

where $\beta = V_{AS}/V_B$ the ratio between the driver's compliance equivalent volume and the rear cavity box volume.

Mid Frequencies

The efficiency expression eq. (6) remains the same as noted before.

High Frequencies

The three HF breakpoint frequencies eqs. (7)-(9) can be shown in the form:

1. Driver moving mass corner,

$$f_{HM} = 2f_{HS} = 2f_S/Q_{TS}; \quad (23)$$

2. Driver voice coil inductance corner remains as before eq. (8); and

3. Front cavity compliance corner,

$$\begin{aligned} f_{HC} &= 2f_{LS}\beta = 2Q_{TS} f_S \beta \\ &= 2Q_{TS} f_S \frac{V_{AS}}{V_{FC}} \end{aligned} \quad (24)$$

where $\beta = V_{AS}/V_{FC}$ the ratio between the driver's compliance equivalent volume and the front cavity volume.

Reactance Annulling

The correct rear cavity volume for reactance annulling eq. (12) can be changed to:

$$V_B = \frac{V_{AS}}{\left(\frac{f_C}{f_{LS}} - 1\right)} = \frac{V_{AS}}{\left(\frac{f_C}{f_S Q_{TS}} - 1\right)}, \quad (25)$$

which is a relatively direct compact form. It must be noted that normally $(f_C/f_{LS}) < 1$ or $f_{LS} < f_C$ which makes V_B finite and positive. If $f_{LS} \approx f_C$ or $f_{LS} > f_C$, the driver is not well suited for operation in a horn at that specific cutoff frequency.

Low-Frequency Maximum Acoustic Output:

The expression for the displacement limited low-frequency output power eq. (16) can be combined with eq. (18) yielding:

$$P_{AR} = \left(\frac{3}{2} \pi \beta c^2\right) \left(\frac{1}{f_S Q_{TS} V_{AS}}\right) f_C^2 V_D^2. \quad (26)$$

For computation in SI metric units $3\pi \beta c^2/2 \approx 6.7 \times 10^5$.

COMPARISON

A comparative listing of some of the horn design equations considered and developed in this paper are shown in Table I.

TABLE I.

A comparison of horn design equations between those which use the fundamental electro-mechanical driver parameters and the Thiele/Small driver parameters.

Symbol	Description	Electromechanical	Thiele/Small
S_T	Horn throat area	$\frac{\rho_0 c R_E S_D^2}{B^2 l^2}$	$\frac{2\pi f_S Q_{TS} V_{AS}}{c}$
V_B	Back cavity volume	$\frac{\rho_0 c^2 R_E S_D^2 C_{MS}}{2\pi f_C B^2 l^2 C_{MS} - R_E}$	$V_{AS} \left/ \left(\frac{f_C}{f_S Q_{TS}} - 1 \right) \right.$
	If $V_{AS} \gg V_B$	$\frac{S_T c}{2\pi f_C}$	$V_{AS} f_S Q_{TS} / f_C$
HF rolloff corner frequencies			
f_{HM}	Due to moving mass	$\frac{B^2 l^2}{\pi R_E M_{MD}}$	$\frac{2 f_S}{Q_{TS}}$
f_{HVC}	Due to voice coil inductance	$\frac{R_E}{\pi L_E}$	Same
f_{HC}	Due to front cavity	$\frac{2 \rho_0 c^2 R_E S_D^2}{B^2 l^2 V_{FC}}$	$2 Q_{TS} f_S \left(\frac{V_{AS}}{V_{FC}} \right)$
P_{AR}	Displacement limited max. acoustic output	$\frac{3\pi^2 B^2 l^2 X_p^2 f_C^2}{R_E}$	$\frac{3\pi \rho_0 c^2}{2 f_S Q_{TS} V_{AS}} f_C^2 V_D^2$

DESIGN EXAMPLE

A low-frequency exponential horn system with cutoff $f_c = 50$ Hz is to be designed for a typical high-efficiency musical instrument driver. Details of horn flaring and selection of proper mouth size will not be considered here but are covered in [3], [4], [8], [9].

Driver Parameters:

The parameters of the 12 inch driver to be used in the horn are listed as follows (all free-air, unenclosed):

Electromechanical Parameters:

$$M_{MS} = 31.4 \text{ g (includes air mass load)}$$

$$C_{MS} = 4.0 \times 10^{-4} \text{ m/N}$$

$$B = 15.2 \text{ Tm}$$

$$R_E = 5.6 \text{ } \Omega$$

Mechanical $Q = 9.5$

$$x_{\max} = 3.3 \text{ mm}$$

$$S_D = 5.0 \times 10^{-2} \text{ m}^2$$

$$L_E = 3.2 \text{ mH}$$

Thiele/Small Parameters:

$$f_s = 45 \text{ Hz}$$

$$Q_{ES} = 0.215$$

$$Q_{MS} = 9.5$$

$$Q_{TS} = 0.210$$

$$V_{AS} = 140 \text{ l} = 0.14 \text{ m}^3$$

$$\eta_0 = 5.8\% \text{ (half-space)}$$

$$V_D = 0.166 \text{ l} = 1.66 \times 10^{-4} \text{ m}^3$$

$$P_E \text{ (max)} = 100 \text{ Watts}$$

Design:

Application of eq. (18) yields for throat area

$$S_T = \frac{2\pi(45)(0.21)(0.14)}{343} = 2.4 \times 10^{-2} \text{ m}^2 \\ = 242 \text{ cm}^2, \text{ and eq. (25)}$$

for back cavity volume

$$V_B = \frac{140}{\frac{50}{45(0.21)} - 1} = 32.6 \text{ l} \\ = 3.26 \times 10^{-2} \text{ m}^3.$$

Analysis:

Small Signal:

The upper and lower bounds of the driver's resistance controlled region are given by eqs. (19 and (20):

$$f_{HS} = f_s / Q_{TS} = 45 / 0.21 \approx 214 \text{ Hz and} \\ f_{LS} = Q_{TS} f_s = 0.21 (45) \approx 9.5 \text{ Hz.}$$

High Frequencies:

The three HF rolloff breakpoints from eqs. (23), (8), and (24) are:

1. Driver moving mass corner,

$$f_{HM} = 2 f_{HS} \approx 430 \text{ Hz;}$$

2. Driver voice coil inductance corner,

$$f_{HVC} = R_E / (\pi L_E) = 5.6 / (\pi \cdot 0.0032) \approx 560 \text{ Hz}; \text{ and}$$

3. Front cavity compliance corner ($V_F = 1.1 \text{ l}$),

$$f_{HC} = 2 f_{LS} \frac{V_{AS}}{V_F} = \frac{2 (9.5) (140)}{1.1} \approx 2400 \text{ Hz}.$$

These breakpoints indicate a 6 dB/octave rolloff starting at 430 Hz, 12 dB/octave at 560 Hz, and a 18 dB/octave rolloff above 2,400 Hz.

Reactance Annulling:

To check for proper reactance annulling the relationship of eq. (10) can be checked:

$$f_{LS} (1+\phi) = f_{LS} \left(1 + \frac{V_{AS}}{V_B} \right) = 9.5 \left(1 + \frac{140}{32.6} \right) \\ \approx 50 \text{ Hz},$$

which is equal to the cutoff frequency as desired.

Large Signal:

The displacement limited LF acoustic output power from eq. (26) is:

$$P_{AR} = \frac{6.7 \times 10^5 (50)^2 (1.66 \times 10^{-4})^2}{45 (0.21) (0.14)} \\ \approx 35 \text{ Watts}.$$

This indicates that the system is capable of generating some 35 acoustic watts or more down to $1.26 f_c \approx 63 \text{ Hz}$ without exceeding the driver's rated maximum displacement of $\pm 3.3 \text{ mm}$ ($\pm 1/8\text{th inch}$). The other limiting mechanism of low-frequency output is the driver's maximum thermal power rating P_{AR} , which is not considered in this analysis.

CONCLUSION

For those who prefer design methods using the Thiele/Small driver parameters, this paper has developed a set of equations for low-frequency horn design which use these parameters. If the Thiele/Small parameters are known for a particular driver, the horn system may be designed and analyzed using these rewritten equations. In some cases, simplifications in design and analysis result from these transformed equations.

It must be pointed out that the transformed design formulas used in this paper are based on traditional low-frequency horn design methods. These traditional methods in some situations may not yield a design which has the optimum combination of response, efficiency and maximum acoustic output. This is primarily due to the fact that traditional horn design dictates a specific value of throat area which maximizes the nominal efficiency. Because a number of the horn's performance characteristics depend heavily on throat area, constraint of this parameter to a specific value removes one valuable degree of design freedom.*

* These last comments resulted from private correspondence with Dr. Richard H. Small of the University of Sydney, Australia.

APPENDIX

CONVERSION BETWEEN ELECTROMECHANICAL DRIVER PARAMETERS AND THIELE/SMALL DRIVER PARAMETERS

The Thiele/Small driver parameters are related to the electromechanical driver parameters by the following relationships [2]:

$$f_s = \frac{1}{2\pi} \sqrt{\frac{1}{M_{MS} C_{MS}}} \quad , \quad (27)$$

$$Q_{ES} = \frac{R_E}{B^2 l^2} \sqrt{\frac{M_{MS}}{C_{MS}}} \quad , \quad (28)$$

$$Q_{MS} = \frac{1}{2\pi f_s C_{MS} R_{MS}} \quad , \quad (29)$$

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}}$$

If $Q_{MS} \gg Q_{ES}$ then $Q_{TS} \approx Q_{ES}$,

$$V_{AS} = \rho_0 c S_D^2 C_{MS}, \text{ and} \quad (30)$$

$$V_D = S_D x_{max}. \quad (31)$$