

LOW-FREQUENCY HORN DESIGN USING THIELE/SMALL DRIVER PARAMETERS

D. B. Keele, Jr.
Klipsch & Associates, Inc.
Hope, Arkansas 71801

The design formulas for low-frequency horns which yield various physical and performance related horn data can be recast in a form which utilizes the Thiele/Small direct-radiator driver parameters. This conversion simplifies computations of items such as required back cavity volume and throat area for desired performance. Performance data such as operating bandwidth, upper rolloff frequencies and low-frequency maximum acoustic output power are easily calculated.

INTRODUCTION

For purposes of direct-radiator loudspeaker system analysis and design, it has been found advantageous to describe the driver in terms of four basic parameters used by Thiele [1] and Small [2] which are related to the fundamental electromechanical driver parameters but are easier to measure and work with. These advantages can be extended to the design and analysis of low-frequency exponential horn systems if the appropriate equations are re-written in a form which utilizes the Thiele/Small driver parameters.

GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
c	velocity of sound in air (≈ 343 m/s)
C_{AB}	acoustic compliance of air in enclosure
C_{MES}	electrical capacitance due to driver mass including rear air load ($=M_{MS}/(B^2 L^2)$)
C_{MET}	electrical capacitance which varies with frequency due to horn throat air load mass ($=\rho c S_D^2 / (2\pi B^2 L^2 S_T f_c)$, for infinite exponential horn, valid for $f \geq f_c$ only)
C_{MS}	mechanical compliance of driver suspension
E_{in}	voltage applied to driver terminals
f	frequency
f_c	horn cutoff frequency
f_c	upper rolloff corner (-3 dB) frequency due to the effects of front cavity compliance acting alone
f_{HM}	upper rolloff corner (-3 dB) frequency due to the effects of driver moving mass acting alone
f_{HS}	upper frequency bound of the driver's resistance controlled region when operated in free air

f_{HVC}	upper rolloff corner (-3 dB) frequency due to the effects of driver voice coil inductance acting alone
f_{LBC}	lower rolloff corner (-3 dB) frequency due to driver suspension and back cavity compliance when driving infinite tube
f_{LC}	lower rolloff corner (-3 dB) frequency due to driver suspension compliance alone when driving infinite tube
f_{LS}	lower frequency bound of the driver's resistance controlled region when operated in free-air
f_s	resonance frequency of driver in free-air
l	length of voice-coil conductor in magnetic field
L_{CEB}	electrical inductance due to compliance of air in back cavity $(=B^2 l^2 V_B / (\rho c^2 S_D^2))$
L_{CEC}	electrical inductance due to compliance of air in front cavity $(=B^2 l^2 V_{FC} / (\rho c^2 S_D^2))$
L_{CES}	electrical inductance due to driver suspension compliance $(=B^2 l^2 C_{MS})$
L_E	inductance of driver voice-coil
M_{MS}	mechanical mass of driver diaphragm assembly including back air load
P_A	acoustic output power
P_{AR}	displacement-limited acoustic power rating
P_E	nominal electrical input power $(=e_{in}^2 / (2R_E))$
Q	ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
Q_{ES}	Q of driver at f_s considering electrical resistance R_E only
Q_{MS}	Q of driver at f_s considering mechanical losses only
Q_{TS}	total Q of driver at f_s including all system resistances $(=Q_{MS} Q_{ES} / (Q_{MS} + Q_{ES}))$
R_E	dc resistance of driver voice coil
R_{ET}	electrical resistance which varies with frequency due to power radiated into horn (proportional to horn throat conductance)
S_D	effective projected surface area of driver diaphragm
S_T	throat area of horn
V_B	net internal volume of rear cavity $(\approx \rho c^2 C_{AB})$
V_D	peak displacement volume ^{of} driver diaphragm $(=S_D x_{MAX})$

V_{AS}	volume of air having same acoustic compliance as driver suspension ($=\rho_0 c^2 C_{MS} S_D^2$)
V_{FC}	net internal volume of front cavity
x_p	peak displacement of driver diaphragm
x_{MAX}	maximum peak linear displacement of driver diaphragm
α	compliance ratio between driver suspension compliance and compliance of air in rear cavity (also= V_{AS}/V_B)
β	compliance ratio between driver suspension compliance and compliance of air in front cavity (also= V_{AS}/V_{FC})
η	efficiency
η_0	reference efficiency (=acoustic output power/nominal electrical input power)
ρ_0	density of ($=1.21$ kg/m ³) air

REVIEW

Driver Parameters

The fundamental electromechanical driver parameters which control system low-frequency performance are [2, p. 387] R_E , (Bl) , S_D , C_{MS} , M_{MS} , R_{MS} , and x_{MAX} which are defined in the glossary of symbols. These parameters are directly related to the drivers' physical characteristics such as diaphragm suspension compliance, total moving mass, the strength of the magnetic field, etc.

Another set of driver descriptors which are related to those above have been gaining increased usage because they are easier to measure and simplify the system design process. These are the parameters f_s , V_{AS} , Q_{TS} ($=Q_{ES} Q_{MS}/(Q_{ES}+Q_{MS})$) and V_D used by Thiele [1] and Small [2] and defined in the symbol glossary. These parameters are more closely associated with directly measurable quantities such as resonance frequency and Q . The conversion between these two sets of parameters is outlined in Appendix.

Low Frequency Horn Design:

Traditional low frequency exponential horn design and analysis using cone type drivers deals with such items as [3], [4], [5], [6], [7], [8], [9], [10]:

1. Selection of horn cutoff frequency and flare rate for desired performance.
2. Selection of throat area to maximize efficiency.
3. Selection of mouth area for best response.
4. Selection of back cavity volume for reactance annulling at horn cutoff.
5. Computation of low-frequency maximum acoustic output power.
6. Computation of high frequency rolloff corner frequencies due to driver moving mass, driver voice coil inductance and front cavity compliance.

This paper will deal only with Items 2, 4, 5, and 6 with emphasis on designs where a horn must be designed for a given driver. For Item 5, only displacement limited maximum output will be analyzed.

Horn Equivalent Circuit

The simplified electrical equivalent circuit of the horn-driver system of Fig. 1 is shown in Fig. 2 [3, p. 262]. Symbols correspond to that used by Small [2]. Driver and box resistive losses are neglected.

Efficiency

The method used in this paper to compute efficiency is similar to that used by Beranek [3, p. 262] and Small [2] and is defined as the acoustic output power divided by the nominal electrical input power delivered by the source into a resistor having a value twice the rated DC voice-coil resistance ($P_{in} = e_{in}^2 / (2R_E)$).

For midband operation, the efficiency is maximized at a value of 50% when the reflected load resistance equals the driver's voice coil resistance i.e. $R_{ET} \approx R_E$. This situation can be attained for a specific throat area given by [6, p. 279], [10, eq. 3] if $n=1$:

$$S_T = \frac{\rho_e R_E S_D^2}{B^2 l^2} \quad (1)$$

It must be noted that the widest bandwidth may not be obtained for this maximum efficiency situation.

Frequency Response

As Beranek indicates [3, pp. 263-266], the frequency response of a horn system can be divided into three distinct regions: low, mid, and high frequencies. If the throat impedance of the horn is assumed to be purely resistive and constant with frequency (simulates a horn with very low cutoff or infinite tube load) the response or nominal efficiency versus frequency can be modeled as shown in Fig. 3. The three frequency bands along with indicated corner points are clearly shown. The three regions indicate respectively compliance, resistance and mass controlled portions of horn operation.

As an aid to later analysis, it helps to define two driver related corner frequencies which indicate respectively the approximate upper and lower bounds of the resistance controlled region of the unmounted driver:

$$\begin{aligned} &\text{Upper bound,} \\ f_{HS} &= \frac{B^2 l^2}{2\pi R_E M_{HS}} \quad ; \text{ and} \end{aligned} \quad (2)$$

$$\begin{aligned} &\text{Lower bound,} \\ f_{LS} &= \frac{R_E}{2\pi B^2 l^2 C_{MS}} \end{aligned} \quad (3)$$

Note that $f_S = \sqrt{f_{LS} f_{HS}}$.

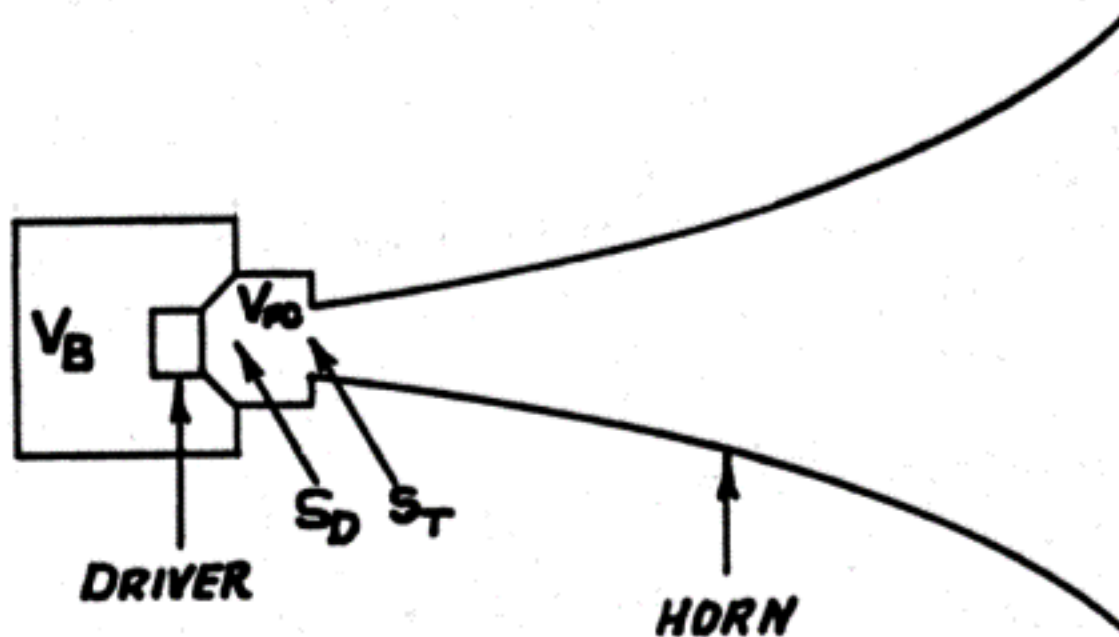


Fig. 1. Depiction of low-frequency horn-driver system. Back cavity V_B , front cavity V_{FC} , diaphragm area S_D , and horn throat area S_T are indicated.

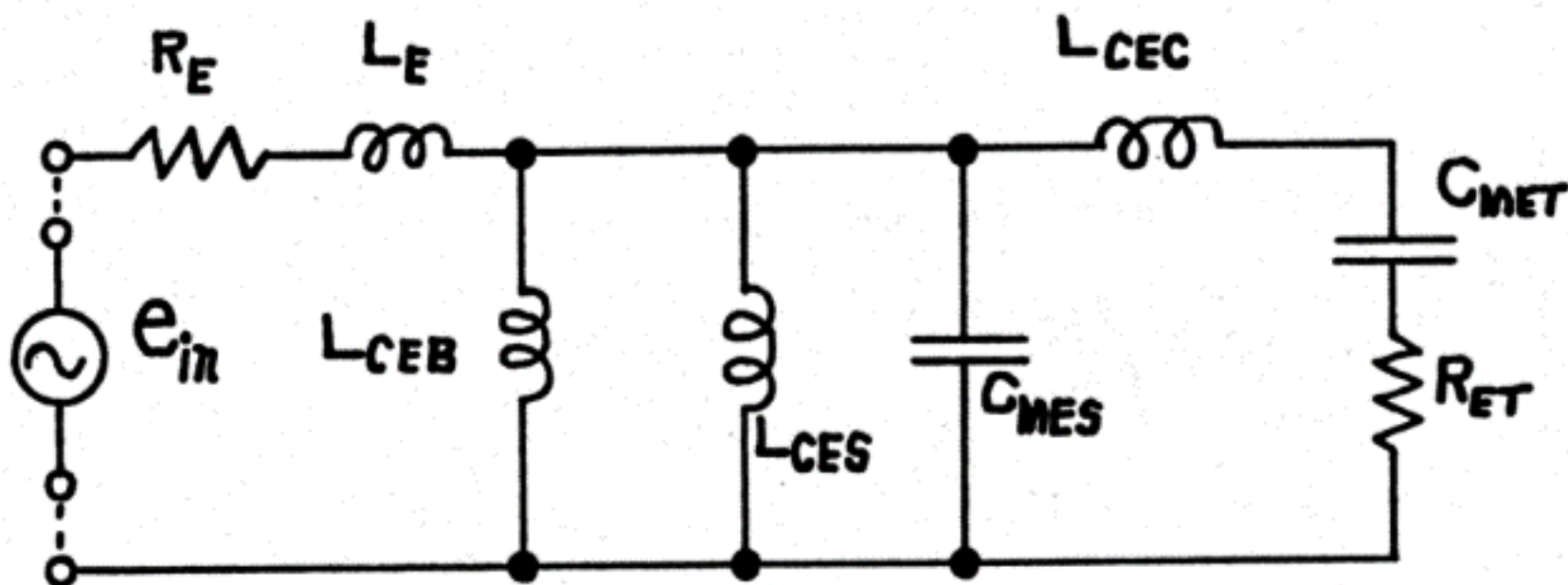


Fig. 2. Simplified lumped electrical equivalent circuit of the low-frequency horn-driver system depicted in Fig. 1. Symbols are defined in glossary of symbols. The effects of driver mechanical resistive losses have been neglected ($Q_{MS} \gg Q_{ES}$). The horn's throat load appears as R_{ET} and C_{MET} which are both non-constant functions of frequency in the general case.

Low Frequencies

At low frequencies, the simplified electrical equivalent circuit reduces to the form shown in Fig. 4a. Examination reveals that the response rolls off at 6 dB per octave below a frequency set by certain driver parameters including suspension compliance, effective circuit resistance, and back cavity compliance.

If the efficiency is maximized by setting the throat area to the value in eq. (1), and the effects of back cavity compliance are neglected ($V_B \rightarrow \infty$), the lower driver compliance corner frequency is given by:

$$f_{LC} = \frac{R_E}{4\pi B^2 l^2 C_{HS}} = f_{LS}/2. \quad (4)$$

For a finite back cavity, the lower corner frequency is increased to:

$$f_{LBC} = \frac{R_E (1+\alpha)}{4\pi B^2 l^2 C_{HS}} = f_{LC} (1+\alpha) = \frac{f_{LS} (1+\alpha)}{2} \quad (5)$$

Where $\alpha = C_{HS}/C_{AB}$, the ratio between the driver suspension compliance and the box compliance.

Mid Frequencies:

At mid frequencies the equivalent circuit reduces to Fig. 4b. Analysis yields a maximum midband nominal efficiency of

$$\eta = \frac{2 R_E R_{ET}}{(R_E + R_{ET})^2} \quad (6)$$

where $R_{ET} = S_T B^2 l^2 / (\rho c S_D^2)$,

which is maximized when $R_{ET} = R_E$ by setting S_T according to eq. (1).

High Frequencies:

At high frequencies the equivalent circuit takes the form shown in Fig. 4c which is a 3rd-order low-pass filter. Three individual rolloff mechanisms are exhibited which are