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Refracting Sound Waves

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Structures are described which refract and focus sound waves. They are similar in principle to certain recently developed electromagnetic wave lenses in that they consist of arrays of obstacles which are small compared to the wave-length. These obstacles increase the effective density of the medium and thus effect a reduced propagation velocity of sound waves passing through the array. This reduced velocity is synonymous with refractive power so that lenses and prisms can be designed. When the obstacles approach a half wave-length in size, the refractive index varies with wave-length and prisms then cause a dispersion of the waves (sound spectrum analyzer). Path length delay type lenses for focusing sound waves are also described. A diverging lens is discussed which produces a more uniform angular distribution of high frequencies from a loud speaker.

INTRODUCTION

In the course of the investigation of artificial dielectric microwave lenses comprising arrays of small conducting objects,[1] much recourse was had to the early analytical work of Lord Rayleigh on the scattering of energy from objects which are small compared to the wave-length. In dealing with this subject, Rayleigh indicated that many of his results were applicable both to aerial (sound) waves and to electric (electromagnetic) waves.[2] It seemed probable, therefore, that the same principles which were applied in the focusing of electromagnetic waves could be applied to sound waves. Preliminary experiments showed that certain existing microwave lenses using rigid elements in an open construction did focus sound waves over a similar range of wave-lengths. An investigation of this subject was therefore begun not only on convergent lenses but also on other optical counterparts such as divergent lenses and prisms. This paper describes the course and results of this investigation.

[1] W. E. Kock, Bell Sys. Tech. J. 27, 58 (1948).
[2] See for example, Lord Rayleigh, Phil. Mag. 43, 259 (1897); *ibid.* 44, 28 (1897); [collected Papers IV, pp. 283 and 305].

OBSTACLE ARRAYS

The Concept of Wave Refraction in Obstacle Arrays

The customary concept of an optical lens is that it is a continuous medium such as glass. One generally associates the term refraction with the continuous

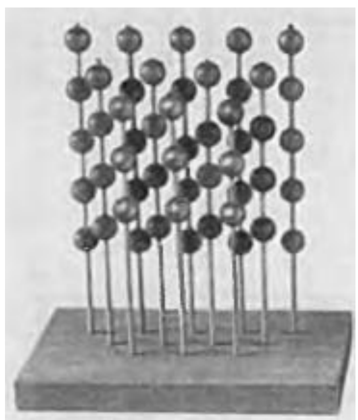


FIG. 1. An array of spherical obstacles.

nature of the medium and associates the term diffraction with non-continuous optical devices such as gratings and Fresnel zone plates. However, refraction can and does occur in media which are assemblages of individual, discrete particles, if the particles and the distances between them are small compared to the wave-length of the wave propagation under consideration. Max Born has presented[3] an elegant proof that the various extensions of Maxwell's equations which describe the behavior of electromagnetic waves in a continuous dielectric medium can be arrived at by assuming the medium to be an assemblage of discrete re-radiating particles (dipoles).

In both the electromagnetic and acoustic cases, the mechanism of refraction can be explained by two approaches: (1) the re-radiation from the individual obstacles, (2) the alteration of the properties of the original medium brought about by the immersion of the obstacles. Figure 1 shows a simple version of an obstacle array of the type we are considering. Here the obstacles are in the form of spheres; for refracting electromagnetic waves, they are made electrically conducting, while for acoustic waves they are assumed to be rigid (immovable). In the re-radiating dipole picture the spheres, under the influence of the impressed electromagnetic (or sound) waves, become small electric (or acoustic) dipoles and the resultant of the original wave and the re-radiated waves manifests itself as a new wave having a lower velocity inside the array. The acoustic dipole action of the spheres can be looked upon in this way. If they were very light and free to move to and fro with the pulsations of the sound wave, they would not influence the progress of the wave. However, when rigidly mounted, air which normally would have passed back and forth through the space occupied by the spheres is prevented from so doing and the re-radiated wave thus produced is equivalent to that generated by spheres moving back and forth in the direction of propagation of the sound waves.

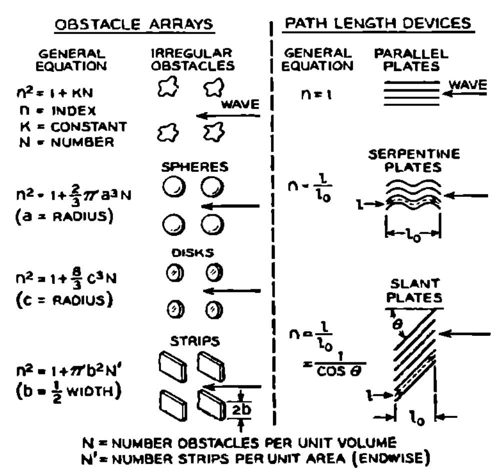


FIG. 2. Refractive index for delay mechanisms using arrays of rigid elements in an open construction.

[3] M. Born, Optik (Verlag Julius Springer, Berlin, 1933), Chapter 7, Article 74.

The second approach lends itself more easily to quantitative evaluation of the effective velocity reduction produced by a given array. In this picture, the extreme nature of the elements (perfectly conducting in the electric case and perfectly rigid in the acoustic case) is used to arrive at a mean refractive effect for the mixture of obstacles and the original medium. In the electromagnetic case, the relative dielectric constant is unity for free space and infinite for a perfect conductor. An array of conducting obstacles in free space thus appears to have a dielectric constant somewhere between unity and infinity. Consequently, its index of refraction is also greater than unity. Similarly, for our acoustic consideration, a perfectly rigid or immovable object has an effectively infinite mass, so that the combination of rigid (infinitely dense) spheres immersed in air (whose relative density is unity) results in a new medium having an index of refraction greater than unity.

The increased effective mass or density of a fluid caused by the immersion of an array of obstacles in it can be visualized in the following way. When one moves a paddle through a fluid, the paddle acquires an effective increased inertia depending upon the amount of fluid moved. Conversely when one holds a paddle rigidly in a moving fluid, the fluid acquires an effective increased inertia or mass. The increased mass of the fluid caused by the presence of an array of obstacles can be used to calculate the increased density of this artificially produced medium. Since the velocity of sound is dependent upon the density of the medium, the velocity of propagation through an obstacle array is less than that in free air.

Evaluation of the Index of Refraction

The increased inertia of a sphere moving through a fluid is known to be equal to $\frac{1}{2}$ the mass of the displaced fluid.[4,5] If, instead, the fluid is in motion and the sphere fixed, the fluid acquires this increased effective mass. A fluid moving past an array of N spheres per unit volume would thus appear to have its original density ρ_0 increased to the value

$$\rho = \rho_0 + \tfrac{1}{2} N \rho_0 V, \tag{1}$$

where V is the volume of one sphere. That is, the ratio of the effective density of the sphere array to that of the free medium is

$$\rho/\rho_0 = 1 + \tfrac{1}{2} N (4\pi/3) a^3, \tag{2}$$

where a is the radius of the spheres.

Since the velocity of propagation of sound in a medium is inversely proportional to the square root of

[4] H. Lamb, *Hydrodynamics* (Cambridge University Press, London, 1916), p. 116.

[5] Lord Rayleigh, *Theory of Sound* (MacMillan Company, London, 1940), Vol. II, p. 248.

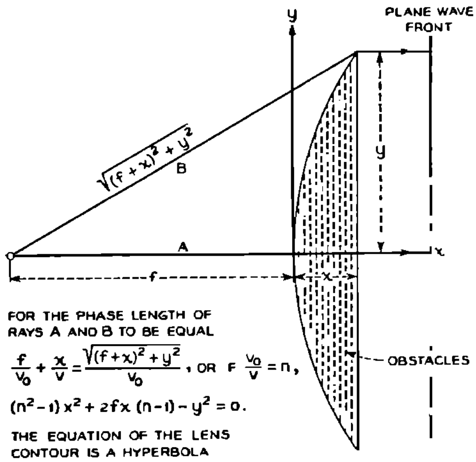


FIG. 3. Profile equation for a delay lens of the obstacle type.

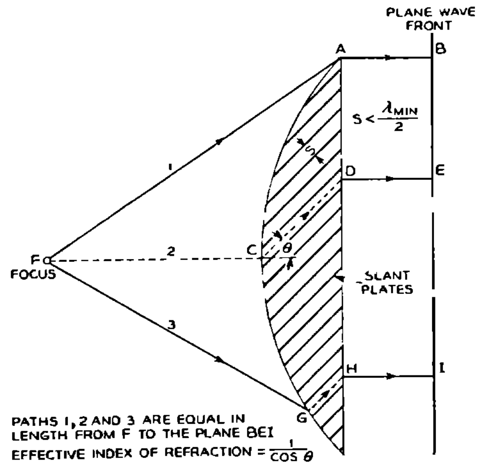


FIG. 4. Geometrical requirements necessary to establish the profile of the slant plate lens.

the density of the medium, the square of the ratio of the sound velocities is

$$(v_0/v)^2 = n^2 = 1 + (2\pi/3)Na^3, \tag{3}$$

where n is the index of refraction. (See Fig. 2.)

For the case of disks, the mass increase per disk is $(2/\pi)\rho_0(4/3)\pi c^3$ where c is the disk radius.[5] This gives

$$n^2 = 1 + \tfrac{8}{3}Nc^3 \tag{4}$$

for a disk array having N disks per unit volume.

For the case of strips, the following equation holds true:[6]

$$n^2 = 1 + \pi b^2 N', \tag{5}$$

where b is the half-breadth of the strip and N' is the number per unit area looking end on at the strips.

Limitations on the Refractive Index Equations

The equations of the preceding section were derived by assuming that the sound field acting on an element was the impressed field alone. This is a satisfactory assumption when the separation between the objects is so large that the elements themselves do not distort the field acting on the neighboring elements. Such is not the case when n exceeds the value of 1.2 or thereabouts.

Furthermore, the above equations are valid only if the size and spacing of the obstacles is small compared to the wave-length. When the obstacle size nears a half-wave-length, resonance effects can occur* and the propagation velocity is strongly affected. Then the expression for n will be modified in a qualitative manner as follows:

$$n^2 \cong 1 + [k/(f_0^2 - f^2)], \tag{6}$$

where k is a constant of the obstacle configuration, f the applied frequency and f_0 the resonant frequency associated with the obstacle.** As f approaches f_0 closely, n begins to increase rapidly, resulting in the familiar phenomenon of dispersion. At $f = f_0$, n becomes quite large and complete reflection occurs. The array then is a better reflector than refractor. Further increase in frequency can result in a value of n less than one and

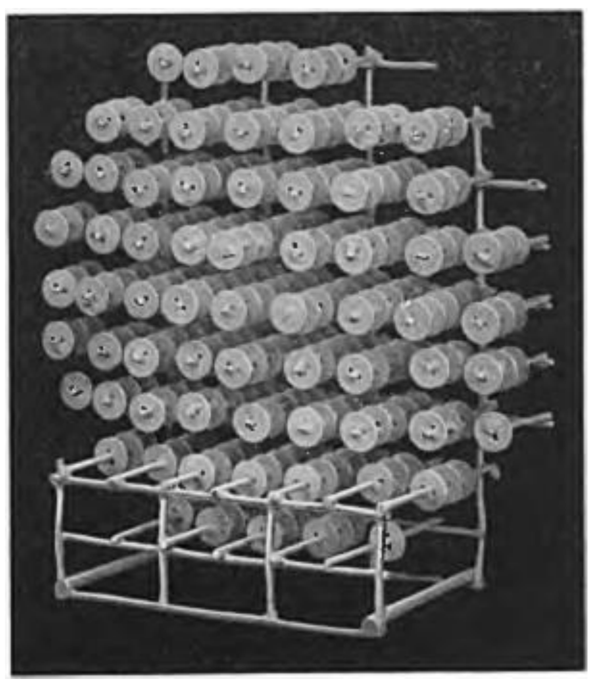


FIG. 5. A disk lens 6 in. in diameter. This lens is convergent and is composed of a double convex array of ½-in. diameter disks.

[6] See reference 4, p. 81, Eq. (11).

* A piston sound source in free air radiates very little energy at long wave-lengths but becomes more effective as the wave-length approaches twice the diameter; similarly a stationary disk reflects very little energy at long wave-lengths but becomes an effective reflector at the shorter wave-lengths.

** This is often called the Sellmeier equation. However, Lord Rayleigh has pointed out that Maxwell had considered it much earlier than Sellmeier. (See Lord Rayleigh, Scientific Papers IV, p. 414, the final equation.)

bring the system into the region of abnormal or anomalous dispersion.

At this point it might be well to mention that many of the experiments to be described later were purposely conducted at frequencies where the obstacle dimensions were not small compared to the wave-length but were approaching or even equaling a half wave-length. This was done in several cases in order to accentuate the focusing effects by increasing the index of refraction and also to make the array dimension contain as many wave-lengths as possible and thereby to become more directive.

PATH LENGTH REFRACTORS

General Description

Another method of slowing down a progressive wave is to guide it through a conduit which provides a longer

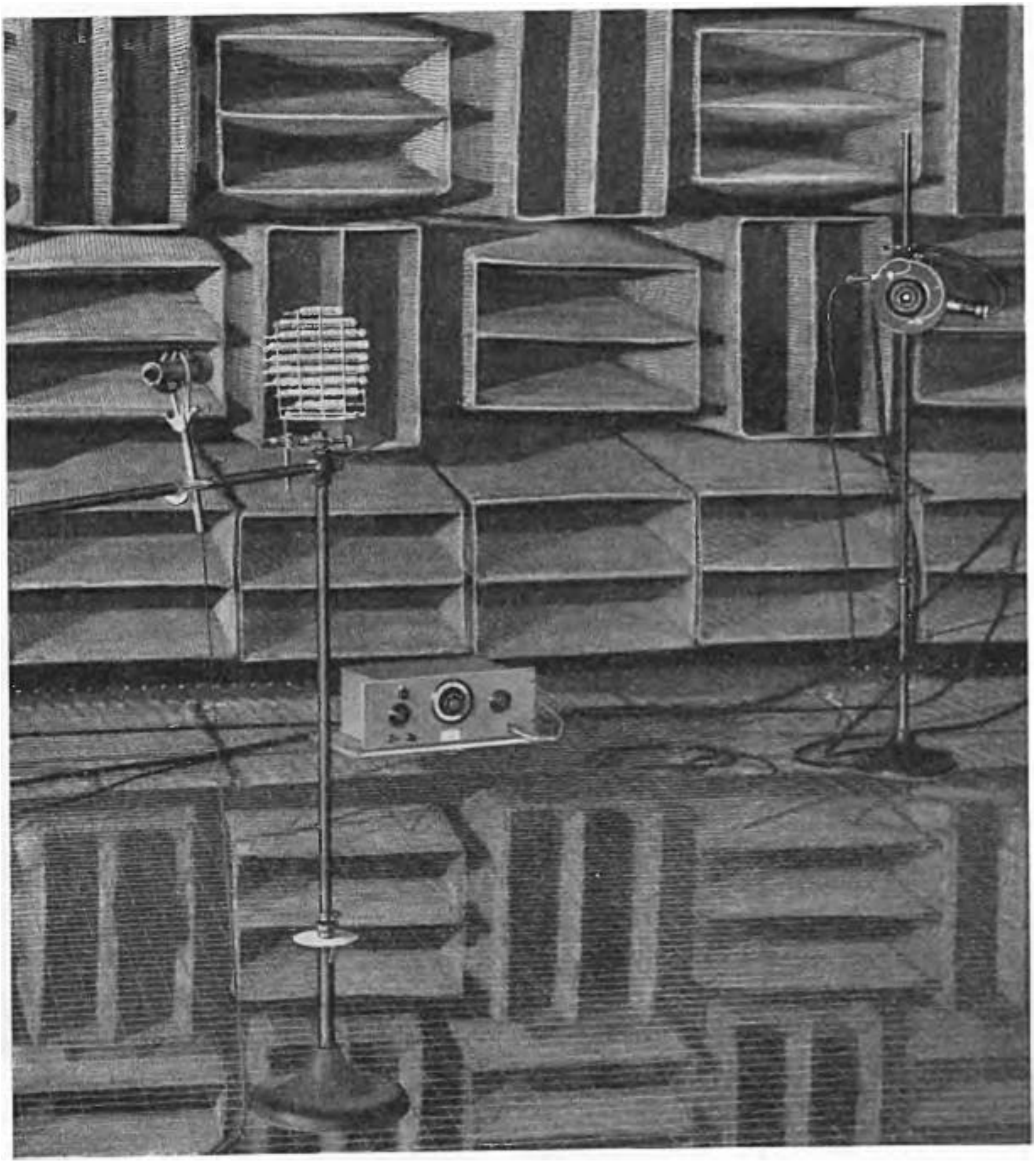


FIG. 6. The disk lens in position between transducers ready for test. Free space conditions are simulated by the use of sound absorbing wedges at all wall, floor, and ceiling surfaces. The apparatus is supported on a raised wire grid, acoustically transparent, floor.

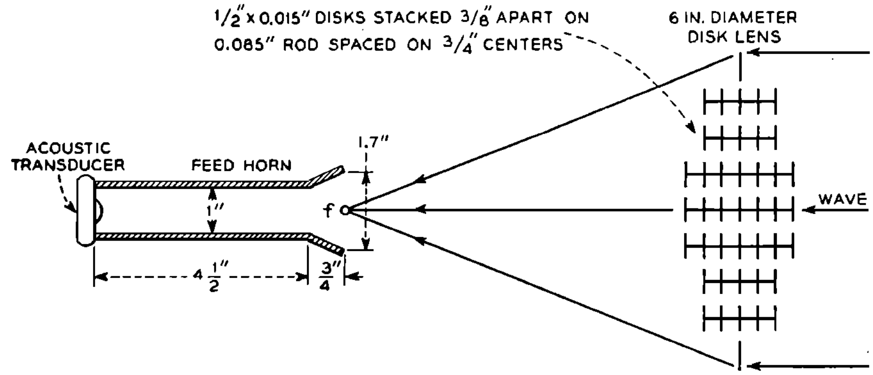


FIG. 7. Construction of the disk lens and its feed horn.

path than the unguided wave would normally take.*** This method of obtaining wave delay has likewise been employed for focusing electromagnetic waves.[7]

The principles of operation of the path length lenses can be described in connection with the right-hand section of Fig. 2. If parallel plates are presented to plane acoustic waves, little effect will be produced on the progress of the waves, providing that the plates are flat and aligned along the direction of propagation, as shown in the top of the figure. If, however, the plates are bent into serpentine shape, as shown in the middle figure, the sinuous path l, inside the plates, will be longer than that outside, l_0, and delay will be produced. Likewise, if the plates are tilted so that they form an angle with the direction of propagation, as shown in the bottom figure, a delay will be produced since the waves will be forced to traverse the longer inclined path.

Evaluation of the Index of Refraction

If planes are drawn perpendicular to the direction of the approaching unguided wave at the entrance and exit points of the conduit, then the distance between these entrance and exit planes is equal to the path length l_0, the unguided wave would normally travel (see Fig. 2). If l is the path length in the conduit, then the index of refraction is

$$n = v_0/v = l/l_0, \tag{7}$$

where v_0 is the velocity of the unguided wave in air and v is the velocity across the conduit structure bounded by the entrance-exit planes.

If the conduit is formed by slanted parallel plates, then,

$$n = l/l_0 = 1/\cos\theta \tag{8}$$

where θ is the angle between the slanted plates and the direction of the oncoming wave.

In such path length devices, n remains constant with frequency up to the point where the plate spacing becomes a half wave-length. A second mode can then be propagated which interferes with the normal action.

*** An acoustic radiator using tubes of varying lengths to obtain the proper phase correction for focusing has been suggested by W. P. Mason, U. S. patent 2,225,312, 1940.

[7] W. E. Kock, Proc. Inst. Radio Eng. 37, 852 (1949).

GENERAL CONSIDERATIONS

Determination of the Lens Profile

Two general classes of construction have now been discussed which act to reduce the velocity of sound waves. Devices using either of these constructions are delay mechanisms and act like the refractors of optics. An example of a design procedure for a convergent lens follows.

The profile of a lens (plano-convex) of the obstacle type can be determined from the desired aperture radius y, the focal length f and the known index of refraction n (see Fig. 3). For the phase length of parallel rays leaving the lens to be equal after starting from a common focal point, the following must hold:

$$f/v_0 + x/v = [(f+x)^2 + y^2]^{\frac{1}{2}}/v_0, \tag{9}$$

where v_0 is the velocity of sound in free air and v the velocity through the lens.

Since $v_0/v = n$, (9) becomes

$$f + nx = [(f+x)^2 + y^2]^{\frac{1}{2}}, \tag{10}$$

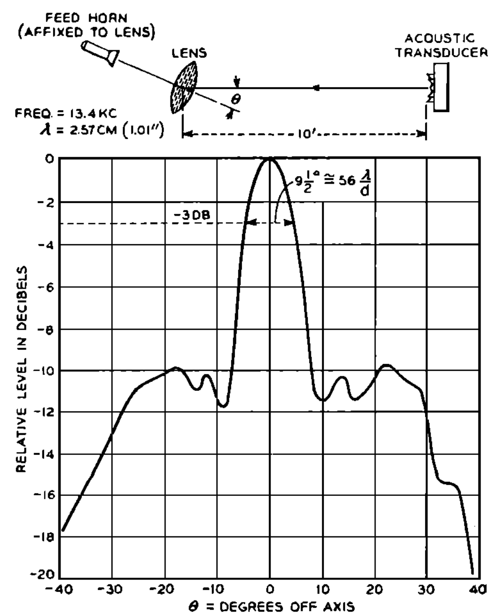


FIG. 8. Directional pattern of the disk lens and horn combination.

and this can be reduced to

$$(n^2-1)x^2+2fx(n-1)-y^2=0. \qquad (11)$$

It can be readily verified that (11) is the equation of a hyperbola. This equation need not be followed exactly; lenses having spherical surfaces approximating the hyperbolic shape will also exhibit appreciable focusing effect.

These formulas can be used on other lens shapes also. For example, a plano-concave lens can be calculated on the basis that f equals the *virtual* focus and x the depth of the concave surface, assuming of course, that parallel rays strike the plane surface. Double convex, double concave, and other lenses can thus be designed in sections.

Although the path length lens can generally be calculated from the formulas of the obstacle lens it is not always safe to do so because of the sidewise displacement of the wave which may accompany transmission through the lens. In the design of a slant plate lens (Fig. 4) the lens contour is adjusted to make the path lengths equal for rays which start at a common focal point and emerge simultaneously from the plane surface of the lens. This provides a plane wave front. In the example shown, the lens turns out to be symmetrical and the lens formulas can be used. If however, the lens should be reversed so that the rays starting from the focal point strike the flat surface, then the contour of the curved surface would have to be readjusted to an asymmetrical shape for a plane wave to emerge from it.

The upper frequency limits for the two types of delay mechanisms have already been suggested in the sections dealing with the index of refraction. The effectiveness of lenses of either type at low frequencies is a function of the size of the array. Since, as discussed below, a lens can only focus energy to a minimum diameter of the order of $\frac{1}{2}$-wave-length it is obvious that when the lens diameter is smaller than $\frac{1}{2}$-wave-length, little focusing action will be exhibited.

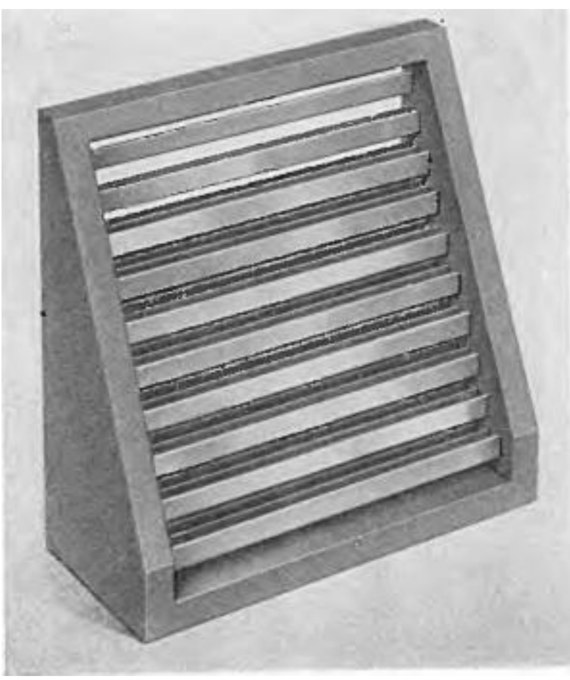


FIG. 9. A strip prism.

The Diameter of the Focal Spot of a Lens

In dealing with acoustic lenses where the wave-lengths used may be measured in feet and inches, it may be more pertinent to talk about a focal area of a

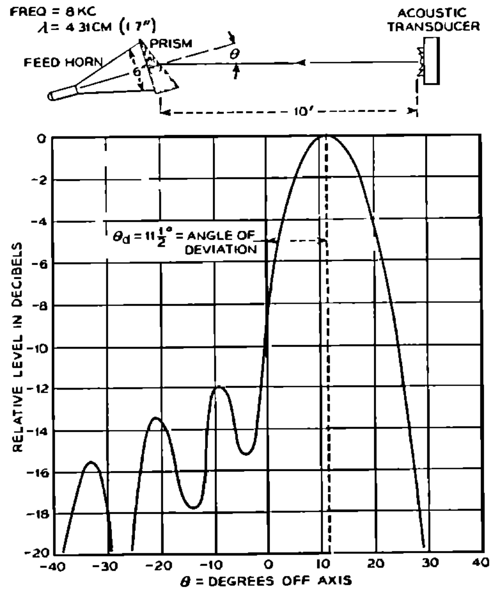


FIG. 10. Angular shift of the directional pattern of a 6-in. aperture horn caused by the prism of Fig. 9.

lens rather than the focal point. In any device which causes plane waves to be brought to a focus, diffraction sets a limit on the focal area.

The effect of diffraction can be obtained as in optics.[8] A properly designed lens will produce a diffraction pattern which consists of alternate concentric rings of minima and maxima surrounding a circular area of maximum energy. The diameter of this focal area as determined by the first minimum is approximately

$$d_1=(2.4F\lambda)/d, \qquad (12)$$

where F is the distance from the lens aperture to the focal area and d the aperture diameter. For a lens with an f 1.0 rating, the diameter of the focal area is 2.4λ with a plane wave incident.

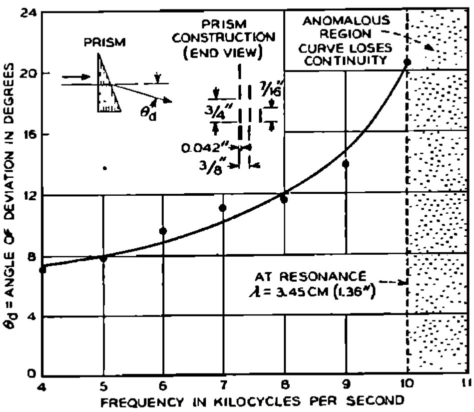
The central circular area receives approximately 80 percent of the lens energy. The maximum intensity at the first ring is 0.0174 (−17.6 db) of that at the center point. From this a directional pattern using a point source receiver at the focus would show the first minor lobe to be 17.6 db down.

[8] Hardy and Perrin, *The Principle of Optics* (McGraw-Hill Book Company, Inc., New York, 1932), p. 128.

An inspection of Eq. (12) shows that even for small f ratings a lens diameter of $\lambda/2$ would yield a focal area comparable to the lens itself. Hence for low frequency sound waves, appreciable focusing action can only be produced with very large lenses.

Reciprocity

According to the law of reciprocity, equivalent directional characteristics will be exhibited whether the lens under test is used as an acoustic radiator or receiver. Consequently, in the measuring techniques to be described, the combination under test is sometimes considered as transmitting and at other times receiving.



11. Dispersion produced by the strip prism of Fig. 9.

Gain Definition

The gain of an acoustic radiator will be defined[9] as the ratio if its maximum radiation intensity (power flow per unit area) to the maximum radiation intensity of a source which radiates uniformly in all directions, i.e., an isotropic radiator. When the gain is compared to that of this isotropic source, it is defined as the absolute gain of the radiator.

A radiator of given aperture area exhibits maximum gain when the energy distribution and phase are uniform across its aperture; the gain of such a "uniphase, uniamplitude" radiator is then

$$G=4\pi A/\lambda^2, \tag{13}$$

where A is the aperture area. This equation is queit accurate for apertures exceeding one or two wavelengths.

EXPERIMENTAL

The Disk Array as a Convergent Lens

One of the first microwave devices to be investigated acoustically was an array of ½-in. disks in the shape of a 6-in. diameter convergent lens (see Fig. 5). The tests were conducted in the free space room of the Bell Telephone Laboratories at Murray Hill, New Jersey. A high frequency radiator was set up in one corner of the room and the lens placed on a stand 10 ft away (see Fig. 6). A microphone was fitted to a small horn to act as a directional pick-up and the combination fastened to an adjustable support for exploring the sound field (see Fig. 7).

A focusing run taken at 13.4 kc showed the focal length at this frequency to be about 13 in., from which the index of refraction would appear to be 1.14. From the formula involving the elementary obstacle dimensions, but without the addition of a resonance correction, $n=1.10$. This is a fair check.

As in the optical case, the lens can be tilted about a diametral axis without much adverse effect on the transmission. Here a tilt of $\pm 40°$ produces only a 3 db change in response at the focal point.

Directional patterns were taken on the lens and horn together by fixing the latter to the lens and then rotating the combination about an axis through the vertical lens diameter. A pattern at 13.4 kc ($\lambda=2.57$ cm) shows the beam width at the 3 db points to be about $9\frac{1}{2}°$ with the minor lobes 10 db down (see Fig. 8). This is roughly equivalent at this wave-length to the theoretical beam width of a uniformly excited aperture 6 inches in diameter, and is another indication that the receiving horn (feed horn) is not sufficiently directive. The large minor lobe "masses" also are evidence of appreciable spill-over which is present due to too small an aperture feed horn.†

FIG. 12. A double convex strip lens 10 in. in diameter. The strip size and spacing is the same as that used in the prism of Fig. 9.

[9] H. Levine and J. Schwinger, Phys. Rev. 73, 383 (1948).

† In most tests of these lenses no serious attempt was made to use a horn at the focus having optimum size and directivity to achieve maximum gain from the particular lens under test. As will be seen below, in the case of the slant plate lens, attention to this detail ensures high gain (comparable to other types of radiators such as parabolic reflectors) and desirable directional characteristics.

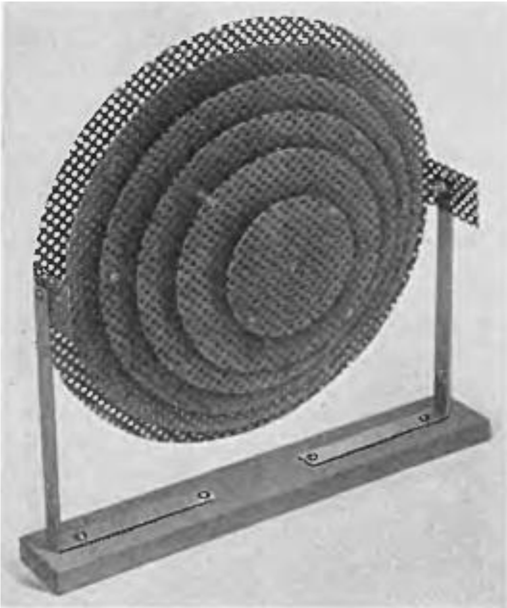


Fig. 13. A plano-convex perforated metal lens 10 in. in diameter. The perforated sheets simulate a crossed strip construction.

The Strip Array as a Prism

An array of $\frac{7}{16}$-in. wide strips made up in prism form has been found very effective in demonstrating the refraction of sound (see Fig. 9). A series of experiments were conducted with the prism placed over the mouth of a 30-in. long pyramidal horn having a 6-in. square aperture and a microphone coupled to its throat. The directional pattern (see Fig. 10) was measured acoustically at various frequencies and the angular position of maximum response plotted as a function of frequency (see Fig. 11). As in the action of dispersion in optics, it is seen that the angle of deviation (and therefore n) varies slowly at low frequencies but increases rapidly as the frequency of resonance is approached. At 10 kc resonance occurs, the strip width corresponding to approximately a half wave-length. Up to the region of resonance, the maximum transmission through the prism, measured at the optimum angle, is fairly constant. Near resonance, the index of refraction rises and reflection loss increases.

It should be noted that this prism produces a true dispersion of airborne acoustic waves and is not to be confused with diffraction devices such as gratings. Diffraction gratings and receivers depending upon path length differences of an integral number of half wave-lengths to differentiate frequencies have been used heretofore but such devices are wasteful of energy in that many spectral "orders" are produced. However, just as acoustic gratings can be used to analyze the

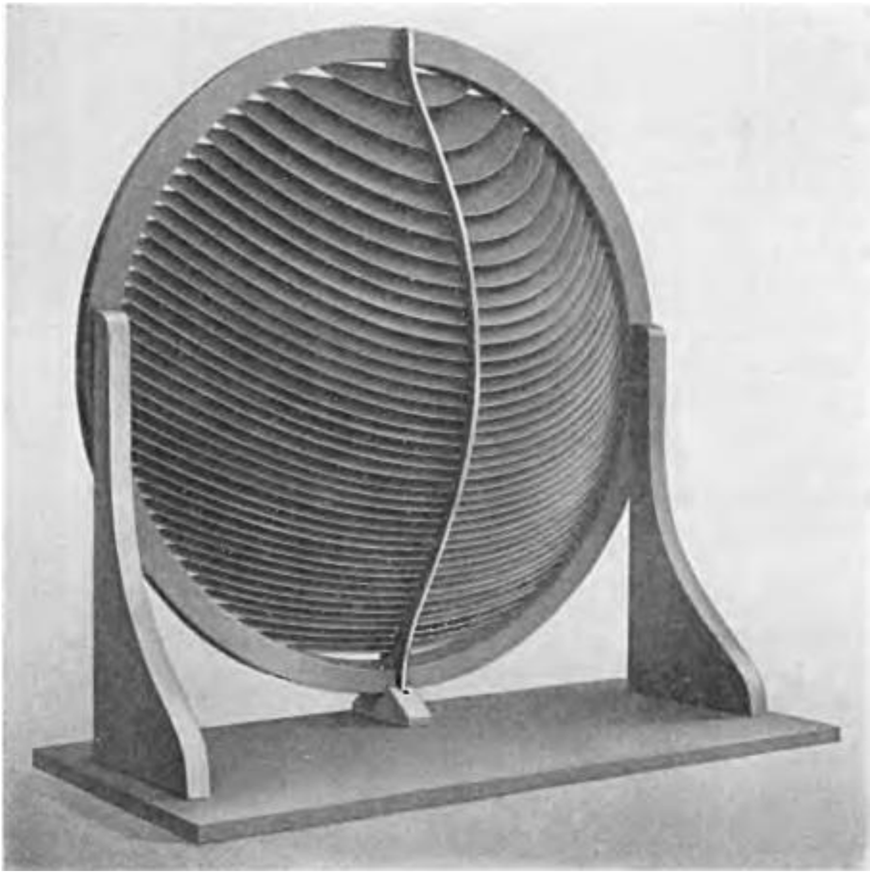


Fig. 14. A plano-convex slant plate lens 30 in. in diameter. The 48.3° tilt of the plates yields a refractive index of 1.5.

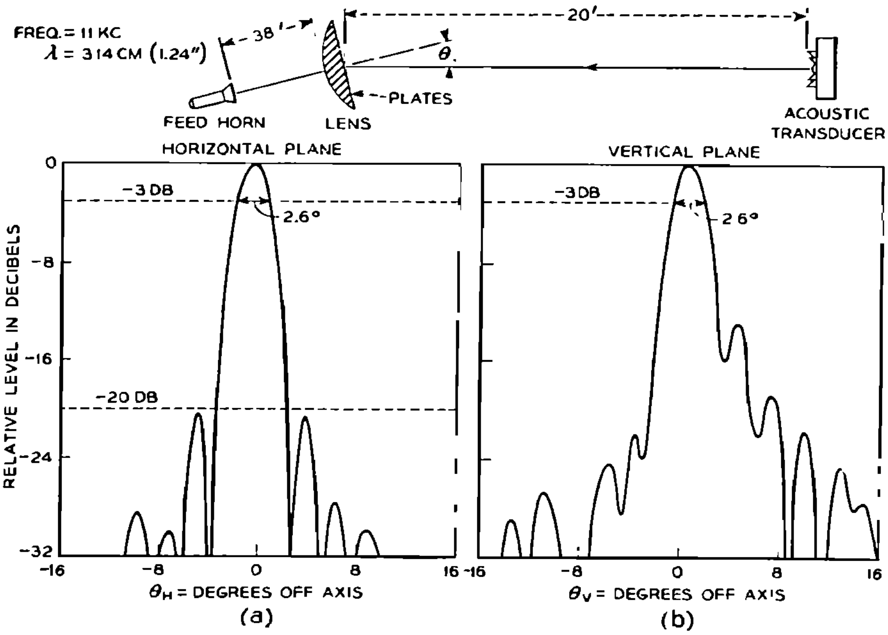


FIG. 15. Directional patterns of the slant plate lens of Fig. 14 using a 3-in. diameter feed horn: (a) Horizontal Plane. (b) Vertical Plane.

Fourier spectrum of a complex sound wave,[10] so a prism of the type described can be used as a spectrum analyzer. Very rapid analyses can be taken since all portions of the wave and all frequency components require approximately equal times to pass through the prism and arrive at the receiving points. This is in contrast to a grating where there is a time delay between the ray arriving from the grating element nearest the receiver and the ray arriving from the most distant element. Even the small prism of Fig. 9 could resolve four or five frequency components located between 4 kc and 10 kc as seen in Fig. 11. Much higher resolution could be obtained with larger prisms.

The index of refraction, as obtained from the measured angle of deviation and the angular width of the prism, was 1.23 at 4 kc but rose gradually to 1.64 at 10 kc. The value of n as obtained from the formula involving the obstacle dimensions (without frequency correction) is 1.24 which corresponds very well with the measured value at 4 kc where the index is still fairly constant.

This prism was originally constructed for 3 cm micro-

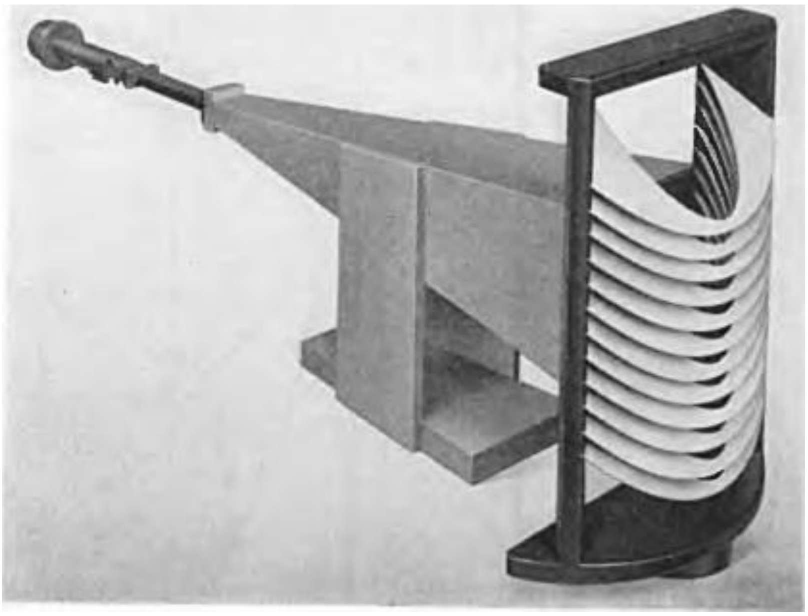


FIG. 16. A divergent slant plate cylindrical lens placed in front of a horn having a 6-in. square aperture.

[10] E. Meyer, *Electro-Acoustics* (G. Bell and Sons, London, 1939), p. 24.

wave experiments but failed to exhibit the expected properties at 3.2 or 3.3 cm. It was then tested acoustically and the reason for its behavior immediately became evident. The size and spacing of the strips were such as to effect resonance at 3.45 cm, and the 3.3-cm wave-lengths were reflected. It was re-examined at microwaves at 3.7 cm and found to operate as expected and to possess the same refractive index for acoustic and microwaves, as predicted from theory. This incident suggests that pertinent information about the electromagnetic behavior of periodic structures can be obtained from the simpler acoustic measurements.

The Strip Array as a Convergent Lens

A 10-in. diameter double convex lens similar to the strip prism just described was constructed for acoustical purposes (see Fig. 12). This lens operates on centimeter microwaves and is a model of the type projected for the New York–Chicago microwave relay circuit of the American Telephone and Telegraph Company. At 9 kc this lens had a focal length of approximately nine inches. As in the prism, transmission cuts off sharply in the neighborhood of 10 kc.

A Modified Strip Lens Using Perforated Metal

A perforated metal plate can be looked upon as a modified strip array with the strips running in two perpendicular directions and having round holes instead of square. Accordingly, perforated metal plates were spaced and stacked to form the 10-in. diameter plano-convex lens shown in Fig. 13. The holes were 0.125 in. in diameter and placed on 0.200-in. centers in a 0.025-in. brass sheet, the sheets spaced 0.375 in. apart. Satisfactory focusing action was observed at a focal length of 18 in. at 11 kc. This lens is, of course, effective only for acoustic waves.

The Slant Plate Array as a Convergent Lens

Tests were made on a 30-in. path length lens composed of an array of slanted plates (see Fig. 14). Its aluminum plates are spaced $\frac{1}{2}$ in. apart and slanted at an angle of 48.3° ($n=1.5$). It was designed to have a focal length of 30 in. for plane waves. However, in order that waves received from a distant point source be flat to within $\frac{1}{16}$ of a wave-length over the 30-in. aperture, the source would have to be 120 ft distant. This was not possible in our test room; therefore, with the source 20 ft distant, a longer focal length (38 in.) was employed to obtain proper focusing. The horizontal directional pattern of this lens at 11 kc is shown in Fig. 15a. The vertical pattern is shown in Fig. 15b. In this plane the slant plates cause an unsymmetrical distribution of energy across the lens face and cause some dissymmetry in the minor lobe structure. The measured beam width of 2.6° checks fairly well with the expected $65\lambda/d$ value of 2.69°. The lens can be rotated almost $\pm 15°$ about a diametral axis (with the feed fixed) before the gain is reduced by 2 db.

From 10 to 13 kc the measured gain of the lens was found to be approximately 2.5 db down from that calculated for uniform illumination ($G=4\pi A/\lambda^2$). This corresponds closely to results on most microwave lenses and paraboloids. This 56 percent "effective area" was maintained to within 2 db of this value over the band from 7.5 kc to 15 kc, falling off at the low end because

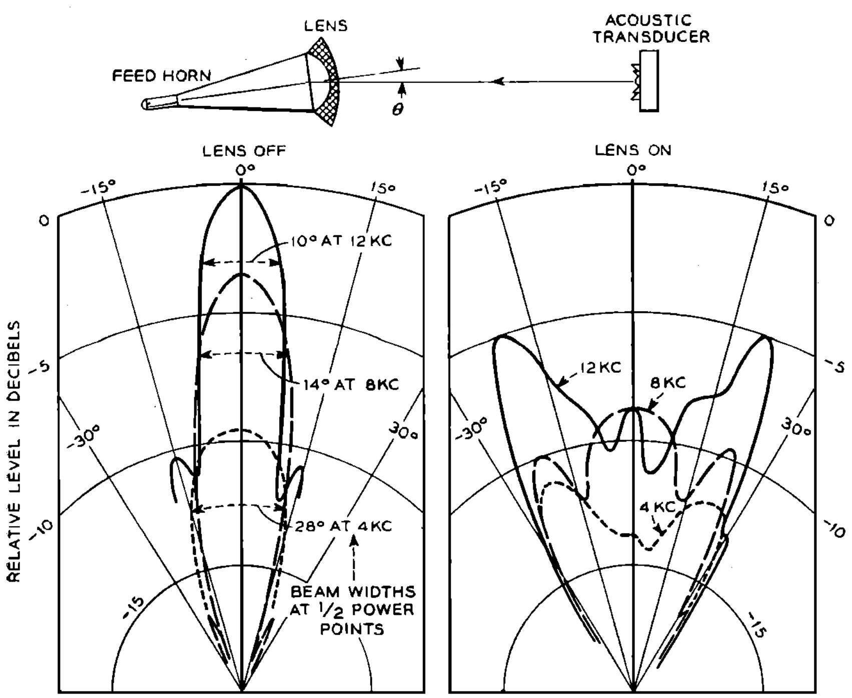


FIG. 17. Horizontal plane directional patterns of the horn of Fig. 16: left, the horn alone, and right, the combination of horn and diffusing lens.

the 3-in. receiving horn failed to intercept the optimum size focal spot and at the high end because the plate spacing reached, at 13.6 kc, the ½-wave-length value where the 2nd order mode can be propagated between the plates. It is interesting to note that at 13 kc the combination of the lens and feed horn is 4500 times more directive than an isotropic radiator.

The Slant Plate Array as a Cylindrical Divergent Lens

The lenses thus far described were mostly focusing devices designed to increase the directivity of acoustic transducers. Divergent or diffusing lenses, however, are also of interest in certain applications. For example, long exponential horns are often used as loud speakers to provide an acoustic impedance match between the narrow aperture of the driver unit and the mouth of the horn. For a satisfactory match to free air, the horn aperture must be of the order of a half-wave-length for the lowest frequency transmitted, and the horn length is also dictated by this lowest frequency since the rate of taper determines the low frequency cut-off. Horns required to handle frequencies from 1 or 2 kc upwards therefore have apertures 6 in. or so in width and may be 2 or 3 ft long. This gradual flare means that the emerging phase fronts are approximately plane and because, at the high frequencies, the 6-in. aperture is quite directive, the energy at these frequencies is sharply beamed along the horn axis. Multiple horns are thus required to distribute adequately spatially these higher frequencies. A diverging lens in front of a single horn could accomplish the same result with an appreciable space saving (see Fig. 16).

Here a slant plate array in the form of a cylindrical convexo-concave lens is employed in conjunction with a 6-in. square feed horn 30 in. long. The plates were spaced ½ in. apart and slanted at an angle of 48.3°. Characteristic directional patterns of the lens and feed horn were taken at representative frequencies throughout the operating range from 4 to 14 kc (see Fig. 17). With the lens on the horn, the beam width appeared fairly constant at around 50° to 60°. With the lens off, the beam width was considerably narrower and decreased as the frequency was raised. Also noticeable was a better equalization of response for equal energy signals. It is apparent that the lens tends to produce the desired circular wave front.

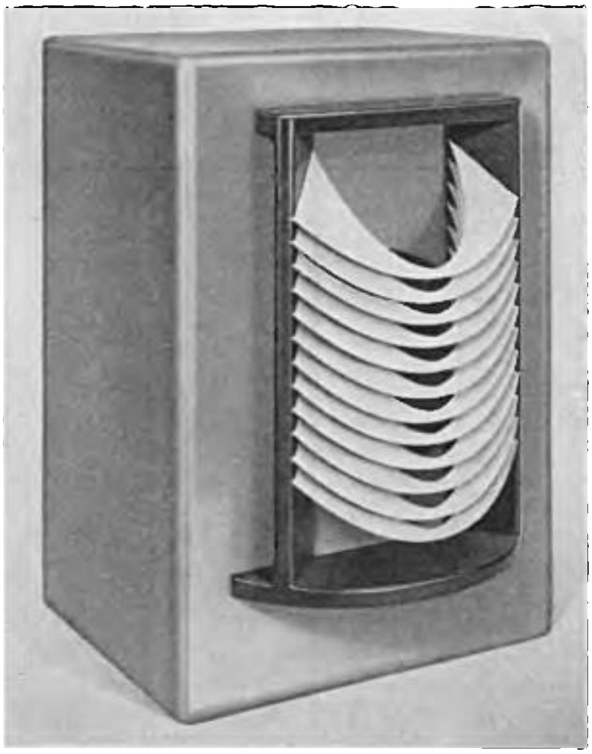


Fig. 18. The divergent lens of Fig. 16 employed in conjunction with a conventional cone type loud speaker.

Listening tests were made using a hiss tone containing frequencies from 5.6 to 11.2 kc sent out from the feed horn with and without the lens. All observers preferred the broader pattern obtained with the lens for high fidelity radio receiver use, since most loud speakers project the higher frequency sounds in a narrow beam directly in front of the receiver. Figure 18 shows a photograph of a cone loud speaker equipped with the cylindrical diffusing lens.

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