

## Optimum Transformer Winding Design

wire height:  $h$  thickness of rectangular (foil) winding

$b$  width of rectangular (foil) winding

$d_{Cu}$  OD of Cu winding (NOTE: does NOT include insulation thickness) NOTE:  $OD_{wire}$  is the outer diameter of the wire, including insulation thickness.  
(which may not be negligible, e.g. thin TIW)

$h = b = d_{Cu} \cdot \sqrt{\frac{\pi}{4}}$  convert round wire into equivalent cross-sectional area square wire  $N_{tpl} = \frac{b_w}{OD_{wire}} - 1$  if the winding start and finish are adjacent, N full turns has N+1 wires in a row.

$F_l = \frac{N_{tpl} \cdot b}{b_w}$   $N_{tpl}$  no. of turns per layer  $b_w$  is the bobbin width. This is, however, wrong (but not by much)

$F_l = \frac{N_{tpl}}{b_a} \cdot OD_{wire} \cdot \sqrt{\frac{\pi}{4}}$  need to use  $OD_{wire}$  and  $b_a$  (length of wound core leg) because  $F_l$  is a correction term for the packing factor

$\phi = \frac{h \cdot \sqrt{F_l}}{\delta(T, f)}$  conductor skin depth  $\delta(f, T)$  F Hz, T degrees C  $\delta(T, f) = \sqrt{\frac{2 \cdot \rho \cdot [1 + \alpha \cdot (T - 20)]}{2 \cdot \pi \cdot f \cdot \mu_0}}$   $\delta_{Cu}(20 \cdot C, f) = \frac{67 \cdot \text{mm}}{\sqrt{f}}$   $\delta_{Cu}(80 \cdot C, f) = \frac{74.098 \cdot \text{mm}}{\sqrt{f}}$

re-writing the expression for  $\phi$ :  $\phi = \frac{d_{Cu} \cdot \sqrt{\frac{\pi}{4}} \cdot \sqrt{\frac{N_{tpl}}{b_a} \cdot OD_{wire} \cdot \sqrt{\frac{\pi}{4}}}}{\delta(T, f)}$  Using the simplification  $\sqrt{\frac{\pi}{4}} \cdot \sqrt{\sqrt{\frac{\pi}{4}}} = \left(\frac{\pi}{4}\right)^{\frac{3}{4}}$  gives:  $\phi = \frac{d_{Cu} \cdot \sqrt{OD_{wire}} \cdot \sqrt{\frac{N_{tpl}}{b_a}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}}}{\delta(T, f)}$

we can also write  $\sqrt{OD_{wire}} = \sqrt{\frac{d_{Cu}}{d_{Cu}} \cdot OD_{wire}} = \sqrt{d_{Cu}} \cdot \sqrt{\frac{OD_{wire}}{d_{Cu}}}$  which then gives:  $\phi = \frac{d_{Cu} \cdot \sqrt{d_{Cu}} \cdot \sqrt{\frac{OD_{wire}}{d_{Cu}}} \cdot \sqrt{\frac{N_{tpl}}{b_a}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}}}{\delta(T, f)}$  where  $d_{Cu} \cdot \sqrt{d_{Cu}} = d_{Cu}^{\frac{3}{2}}$

we can therefore write:  $\phi = \frac{d_{Cu}^{\frac{3}{2}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{\frac{N_{tpl}}{b_a}} \cdot \sqrt{\frac{OD_{wire}}{d_{Cu}}}}{\delta(T, f)}$  where  $\frac{OD_{wire}}{d_{Cu}} > 1$ . When using magnet wire,  $\frac{OD_{wire}}{d_{Cu}}$  is  $\sim 1$ , and  $\sqrt{\frac{OD_{wire}}{d_{Cu}}}$  is even closer to 1.

Using  $N_{tpl} = \frac{b_w - OD_{wire}}{OD_{wire}}$  gives  $\phi = \frac{d_{Cu}^{\frac{3}{2}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{\frac{1}{b_a} \cdot \frac{b_w - OD_{wire}}{OD_{wire}}} \cdot \sqrt{\frac{OD_{wire}}{d_{Cu}}}}{\delta(T, f)}$  therefore:  $\phi = \frac{d_{Cu}^{\frac{3}{2}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{\frac{b_w - OD_{wire}}{b_a}}}{\delta(T, f)}$

Or using  $OD_{wire} = d_{Cu} + 2 \cdot t_{insulation}$  to get  $\sqrt{\frac{OD_{wire}}{d_{Cu}}} = \sqrt{1 + \frac{2 \cdot t_{insulation}}{d_{Cu}}}$  gives:  $\phi = \frac{d_{Cu}^{\frac{3}{2}} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{\frac{N_{tpl}}{b_a}} \cdot \sqrt{1 + \frac{2 \cdot t_{insulation}}{d_{Cu}}}}{\delta(T, f)}$

for  $2 \cdot t_{insulation} \ll d_{Cu}$ ,  $\sqrt{1 + \frac{2 \cdot t_{insulation}}{d_{Cu}}} \sim 1$  and  $OD_{wire} \sim d_{Cu}$  so  $N_{tpl} = \frac{b_w - d_{Cu}}{d_{Cu}}$  defining  $b_a = b_w \cdot k$  gives:  $\phi_{approx} = \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \frac{d_{Cu}^{\frac{3}{2}} \cdot \sqrt{\frac{b_w - d_{Cu}}{d_{Cu}} \cdot \frac{1}{b_w \cdot k}}}{\delta(T, f)}$

which reduces to:  $\phi_{approx} = \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \frac{d_{Cu}^{\frac{3}{4}} \cdot \sqrt{1 - \frac{d_{Cu}}{b_w}} \cdot \sqrt{\frac{1}{k}}}{\delta(T, f)}$   $k$  is fairly close to one. for ETD cores,  $\sqrt{\frac{1}{k}} = 0.95 \pm 1\%$  as they have a long center leg, whereas for PQ cores, which have a much shorter center leg,  $\sqrt{\frac{1}{k}} = 0.925 \pm 5\%$

if we assume 5 to 20 turns per layer  $\frac{d_{Cu}}{b_w} = 0.2 \dots 0.05$  then choose  $\sqrt{\frac{1}{k}} = 0.93$  we can then write  $\phi_{approx} = \left[\left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{1 - \frac{d_{Cu}}{b_w}} \cdot \sqrt{\frac{1}{k}}\right] \cdot \frac{d_{Cu}^{\frac{3}{4}}}{\delta(T, f)}$

$\frac{d_{Cu}}{b_w} = 0.2$  gives  $0.93 \cdot \sqrt{1 - 0.2} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} = 0.694$  and  $\frac{d_{Cu}}{b_w} = 0.05$  gives  $0.93 \cdot \sqrt{1 - 0.05} \cdot \left(\frac{\pi}{4}\right)^{\frac{3}{4}} = 0.756$  note that  $0.756^{\frac{4}{3}} = 0.689$  and  $0.694^{\frac{4}{3}} = 0.614$

these are both pretty close to 2/3. Noting that  $\left(\frac{2}{3}\right)^{\frac{3}{4}} = 0.738$  choose  $\left[\left(\frac{\pi}{4}\right)^{\frac{3}{4}} \cdot \sqrt{1 - \frac{d_{Cu}}{b_w}} \cdot \sqrt{\frac{1}{k}}\right] = 0.738 = \left(\frac{2}{3}\right)^{\frac{3}{4}}$  we **finally** arrive at  $\phi_{approx} = \frac{\left(\frac{2}{3} \cdot d_{Cu}\right)^{\frac{3}{4}}}{\delta(T, f)}$

but  $\delta_{Cu}(80 \cdot C, f) = \frac{74.098 \cdot \text{mm}}{\sqrt{f}}$  giving  $\phi_{approx} = \frac{\sqrt{f}}{74.098 \cdot \text{mm}} \cdot \left(\frac{2}{3} \cdot d_{Cu}\right)^{\frac{3}{4}}$  which we can write as  $\phi_{approx} = \frac{\sqrt{f}}{74.098 \cdot \text{mm}} \cdot \left(\frac{2}{3}\right)^{\frac{3}{4}} \cdot d_{Cu}^{\frac{3}{4}}$  where  $\left(\frac{2}{3}\right)^{\frac{3}{4}} = 0.7378$

and subsuming this into the Copper skin-depth constant gives  $\phi_{approx} = \frac{\sqrt{f}}{74.098 \cdot \text{mm}} \cdot 0.7378 \cdot d_{Cu}^{0.75}$  where the magic number 0.7378 has the units  $\frac{\text{mm}}{\sqrt{\text{Hz}}}$

the constant terms evaluate as  $\frac{0.7378}{74.098 \cdot \text{mm}} = 9.957$  therefore the final result is:  $\phi_{approx} = 10 \cdot \sqrt{f} \cdot d_{Cu}^{0.75}$  (which is a dimensionless number)

flipping it around to solve for approximate wire diameter given  $\phi$  we get  $d_{Cu\_approx} = \left(\frac{10 \cdot \phi}{\sqrt{f}}\right)^{1.333}$

The equation for  $F_R(\phi, p)$  can be shown to be:

$$F_{Rh}(\phi, p) := \phi \cdot \left[ \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} + 2 \cdot \frac{(p^2 - 1)}{3} \cdot \frac{\sinh(\phi) - \sin(\phi)}{\cosh(\phi) + \cos(\phi)} \right] \quad \text{From the paper by Hurley et al}$$

$$F_{Rm}(\phi, p) \equiv \phi \cdot \left[ \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} + 2 \cdot \frac{(p^2 - 1)}{3} \cdot \left( \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} - 2 \cdot \frac{\sinh(\phi) \cdot \cos(\phi) + \cosh(\phi) \cdot \sin(\phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} \right) \right] \quad \text{Maksimovic p.518}$$

Note: The expression given in Petkov's paper is WRONG.

Plotted both curves - identical. Just use:

$\phi := 0.1, 0.11 \dots 10$

Hurley:

$$F_{Rh}(\phi, p) = \phi \cdot \left[ \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} + 2 \cdot \frac{(p^2 - 1)}{3} \cdot \frac{\sinh(\phi) - \sin(\phi)}{\cosh(\phi) + \cos(\phi)} \right]$$

Maksimovic

$$F_{Rm}(\phi, p) = \phi \cdot \left[ \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} + 2 \cdot \frac{(p^2 - 1)}{3} \cdot \left( \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} - 2 \cdot \frac{\sinh(\phi) \cdot \cos(\phi) + \cosh(\phi) \cdot \sin(\phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} \right) \right]$$

Are these related by hyperbolic trigonometric identities? YEP

$$\left( \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} - 2 \cdot \frac{\sinh(\phi) \cdot \cos(\phi) + \cosh(\phi) \cdot \sin(\phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} \right) - \left( \frac{\sinh(\phi) - \sin(\phi)}{\cosh(\phi) + \cos(\phi)} \right) \text{ simplify } \rightarrow 0$$

yeah, I'm cheating. I did look at the trig identities, but this is how I check I haven't made any mistakes, and its a lot quicker :)

Might as well use the simple one!

$$F_R(\phi, p) := \phi \cdot \left[ \frac{\sinh(2 \cdot \phi) + \sin(2 \cdot \phi)}{\cosh(2 \cdot \phi) - \cos(2 \cdot \phi)} + 2 \cdot \frac{(p^2 - 1)}{3} \cdot \frac{\sinh(\phi) - \sin(\phi)}{\cosh(\phi) + \cos(\phi)} \right]$$

$$F_{Ra}(\phi, p) := 1 + \frac{5 \cdot p^2 - 1}{45} \cdot \phi^4 \quad \text{Is a commonly used approximation! Of course its a lousy one.....}$$

$$F_{Rb}(\phi, p) := \phi \cdot \frac{2 \cdot p^2 + 1}{3} \quad \text{works pretty well for } \phi > 2, \text{ but thats about where proximity effect tapers off - i.e. the losses have already been maximised!}$$

$$F_{rc}(\phi) := \max(1, \phi) \quad \text{is a reasonably good approximation for 1 layer}$$