

3 MODES OF VIBRATION IN A TUBE

3.1 Solutions of the Wave Equation

As the name itself implies, the propagation of sound inside a plane-wave tube takes place mainly through plane waves. Yet the plane wave is by no means the only possible way of propagation inside a cylinder [4]. There is an infinity of higher modes of vibration, each characterized by an integer number n of nodal cylinders¹ parallel to the axis and an integer number m of nodal planes through the axis. These higher modes are also referred to as "transverse modes" because the particle velocity in every point is no longer parallel to the tube axis, as in the plane-wave mode.

The solution of the wave equation in cylindrical coordinates can be written as

$$p_{mn}(r, \varphi, x) = p_0 \cos(m\varphi) J_m \left(b_{mn} \frac{r}{a} \right) \exp[j(-k_{mn}x + \omega t)] \quad (1)$$

for a single mode having m nodal planes and n nodal cylinders, where x is the axial coordinate, r and φ are the polar coordinates, a is the tube radius (Fig. 5), and J_m is the Bessel function of order m . The scale factor b_{mn} in the argument of the Bessel function is determined by the constraint that the radial derivative of the pressure—hence of the Bessel function—be zero for $r = a$, that is, no flow through the tube wall; b_{mn} is therefore the argument value corresponding to the n th zero of the derivative of J_m .

The wave number along the axis k_{mn} , in turn, is deter-

mined by the condition [1]

$$k_{mn}^2 = \left(\frac{\omega}{c} \right)^2 - \left(\frac{b_{mn}}{a} \right)^2 \quad (2)$$

It is customary to call "symmetrical" the modes having nodal cylinders only ($m = 0$, $n \neq 0$, they have axial symmetry), "asymmetrical" the ones having nodal planes only ($m \neq 0$, $n = 0$), and "mixed" the ones having nodal planes and cylinders ($m, n \neq 0$).

3.2 Cutoff Frequencies

Eq. (2) has two important consequences. The first is that the wave number k_{mn} is purely imaginary for frequencies less than the cutoff value,

$$\omega_c = \frac{b_{mn}c}{a} \quad (3)$$

Therefore those frequencies cannot propagate in the (m, n) mode and are exponentially attenuated (they *can*, of course, propagate as plane waves or in modes of lower cutoff frequency). It is possible to arrange the higher modes of propagation in the order of increasing cutoff frequency with the aid of a table of Bessel functions (see, for example, [5]). Table 1 shows the first six modes of this ordering, with the values of b_{mn} and the corresponding cutoff frequencies $f_c = \omega_c/2\pi$ calculated for $c = 344$ m/s and a tube radius $a = 12.7$ mm (0.5 in).

The second consequence of Eq. (2) is that the wave number of the higher modes inside the tube is always less than the wave number in free air for the same frequency (Fig. 6). This means that for frequencies above cutoff, the wavelength and the phase velocity are larger than in free air and tend to infinity at cutoff. The input impedance of the transverse modes is also infinite at cutoff, and this is seen as a notch in the frequency response at f_c .

¹ A nodal surface is a locus of points where the sound pressure is zero for every time t in a given mode of vibration. The planar patterns of nodal lines in the cross section of a cylinder are similar to those of a circular membrane [1].

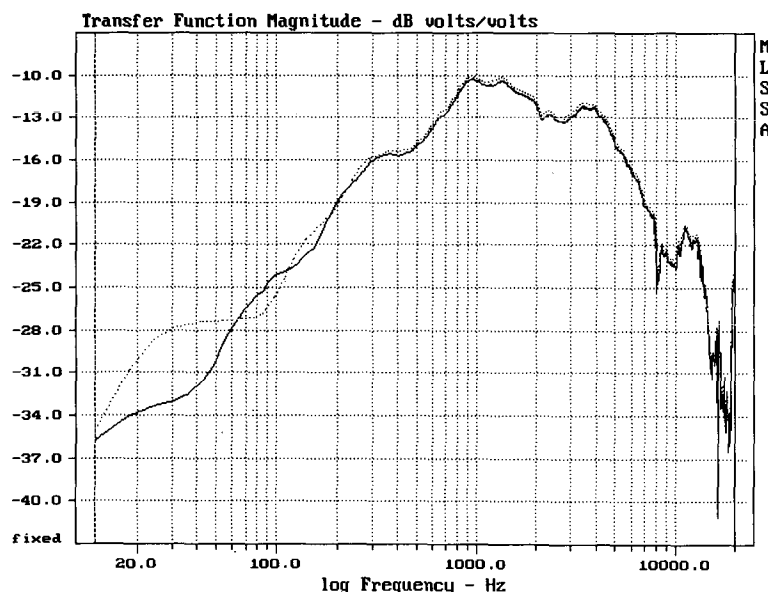


Fig. 2. Frequency response of 25.4-mm (1-in) driver on plane-wave tube with different far-end conditions. ... open end; — closed end.