

Smith then analyzes the case wherein the horn throat is an annular ring at the air chamber as shown in Figure 8. Again the mode shapes inside the air chamber are found to be complex functions of Bessel functions. It is further found that the first null can be suppressed by properly choosing the radius of the horn throat annulus. To suppress the  $j$ th node it is necessary to choose  $r_1$  such that  $k_j r_1$  is a root of  $J_0$ . The parameter  $k_j$  has already been chosen such that  $k_j a$  is a root of  $J_1$ .

The radius of each of  $m$  annuli is chosen to suppress each of the first  $m$  modes that exist in the air chamber by finding the first  $m$  roots to the  $J_0$  function. Table 1 lists the first five roots of the  $J_0$  and  $J_1$  functions.

Table 1  
Roots of the Functions  $J_0(m)$  &  $J_1(m)$

$m$	$J_0(m)$	$J_1(m)$	$k_j$
1	2.41	3.83	$3.83/a$
2	5.52	7.02	$7.02/a$
3	8.65	10.17	$10.17/a$
4	11.79	13.32	$13.3/a$
5	14.93	16.47	$16.47/a$

The procedure, then, is to choose a number  $m$  corresponding to the number of concentric rings desired. If this number is five then the  $m$ th root of  $J_1$  is 16.470. The radius of each of the rings is obtained by dividing each of the first five roots of  $J_0$  by this fifth root of  $J_1$  and multiplying by the radius of the air chamber. Note: If the air chamber is not flat, i.e. spherical, then the radius of the chamber is the distance from the center to the outside edge measured along the curve defining the chamber, i.e. arc length.

## Bessel Function Roots (Zeros)

m	$J_0(m)$	$J_1(m)$
1	2.404825577	3.8317059702
2	5.5200781103	7.0155866698
3	8.6537279129	10.1734681351
4	11.7915344391	13.3236919363
5	14.9309177086	16.4706300509

Note: Use of these values, instead of those provided in the paper, will improve the accuracy of the calculations, particularly those related to determination of slit area.  
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For the five ring case under consideration

$$a_1 = \frac{2.405a}{16.47} = 0.146a$$

$$a_2 = \frac{5.520a}{16.47} = 0.335a$$

$$a_3 = 0.525a$$

$$a_4 = 0.716a$$

$$a_5 = 0.907a$$

Up to this point the ratio of horn throat area to the diaphragm area has been arbitrary. Indeed, if additional concentric annuli are added their total area is still the choice of the designer. However, the area allotted to each additional annulus now becomes defined by the solution to the problem. To determine the width of each of these annular slots we must solve  $m^{-1}$  simultaneous equations and one simple equation to find the areas of each of the slots. Smith's equation 25 defines the simultaneous equations and for the case of the five rings they are written as:

$$\begin{array}{ccccccc}
 S_1 J_0(k_1 a_1) & . & . & . & S_5 J_0(k_1 a_5) = 0 & & \\
 . & . & . & . & . & & \\
 . & . & . & . & . & & \\
 S_1 J_0(k_4 a_1) & . & . & . & S_5 J_0(k_4 a_5) = 0 & & 
 \end{array} \quad (3)$$

The fifth required equation is obtained when the designer chooses the total area of the rings  $S_\delta$

$$S_1 + S_2 + S_3 + S_4 + S_5 = S_\delta \quad (4)$$

As before the values of  $k_j$  are defined by the roots of  $J_1$  and are listed in the last column of Table 1. The five unknowns in the four equation matrix may be reduced to four by dividing each of the  $S_j$  values by  $S_1$  to produce the four unknowns  $\delta_j = S_j/S_1$

The matrix then becomes:

$$\begin{array}{l}
 0.629 \delta_2 + 0.217 \delta_3 - 0.161 \delta_4 - 0.377 \delta_5 + 0.923 = 0 \\
 0.029 \delta_2 - 0.398 \delta_3 - 0.170 \delta_4 + 0.236 \delta_5 + 0.754 = 0 \\
 - 0.366 \delta_2 - 0.062 \delta_3 + 0.269 \delta_4 - 0.143 \delta_5 + 0.520 = 0 \\
 - 0.328 \delta_2 + 0.300 \delta_3 - 0.200 \delta_4 + 0.066 \delta_5 + 0.256 = 0
 \end{array} \quad (3a)$$

Solving for  $\delta_j$  gives

$$\delta_2 = 2.142$$

$$\delta_3 = 3.463$$

$$\delta_4 = 4.917$$

$$\delta_5 = 5.929$$

$$\begin{aligned} \text{If } S_\delta &= S_D \\ &= \frac{\pi a^2}{10} \end{aligned} \tag{5}$$

then

$$S_1 (1 + 2.142 + 3.463 + 4.917 + 5.929) = \frac{\pi a^2}{10}$$

$$S_1 = 0.018a^2$$

$$S_2 = 0.0386a^2$$

$$S_3 = 0.0623a^2$$

$$S_4 = 0.0885a^2$$

$$S_5 = 0.1067a^2$$

If  $\Delta a_j$  is taken as  $1/2$  the width of the  $j$ th slot (Figure 8) then it can be shown that:

$$w_j = 2\Delta a_j = s_j/2\pi a_j \quad (6)$$

giving  $w_1 = 0.0196a$

$$w_2 = 0.0183a$$

$$w_3 = 0.0189a$$

$$w_4 = 0.0197a$$

$$w_5 = 0.0187a$$

This completes the review of Smith's article. The transitions between these annular rings and the throat of the horn are left to the designer. The main requirement is for a smooth transition. An exponential area function in this transition is exotic, but will frequently be found to be too short to justify its economic impact.