant r\leqslant r_1,$$
$$f(r) = 0, \quad r_1 < r \leqslant a.$$

Equation (3) then becomes

$$p \simeq j\rho\frac{c}{kl}\left[-u_0 + \frac{2}{a^2}\int_0^{r_1}\frac{p_\tau}{\rho c}rdr + \sum_{n=1}^{\infty}\frac{2J_0(k_n r)}{[1-(f_n/f)^2]a^2J_0^2(p_n')}\int_0^{r_1}\frac{p_\tau}{\rho c}J_0(k_n r)rdr\right]. \tag{5}$$

If one performs the indicated integration,

$$p \simeq j\frac{1}{kl}\left\{-u_0\rho c + p_\tau\frac{A_t}{A_D}\left[1 + \sum_{n=1}^{\infty}\frac{2J_1(k_1 r_1)J_0(k_n r)}{[1-(f_n/f)^2]J_0^2(p_n')k_n r_1}\right]\right\} \tag{6}$$

at $r=0$ $p=p_\tau$,

$$u_0 = p_\tau\left[j\frac{\omega l}{\rho c^2} + \frac{1}{\rho c}\frac{A_t}{A_D} \times \sum_{n=1}^{\infty}\frac{2A_tJ_1(k_n r_1)}{\rho c A_D k_n r_1 J_0^2(p_n')[1-(f_n/f)^2]}\right]. \tag{7}$$

It is apparent from Eq. (7) that at the resonant frequency of each mode the throat pressure, p_t is zero. Since $u_t = p_t/\rho c$, the throat velocity also is zero at these resonances, and therefore, along $z=l$ there is no axial component of velocity.

A physical picture of the wave propagation within the air chamber can be obtained by solving for the axial and radial velocities. The velocity potential is

$$\Phi = -\frac{u_0}{k}(\sin kz + \cot kl\cos kz) + \frac{u_\tau A_t}{k^2 l A_D}\left[\cos kz + \sum_{n=1}^{\infty}\frac{2J_1(k_n r_1)J_0(k_n r)\cos k[1-(f_n/f)^2]^{\frac{1}{2}}z}{[1-(f_n/f)^2]J_0^2(p_n')k_n r_1}\right]. \tag{8}$$

Thus,

$$u_z = -\frac{\partial \Phi}{\partial z} = u_0(\cos kz - \cot kl \cos kz) + \frac{u_r A_t}{klA_D}\Bigg[\sin kz + \sum_{n=1}^{\infty} \frac{2J_1(k_n r_1)J_0(k_n r)\sin k[1-(f_n/f)^2]^{\frac{1}{2}} z}{[1-(f_n/f)^2]^{\frac{1}{2}} J_0^2(p_n')k_n r_1}\Bigg],$$

and

$$u_r = -\frac{\partial \Phi}{\partial r} \simeq \frac{\dfrac{2u_0 A_t}{k^2 l A_D}\displaystyle\sum_{n=1}^{\infty} \frac{J_1(k_n r_1)J_1(k_n r)}{[1-(f_n/f)^2]J_0^2(p_n')r_1}}{jkl + \dfrac{A_t}{A_D}\Bigg[1+\displaystyle\sum_{n=1}^{\infty} \frac{2J_1(k_n r_1)}{k_n r_1 J_0^2(p_n')[1-(f_n/f)^2]}\Bigg]}. \qquad (9)$$

Let f_p be the resonant frequency of any one of the radial modes. Take the limit of the above expressions as f approaches f_p, and the components of velocity become

$$u_z\bigg|_{f=f_p} = u_0 \sin k_p z(\cot k_p z - \cot k_p l) \simeq u_0[1-(z/l)], \qquad (10)$$

$$u_r\bigg|_{f=f_p} = \frac{u_0}{k_p l} J_1(k_p r) = \frac{u_0 \lambda p}{2\pi l} J_1(k_p r). \qquad (11)$$

Equation (10) says that the axial component of velocity decreases linearly from that of the diaphragm to zero at the horn throat. Equation (11) says that a radial standing wave of very large amplitude is excited. Since the wavelength of the first few modes for practical loudspeakers is of the order of 5000 times the air chamber thickness, it is apparent that the radial component of velocity at the maxima of J_1 is of the order of 1000 times the axial velocity. Resonance can be visualized as a condition in which the velocity of the diaphragm excites only the radial mode and does not couple an axial component to the horn throat.

If the particle velocity is related to the throat pressure using the classical lumped constant analysis, the following relation between diaphragm velocity and throat pressure is obtained:

$$u_0 = p_r\left[j\frac{\omega l}{\rho c^2} + \frac{1}{\rho c}\frac{A_t}{A_D}\right]. \qquad (12)$$

If this is compared with Eq. (6), it is evident that the lumped constant solution can be derived from the wave solution by neglecting the higher order modes. The modes with resonances in the audiospectrum are ordinarily not negligible since they cause nulls in the output pressure of the loudspeaker.

The resonant frequencies of the higher modes are

$$f_n = p_n' c / 2\pi a,$$

and the resonant wavelengths are $\lambda_n = 2\pi a / p_n'$,

$$\lambda_1 = 1.64a,\quad \lambda_2 = 0.896a,\quad \lambda_3 = 0.618a,\quad \lambda_4 = 0.471a. \qquad (13)$$

In order to obtain numerical values for the resonant frequencies, the speed of sound within the air chamber must first be determined. It is lower than the free space value of 1130 ft per second because of the viscous forces acting in the confined space of the air chamber.

If the viscous forces are considered rigorously the problem becomes much more complex because they require modification of the wave equation itself. However, it has been shown[4,5] that the effects of viscosity can be accounted for by suitably modifying the propagation constant. This leads to attenuation and a change in the phase velocity of the waves. Applied to an air chamber the attenuation per unit length and the phase velocity are

$$\alpha = \frac{2.172}{cl}\left[\frac{\omega\mu}{2\rho}\right]^{\frac{1}{2}} \text{ db/unit length}, \qquad (14)$$

$$c' = \frac{c}{1+(1/2l)[\mu/2\omega\rho]^{\frac{1}{2}}}. \qquad (15)$$

For an air chamber with a thickness of 0.020 inches the attenuation and phase velocity become 1.54 db per inch and $0.825c$ at 5000 cps and 2.67 db per inch and $0.888c$ at 15 000 cps. Since the distances within the air chamber are small, the viscous attenuation only amounts to a few decibels. The change in the phase velocity reduces the frequency of the higher modes correspondingly.

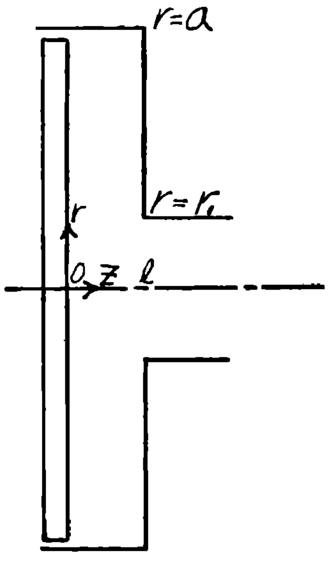
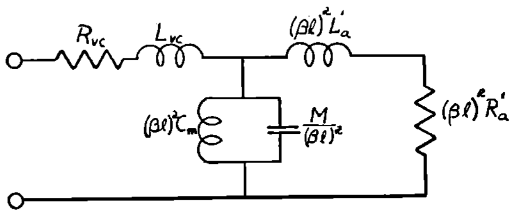


FIG. 2. Since the thickness of the air chamber is small compared with its radius of curvature it may be represented as a simple circular cavity for analytical purposes. r_1 is the radius of the horn throat, a the radius of the outer boundary of the air chamber, and l its thickness. In this case the horn throat enters as a simple orifice.

[4] See reference 1, pp. 114–120.

[5] L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics* (John Wiley and Sons, Inc., New York, 1950), pp. 238–245.



$$j\omega L'_a = X_a + \frac{\frac{A_t^2}{\rho c A_D^3}\sum_{n=1}^{\infty}\frac{K_n}{kl}}{1+\left(\frac{A_t}{A_D}\sum_{n=1}^{\infty}\frac{K_n}{kl}\right)^2}$$

$$R'_a = \frac{R_a}{1+\left(\frac{A_t}{A_D}\sum_{n=1}^{\infty}\frac{K_n}{kl}\right)^2}$$

FIG. 3. The equivalent circuit of a horn type loudspeaker.

For an air-chamber spacing of 0.020 inch the resonant frequencies of the first few modes are

$$f_1=6940/a, \quad f_2=13\,300/a, \quad f_3=20\,400/a,$$
$$f_4=28\,500/a, \quad a \text{ in inches.}$$

Let us now determine the force exerted upon the diaphragm by the air chamber. This procedure leads to a new equivalent circuit for the horn type loudspeaker which will include the effects of the higher modes.

The force at the diaphragm is

$$F=\int_0^a 2\pi p r dr. \tag{16}$$

From Eq. (6) the pressure is

$$p=j\frac{1}{kl}\left\{-u_0\rho c+p_r\frac{A_t}{A_D}\left[1+\sum_{n=1}^{\infty}K_nJ_0(k_nr)\right]\right\},$$
$$K_n=\frac{2J_1(k_nr_1)}{[1-(f_n/f)^2]k_nr_1J_0^2(p_n')}, \tag{17}$$

$$F=j\frac{1}{kl}\left\{-u_0\rho cA_D\right.$$
$$\left.+p_r\frac{A_t}{A_D}\left[A_D+\sum_{n=1}^{\infty}\int_0^a 2\pi K_nJ_0(k_nr)rdr\right]\right\}. \tag{18}$$

The above integral is zero. Substituting p_t from (7) the force becomes

$$F=j\frac{u_0\rho c}{kl}\left\{-A_D+\frac{A_t}{jkl+\frac{A_t}{A_D}\left(1+\sum_{n=1}^{\infty}K_n\right)}\right\}. \tag{19}$$

The quantity analogous to electrical impedance, when the force-current analogy is used, is the ratio of velocity to force. If one denotes this quantity as Z and rearranges Eq. (19),

$$\frac{u_0}{F}=Z=\frac{R_a}{1+\left(\frac{A_t}{A_D}\sum_{n=1}^{\infty}\frac{K_n}{kl}\right)^2}$$
$$+j\left\{X_a+\frac{\frac{A_t^2}{\rho cA_D^3}\sum_{n=1}^{\infty}\frac{K_n}{kl}}{1+\left(\frac{A_t}{A_D}\sum_{n=1}^{\infty}\frac{K_n}{kl}\right)^2}\right\} \tag{20}$$

in which

$$R_a=A_t/\rho cA_D^2, \quad X_a=j\omega l/A_D.$$

R_a and X_a are the quantities used in the classical equivalent circuit of the horn type loudspeaker to represent the air chamber and horn. Equation (20) reduces to these values if the mode terms are neglected. The equivalent circuit of the horn type loudspeaker is shown in Fig. 3.

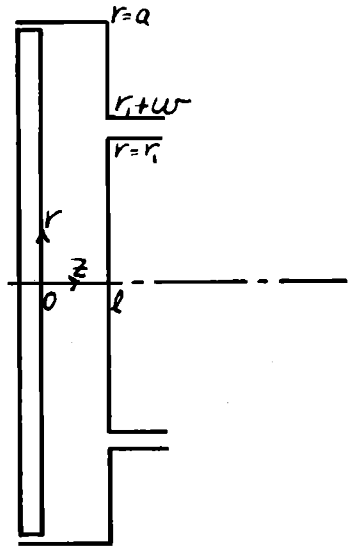


FIG. 4. The case in which the horn throat enters the air chamber as a single annulus. The normal component of velocity along the diaphragm is u_0. It is zero along the rigid boundary of the air chamber and u_1 at the horn throat.

THE CASE IN WHICH THE HORN THROAT ENTERS THE AIR CHAMBER BY MEANS OF A SINGLE ANNULUS

Now let us consider a slightly more complex air chamber[6] (see Fig. 4). For this type of air chamber, $f(r)$ may be defined as follows:

$$f(r)=0, \quad 0\leqslant r<r_1, \quad z=l,$$
$$f(r)=0, \quad r_1+w<r<a, \quad z=l,$$
$$f(r)=u_r=p_r/\rho c, \quad r_1\leqslant r\leqslant r_1+w_1, \quad z=l.$$

Substituting these values into Eq. (3), the pressure

[6] E. C. Wente and A. L. Thuras, Bell System Tech. J., 7, 140 (1928).

becomes

$$p\simeq j\frac{\rho c}{kl}\Bigg[-u_0+u_r\frac{A_t}{A_D}$$
$$+\sum_{n=1}^{\infty}\frac{2J_0(k_n r)u_r}{[1-(f_n/f)^2]a^2J_0^2(p_n')}\int_{r_1}^{r_1+w}J_0(k_n r)rdr. \quad (21)$$

Assuming that w is small compared with r_1:

$$p\simeq j\frac{\rho c}{kl}\Bigg\{-u_0$$
$$+u_r\frac{A_t}{A_D}\left[1+\sum_{n=1}^{\infty}\frac{J_0(k_n r_1)J_0(k_n r)}{[1-(f_n/f)^2]J_0^2(p_n')}\right]\Bigg\}. \quad (22)$$

Any mode, the "j"th, for example, can be suppressed by choosing $k_j r_1$ to be a root of J_0. Logically, one would choose to suppress the first mode, and that requires

$$r_1=0.628a.$$

Physically, when this choice of r_1 is made, the perturbation of the horn throat is being placed at the node of the first mode, and for this reason it is not excited. If, however, some other unavoidable perturbation should excite the mode, the horn throat being at the node would not be coupled to it. This air chamber has the property that it neither excites nor couples the horn to the suppressed mode. Therefore, it provides extremely good mode suppression.

The equivalent circuit of this type of air chamber is the same as that for the air chamber with the horn coupled by means of a center hole except that K_n takes on a new value. It is

$$K_n=\frac{J_0^2(k_n r_i)}{[1-(f_n/f)^2]J_0^2(p_n')}. \quad (23)$$

THE CASE IN WHICH THE HORN THROAT ENTERS THE AIR CHAMBER AS "M" ANNULUSES[7]

$$f(r)=0 \text{ if } 0\leqslant r<r_1,\quad r_1+w_1<r<r_2\cdots r_m+w_m<r<a,$$
$$f(r)=u_i \text{ if } r_i\leqslant r\leqslant r_i+w_i \quad i=1,2,\cdots m.$$

Substituting these values into Eq. (3) and integrating, one gets

$$p=j\frac{\rho c}{kl}\Bigg\{-u_0$$
$$+\sum_{i=1}^{m}\frac{u_iA_i}{A_D}\left[1+\sum_{n=1}^{\infty}\frac{J_0(k_n r_i)J_0(k_n r)}{[1-(f_n/f)^2]J_0^2(p_n')}\right]\Bigg\}. \quad (24)$$

In order to suppress the "j"th mode,

$$\sum_{i=1}^{m}A_iJ_0(k_j r_i)=0.$$

The first a modes can be suppressed by letting "j" take on integral values from 1 to m. This produces a set of simultaneous equations:

$$\begin{array}{cccc} A_1J_0(k_1r_1) & \cdot\ \cdot\ \cdot & A_mJ_0(k_1r_m)=0 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ A_1J_0(k_mr_1) & \cdot\ \cdot\ \cdot & A_mJ_0(k_mr_m)=0. \end{array} \quad (25)$$

Any set of annulus areas and radii which satisfy Eq. (25) will suppress the first m modes. One way of doing this is to choose the radii such that

$$J_0(k_m r_i)=0 \quad i=1,\cdots m, \quad (26)$$

i.e., choose the radii to be at the nodes of the "m"th mode of J_0. This reduces Eq. (25) to "m–1" equations. These equations can be solved simultaneously for the area of each annulus. For the case of one, two, or three annuluses the proper radii and widths of annulus are

for $m=1$,
$r_1=0.628a$ $\quad w_1$ arbitrary;
for m=2,
$r_1=0.334a$ $\quad r_2=0.788a$,
w_1 arbitrary $\quad w_2=1.004w_1$;
for $m=3$,
$r_1=0.238a$ $\quad r_2=0.543a$ $\quad r_3=0.853a$,
w_1 arbitrary $\quad w_2=1.025w_1$ $\quad w_3=1.065w_1$.

A physical picture of the mode suppression in multiple annulus air chamber can be obtained by considering the case of two annuluses. For this case the two an-

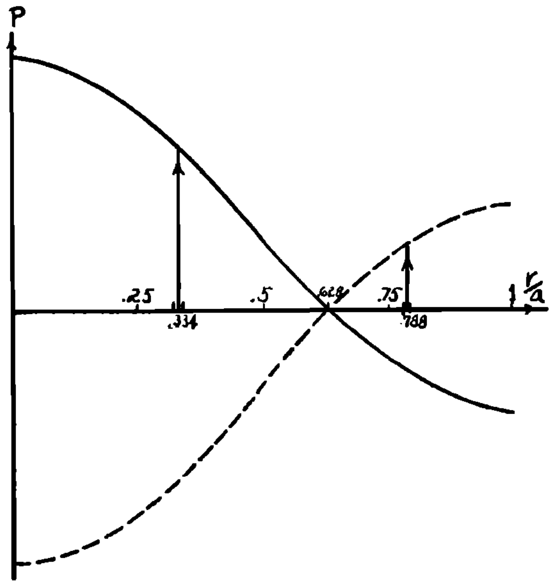


FIG. 5. If two annuluses are used, and placed at the nodes of the second mode, the second mode is not excited. Each annulus does excite a component of the first mode, but the two components are out of phase. Thus, if the relative annulus areas are properly chosen the two components will cancel, suppressing the first mode. Such an air chamber excites neither the first nor the second mode. In general, "m" annuluses properly designed will suppress the first "m" modes.

[7] E. C. Wente and A. L. Thuras, Trans. Am. Inst. Elec. Engrs. 53, 17 (1934).

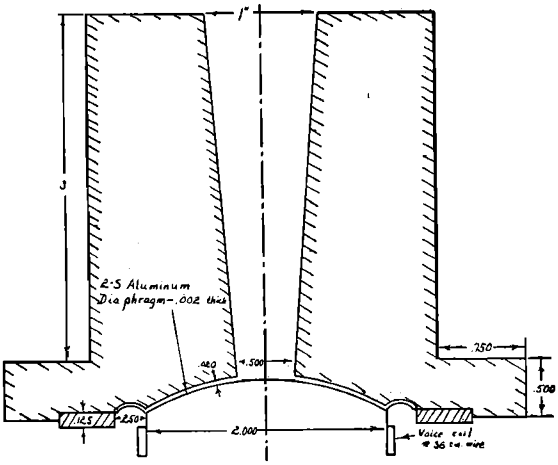


FIG. 6. Air chamber number one.

nuluses are placed at the nodes of the second mode, and hence, this mode is not excited. However, neither annulus is at the node of the first mode, and hence each excites a component of the first mode. The second annulus is at a greater radius than the node of the first mode, while the first annulus is at a smaller radius than the node. Therefore, the component of the first mode excited by the second annulus is out of phase with the component of first mode excited by the first annulus. The areas of these annuluses may be chosen in such a way that the two components of the first mode cancel. This is illustrated in Fig. (5).

EXPERIMENTAL RESULTS

The experimental work was carried on simultaneously with the theoretical work to serve not only as a confirmation of the latter but also to indicate which factors were important and which were negligible. In this way the experimental work aided the development of the theory.

The construction of the first three air chambers was started before the theory had been developed. As a

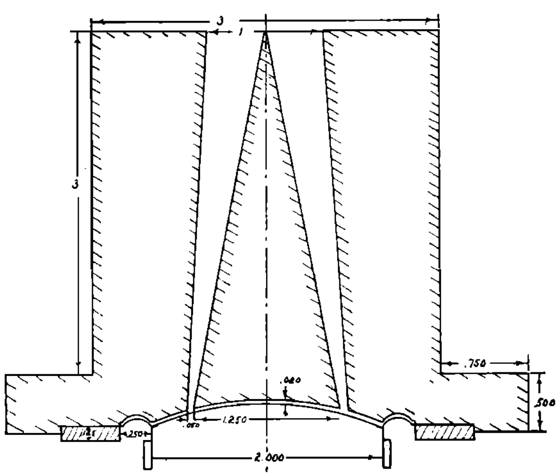


FIG. 7. Air chamber number two. r_1/a equals 0.463.

result the choice of annulus position was made according to the best commercial loudspeaker practice known at the time. The subsequent work of this investigation indicates that the annulus radii chosen would not suppress the higher modes.

Air chamber number one (Fig. 6) had the horn throat entering simply—by means of a center hole. Air chamber number two (Fig. 7) had a horn throat consisting of a single annulus for which $r_1=0.463a$. In order to suppress the first mode, r_1 should have been $0.628a$. Air chamber number three (Fig. 8) had a horn throat consisting of two annuluses and a center hole. These were equally spaced except that the distance from the outer annulus to the outer boundary of the air chamber was half of the distance between the other annuluses.

The frequency response curves of these air chambers was obtained and is shown in Fig. 9. (A machine was

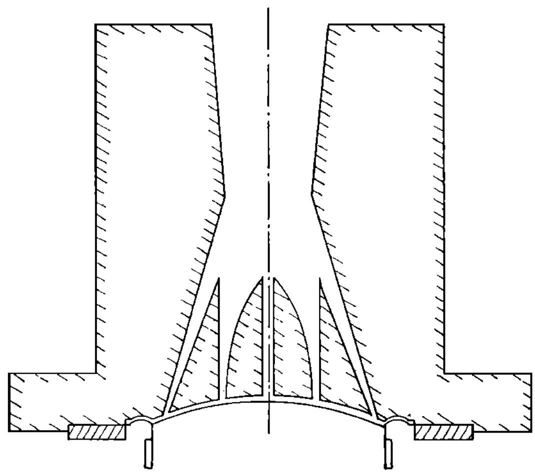


FIG. 8. Air chamber number three has a complex horn throat consisting of two annuluses and one center hole. The annuluses are not placed properly to suppress the higher modes.

built to obtain frequency response curves automatically.) It is apparent that the output pressure does drop to zero (the noise level) at each of the higher modes. Air chamber number two shows some suppression of the first mode. All of the modes are present in air chamber number three.

The radial length of these air chambers is approximately 1.375 inches. If the resonant frequencies of the higher modes are computed and compared with the measured values, a small discrepancy is observed. For example, consider air chamber number three. The following are the computed and measured values of the mode resonant frequencies:

Mode	Computed frequency	Measured frequency	Percent difference
1	5050	5500	8.9
2	9670	8000	17.5
3	14 800	14 000	5.7

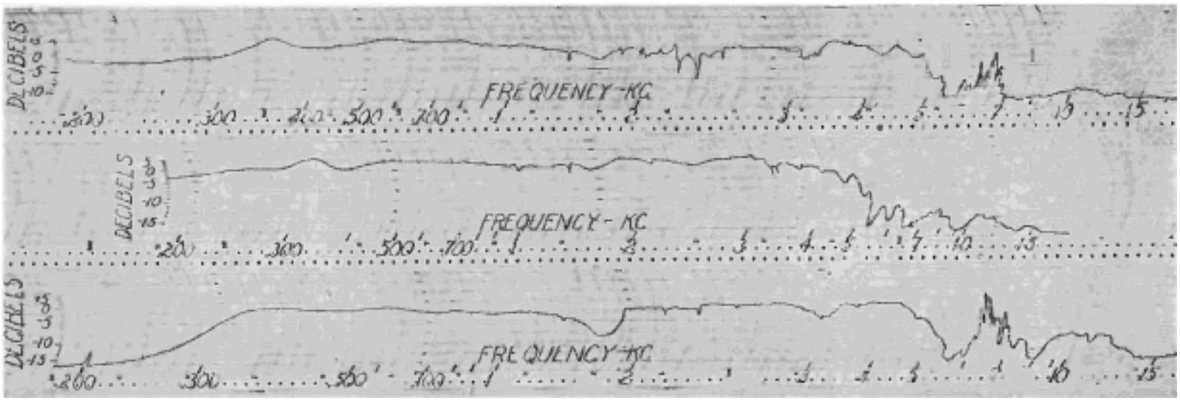


FIG. 9. Frequency response curves. Top—air chamber number one, middle—air chamber number two, bottom—air chamber number three. The nulls in the output pressure caused by the mode resonances are clearly evident at 5.5 kc (first mode), 8 kc (second mode), and 14 kc (third mode). Some first mode suppression is evident in number two, but the radius of the annulus is not correct for complete suppression.

The difference between these quantities cannot be attributed to experimental error; the audio-oscillator was compared with a frequency standard, and the measured results were suitably corrected. All of the dimensions of the air chamber except the thickness are known within one percent. The air-chamber thickness was designed to be 0.020 inch, but the tolerance is about plus or minus 0.005 inch. The tolerance might cause as much as a ten percent difference in the velocity of propagation within the air chamber and, hence, a corresponding error in the resonant frequency of the modes. However, this effect should be the same for all modes, i.e., if the measured value of the first mode were high, the second and third modes should be high also. The experimental work shows that this is not the case; the measured value of the first mode is high, while that of the second mode is low.

The displacement of the resonant frequency of the higher modes can be explained in the following way: The theory assumes that the diaphragm velocity is independent of radius. In an actual loudspeaker it varies slightly with radius due to the diaphragm compliance and the acoustical load. Moreover, since the acoustical load is a function of frequency, so is the radial distribution of diaphragm velocity. The diaphragm compliance and associated loads change the acoustical length of the air chamber in the same way that series inductance and shunt loads change the electrical length of resonant transmission lines. In other words, these quantities lengthen or decrease the effective air chamber radius, thus displacing the resonant frequencies of the higher modes.

The experimental work described thus far confirms the nulls in the output pressure of the loudspeaker at the resonant frequencies of the higher modes. In addition, it shows the influence of the diaphragm compliance and thus contributes to the theory.

After the theoretical work was complete, it was decided to construct a loudspeaker in which the first mode would be suppressed. The diaphragm diameter (1.69 inches) was chosen to place the second mode at 15.7 kc and the first mode at 8.2 kc. A single annulus was chosen for the horn throat and placed at approximately 62.8 percent of the air chamber radius.

In order for the loudspeaker to have a uniform frequency response up to 15 kc, the air-chamber thickness and the mass of the moving system would have to be too small for any practical future application of the unit. It was decided, therefore, to allow the frequency response to roll off smoothly above 5 kc. Uniform frequency response above 5 kc can easily be obtained by means of a resistance capacitance equalizer in the amplifier. This allows the moving system to have a mass of 400 mg and an air-chamber thickness of 0.015 inch. (See Fig. 10.)

Figure 11a shows the frequency response of the loudspeaker. The first mode, which would resonate at 8.2 kc, is suppressed, and the second mode resonates at 15 kc. Figure 11b shows the frequency response of the loud-

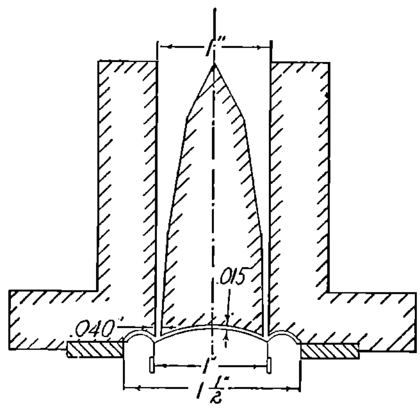


FIG. 10. For this air chamber r_1/a is 0.62 in order to suppress the first mode. The air-chamber radius has been chosen to place the second mode resonance at 15 kc. The diaphragm was spun from 0.001 inch aluminum foil, and the voice coil consists of 25 turns of number 36 copper wire. The moving mass is 400 mg.

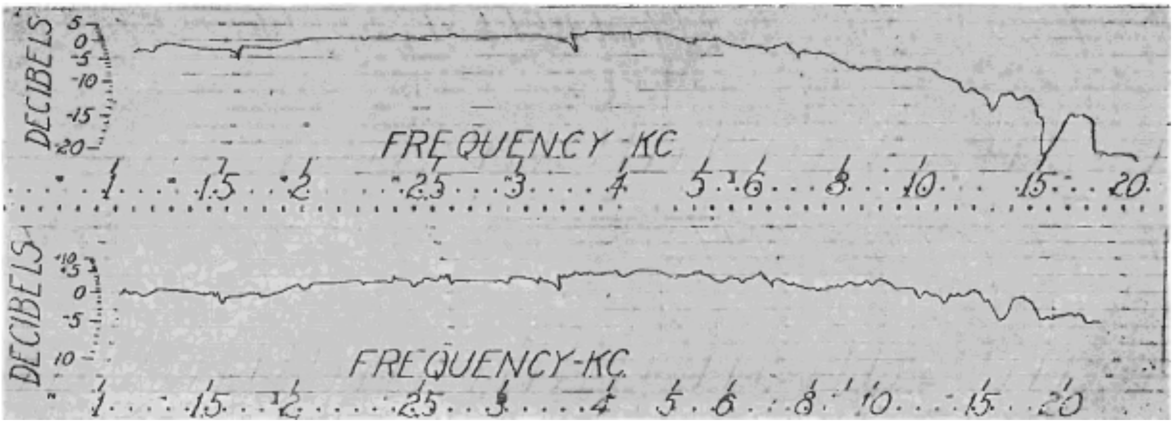


FIG. 11. (a). (top) The frequency response of the loudspeaker of Fig. 10. This loudspeaker was designed to have a uniform response up to 5 kc and to drop off smoothly until the second mode becomes effective at 15 kc. This makes possible a sufficiently heavy moving system to deliver a considerable amount of power. (b). (bottom) Since the response of the loudspeaker of Fig. 10 declines smoothly above 5 kc, it may be corrected with a simple resistance capacitance equalizer (three section). This curve is the response of the equalizer, amplifier, and loudspeaker.

speaker, amplifier, and equalizer; it is within plus or minus 3.75 db from 1 kc to 14.5 kc.

CONCLUSION

Treatment of the air chamber as a boundary value problem reveals the following physical picture. If there is no radial perturbation as in the case of the air chamber with no horn throat, there is no radial propagation and the pressure is independent of radius. When the air chamber is connected to a horn and the horn throat has smaller dimensions than the air chamber, the particles must move radially. This generates radial waves which are reflected by the outer boundary of the air chamber. These waves, or higher modes, become resonant for certain frequencies. At these frequencies the throat pressure becomes zero and the loudspeaker does not radiate.

Any one of the modes may be suppressed by making the horn throat an annulus which is located at the node, of this mode. If it is necessary to suppress two modes, two annuluses are required. These annuluses can be located at the nodes of the second mode and thus do not excite it. Each annulus does excite the first node, but the excitation by the second annulus is out of phase with that of the first annulus. By suitable choice of annulus widths, complete cancellation of the first mode results. Thus, the first two modes are suppressed. The process can be carried out for any number of annuluses, i.e., in the general case of "m" annuluses the first "m" modes can be suppressed.

The air chamber theory developed here suggests the following design procedure: The diaphragm size is selected by the power requirements of the loudspeaker. One then computes the frequencies of the modes associated with this diaphragm from Eq. (13), decides how many modes have to be suppressed, and chooses this number of annuluses. The radii of these annuluses are determined from Eq. (26) and the relative widths from the set of Eqs. (25). Next the constants of the loudspeaker are determined from the equivalent circuit (Fig. 3). For details of this last step see references 1 and 3. The total throat area is distributed according to the solution of Eqs. 25.

TABLE OF SYMBOLS

u_0	diaphragm velocity—meters per second
$\bar{u}$	particle velocity—meters per second
u_r	radial component of particle velocity
u_z	axial component of particle velocity
p_n'	"n"th root of $J_1(x)=0$
p	pressure—newtons per square meter
ρ	density of air—1.21 kg per cubic meter
c	velocity of sound
l	air-chamber thickness—meters
a	air-chamber radius—meters
r_j	radius of "j"th annulus—meters
w_j	width of "j"th annulus—meters
μ	coefficient of viscosity—1.84×10^{-3} kg/meter sec
ω	$2\pi f$
f	frequency cycles per second
λ	wavelength—meters
F	force—newtons
Φ	velocity potential
A_t	area of the horn throat—sq meters
A_D	area of the diaphragm sq meters
f_n	resonant frequency of the "m"th mode
$k=\omega/c$	
$k_n=2\pi f_n/c$	