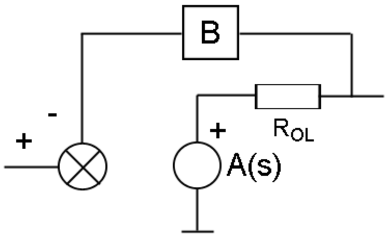
Output effective impedance of negative feedback audio amplifiers

Andy_c, DIY Audio, May/2007



The circuit at left represents global negative feedback amplifier, where:

$A(s) = \dfrac{A_0 \omega_0}{s + \omega_0}$ Open loop gain, assumed single pole with DC gain A_0, 3 dB bandwidth ω_0 and output equivalent resistance R_{OL}. Closed loop Thèvenin equivalent output impedance is then:

$$Z_{OUT}(s) = \frac{V_{OC}}{I_{CC}} = \frac{R_{OL}}{1 + BA(s)} = \frac{R_{OL}}{1 + \dfrac{BA_0\omega_0}{s + \omega_0}} = \frac{R_{OL}(s + \omega_0)}{s + (1 + A_0 B)\omega_0}$$

Letting $\omega_1 = (1 + A_0 B)\omega_0$, be the closed loop bandwidth then: $Z_{OUT}(s) = \dfrac{R_{OL}(s + \omega_0)}{s + \omega_1}$

Foster's expansion of RL impedance steps:

1) Divide $Z_{OUT}(s)$ by s
2) Expand in partial fractions
3) Multiply back by s.
4) Interpret each term in expansion as:
    a) Resistor or,
    b) Inductor or,
    c) Parallel RL circuit

$$\frac{Z_{OUT}(s)}{s} = \frac{R_{OL}(s + \omega_0)}{s(s + \omega_1)} = \frac{k_1}{s} + \frac{k_2}{s + \omega_1}$$

Where:

$$k_1 = \left.\frac{R_{OL}(s + \omega_0)}{s + \omega_1}\right|_{s=0} = \frac{R_{OL}\omega_0}{\omega_1}$$

$$k_2 = \left.\frac{R_{OL}(s + \omega_0)}{s}\right|_{s+\omega_1=0} = \frac{R_{OL}(\omega_1 - \omega_0)}{\omega_1}$$

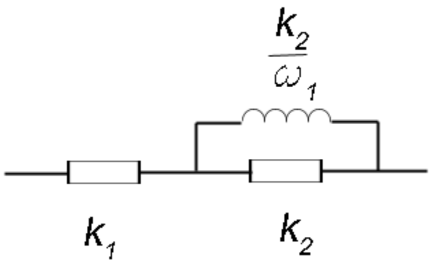
So:

$$Z_{OUT}(s) = k_1 + \frac{k_2 s}{s + \omega_1}$$

Where the first term corresponds to a resistor, while the second is a resistor in parallel with an inductor.

Let:

$$Z_1(s) = \frac{k_2 s}{s + \omega_1} \text{ so } Y_1(s) = \frac{s + \omega_1}{k_2 s} = \frac{1}{k_2} + \frac{1}{{}^{k_2}\!/_{\omega_1} s}$$



Z_{OUT} now looks as the circuit at left.

Recalling the closed loop bandwidth is given by:

$$\omega_1 = (1 + A_0 B)\omega_0$$

Then:

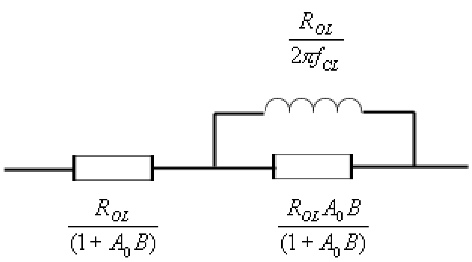
$$k_1 = \frac{R_{OL}\omega_0}{\omega_1} = \frac{R_{OL}\omega_0}{(1 + A_0 B)\omega_0} = \frac{R_{OL}}{(1 + A_0 B)}$$

$$k_2 = \frac{R_{OL}(\omega_1 - \omega_0)}{\omega_1} = \frac{R_{OL} A_0 B \omega_0}{(1 + A_0 B)\omega_0} = \frac{R_{OL} A_0 B}{(1 + A_0 B)}$$

$$\text{But : } L = \frac{k_2}{\omega_1} = \frac{R_{OL} A_0 B}{(1 + A_0 B)^2 \omega_0} \approx \frac{R_{OL}}{(1 + A_0 B)\omega_0} \quad \text{for} \quad \frac{A_0 B}{(1 + A_0 B)} \approx 1$$

So *L* is given in terms of open loop impedance and closed loop bandwidth in Hz as:

$$L =\approx \frac{R_{OL}}{(1 + A_0 B)\omega_0} = \frac{R_{OL}}{2\pi f_{CL}}$$



And the equivalent closed loop output impedance may be represented as a series resistor equal to the DC output resistance divided by the loop gain, and a parallel combination of a resistor nearly equal to the DC output resistance and an inductor whose value is also the DC output resistance but divided by the closed loop bandwidth.

Note from andy_c: I'd like to thank Rodolfo (ingrast) for putting this PDF document together and clarifying the ideas from my messy handwritings in the process.