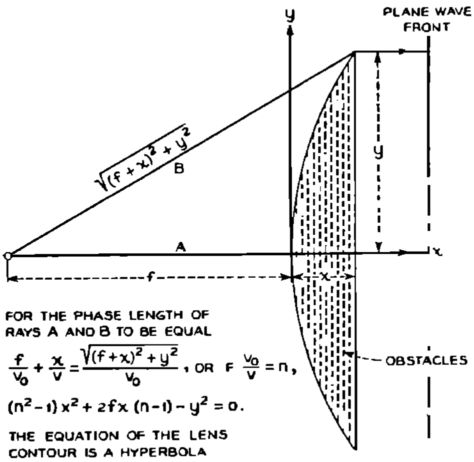

Fig. 3. Profile equation for a delay lens of the obstacle type.

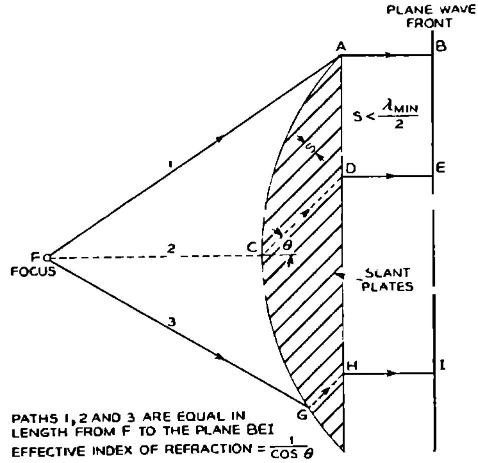

Fig. 4. Geometrical requirements necessary to establish the profile of the slant plate lens.

the density of the medium, the square of the ratio of the sound velocities is

$$(v_0/v)^2 = n^2 = 1+(2\pi/3)Na^3, \tag{3}$$

where n is the index of refraction. (See Fig. 2.)

For the case of disks, the mass increase per disk is $(2/\pi)\rho_0(4/3)\pi c^3$ where c is the disk radius.[5] This gives

$$n^2 = 1+\tfrac{8}{3}Nc^3 \tag{4}$$

for a disk array having N disks per unit volume.

For the case of strips, the following equation holds true:[6]

$$n^2 = 1+\pi b^2 N', \tag{5}$$

where b is the half-breadth of the strip and N' is the number per unit area looking end on at the strips.

Limitations on the Refractive Index Equations

The equations of the preceding section were derived by assuming that the sound field acting on an element was the impressed field alone. This is a satisfactory assumption when the separation between the objects is so large that the elements themselves do not distort the field acting on the neighboring elements. Such is not the case when n exceeds the value of 1.2 or thereabouts.

Furthermore, the above equations are valid only if the size and spacing of the obstacles is small compared to the wave-length. When the obstacle size nears a half-wave-length, resonance effects can occur* and the propagation velocity is strongly affected. Then the expression for n will be modified in a qualitative manner as follows:

$$n^2 \cong 1+[k/(f_0^2-f^2)], \tag{6}$$

where k is a constant of the obstacle configuration, f the applied frequency and f_0 the resonant frequency associated with the obstacle.** As f approaches f_0 closely, n begins to increase rapidly, resulting in the familiar phenomenon of dispersion. At $f=f_0$, n becomes quite large and complete reflection occurs. The array then is a better reflector than refractor. Further increase in frequency can result in a value of n less than one and

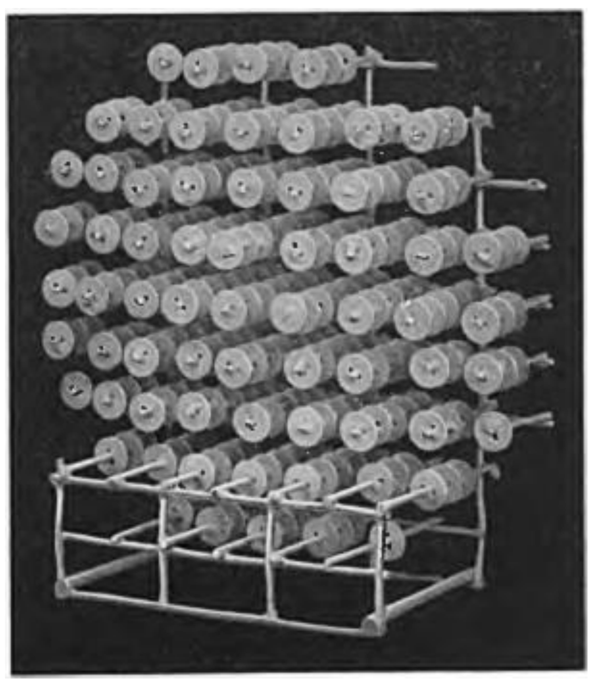

Fig. 5. A disk lens 6 in. in diameter. This lens is convergent and is composed of a double convex array of ½-in. diameter disks.

[6] See reference 4, p. 81, Eq. (11).

* A piston sound source in free air radiates very little energy at long wave-lengths but becomes more effective as the wave-length approaches twice the diameter; similarly a stationary disk reflects very little energy at long wave-lengths but becomes an effective reflector at the shorter wave-lengths.

** This is often called the Sellmeier equation. However, Lord Rayleigh has pointed out that Maxwell had considered it much earlier than Sellmeier. (See Lord Rayleigh, Scientific Papers IV, p. 414, the final equation.)

bring the system into the region of abnormal or anomalous dispersion.

At this point it might be well to mention that many of the experiments to be described later were purposely conducted at frequencies where the obstacle dimensions were not small compared to the wave-length but were approaching or even equaling a half wave-length. This was done in several cases in order to accentuate the focusing effects by increasing the index of refraction and also to make the array dimension contain as many wave-lengths as possible and thereby to become more directive.

PATH LENGTH REFRACTORS

General Description

Another method of slowing down a progressive wave is to guide it through a conduit which provides a longer

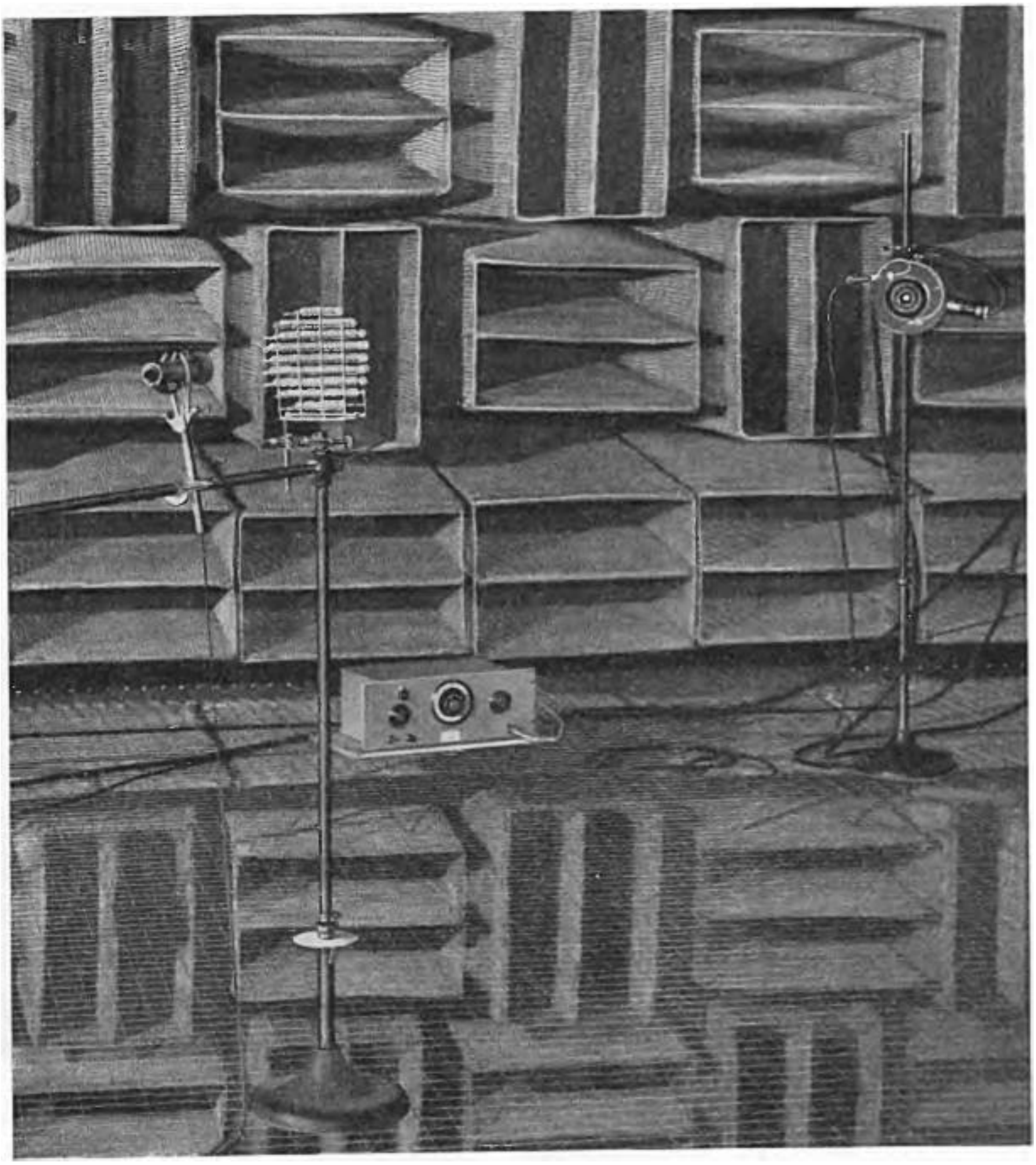

FIG. 6. The disk lens in position between transducers ready for test. Free space conditions are simulated by the use of sound absorbing wedges at all wall, floor, and ceiling surfaces. The apparatus is supported on a raised wire grid, acoustically transparent, floor.