

Acoustical Studies of the Tractrix Horn. II*

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Experimental investigations have been carried out on the tractrix horn structure to determine its "free-field" radiation characteristics. Axial, off-axis, and polar response characteristics, as well as throat impedance data on a single cell horn, are presented for both small and large baffle mounting. Pertinent data on a two cell structure are also presented. These data show the tractrix performance to be comparable with that of the well-known exponential horn. A multicellular structure, while showing definite improvement in uniformity of angular distribution at high frequencies, exhibits undesirable band rejection characteristics within the useful frequency range of the horn.

INTRODUCTION

IMPORTANT applications for acoustic horns are producing desired radiation patterns and effecting good impedance match between loudspeaker and air medium over a wide frequency range. For many years the comparison standard for horn performance has been the Webster contours¹ predicted from plane wave propagation theory.

The tractrix structure defined in a companion paper differs somewhat from the Webster horns in that it flares rapidly near the mouth. At its throat, however, the horn tapers gradually, becoming exponential as the length is increased. Professor H. E. Hartig suggested that experimental information about the tractrix would be of aid in evaluating the performance of rapidly flaring horns. It seemed reasonable that the flare and spherical symmetry associated with the tractrix might provide good impedance match over a wide range of frequencies. Because of the dearth of material on this subject, experimental measurements would also be very valuable in theoretical studies of propagation and radiation in such coupling devices.

This paper discusses the more pertinent information obtained from excess pressure response and throat impedance measurements on single and multicellular tractrix horn structures. Axial pressure response, off-axis response, polar response, and throat impedance characteristic are presented for a tractrix structure of length 23.5 inches, mouth diameter 20 inches, having a lower cutoff frequency of about 180 cps.

EXPERIMENTAL APPARATUS AND THE TECHNIQUE OF MEASUREMENT

When embarking on the problem of determining the radiation response of a horn, the first obstacle that must

be overcome is securing a "free-field" space if one does not have access to an anechoic chamber of suitable size. The out-of-doors appears to yield the simplest solution to this problem, especially for measurements over a wide frequency range.

The first site chosen for making the required measurements was a window on the third floor of the Electrical Engineering Building. This window is located in a practically continuous brick wall opening onto a large court of dimensions approximately 90 feet wide by 150 feet long by 50 feet deep. It was thought that the brick wall together with an appropriate horn flange would serve as an infinite baffle and that the court was large enough to minimize reflection problems. Other factors in the selection of this site were that it afforded some degree of protection from wind gusts and allowed the equipment to be housed indoors. Experiments performed at this location established certain measurement techniques which were used throughout the remainder of the tests. The results of some of these experiments showed that monitoring either the input current or voltage to the W.E. 555 w, which was used to drive the horn during frequency response measurements, gave almost identical results as far as axial pressure response was concerned. It was decided to hold the driver voice coil current constant with the thought in mind that this technique might facilitate theoretical horn studies hypothesizing constant throat particle velocity. A calibrated 0.254-ohm Ayrton wound resistor was used as a current indicator. "Q" measurements performed on this resistor showed it to be practically a pure resistance over the frequency range of 100 cps to 100 k cps. A calibrated resistor was used in preference to a thermocouple because of its lower resistance and greater ruggedness.

In these experiments particular attention was given to such things as amplifier stability and response, signal distortion, frequency calibration and stability, and microphone cable response. The secondary frequency standard was a W.E. 6010 B oscillator. In order to reduce nonlinearity in the driving and measuring equipment to a tolerable level, it was found necessary to limit the input power to the W.E. 555 w to approximately 0.3 watt. Viewing the driving current wave

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¹ P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), pp. 265-85.

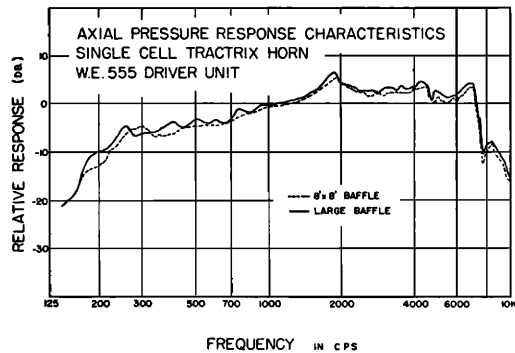


FIG. 1. Axial response of single cell tractrix horn in a large baffle and in an 8-foot by 8-foot baffle.

form and the W.E. 640 A condenser microphone output voltage wave form on an oscilloscope showed that the effects of nonlinearities (due to driver,² horn,³ or detector⁴) were negligible at this power level. The effects of varying bias voltage on the condenser microphone and varying driver field current were also investigated. It was found that when the bias voltage was held at 200 volts ± 2 percent the relative pressure indication varied by less than 0.25 db. The driver field current was held at 1.5 amperes. However, it was found that the relative pressure response measurements were insensitive to variations of ± 25 percent in the driver field current.

The day-to-day stability of the entire driving and indicating system was examined before measurements were taken by starting any series of measurements from a set of fixed conditions. The equipment was allowed to warm up for at least two hours; frequency was set at 200 cps; driver voice coil current was adjusted to 138 ma; and with the microphone placed at a fixed distance from the horn mouth the signal response was noted. Variations in the recorded response of more

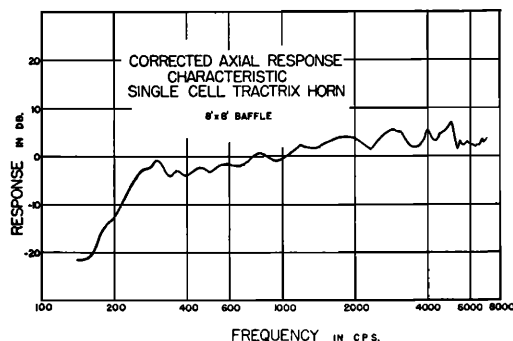


FIG. 2. Axial response of single cell tractrix horn in 8-foot by 8-foot baffle corrected for W. E. 555 Driver, 640 AA microphone, microphone cable, and amplifier response.

² E. C. Wentz and A. L. Thuras, *Bell System Tech. J.* 7, 140 (1928).

³ L. L. Beranek, *Acoustics* (McGraw-Hill Book Company, Inc., New York, 1954), pp. 272-76.

⁴ L. L. Beranek, *Acoustic Measurements* (John Wiley and Sons, Inc., New York, 1949), p. 220.

than 0.5 db from day to day under these standard conditions were regarded as indicating instability in some phase of the measuring equipment or radical changes in climatic conditions. The measurements were discontinued until climatic conditions improved or electronic troubles were located.

Tests of axial response under different wind, noise, and temperature conditions were made to determine the effect of these random variants upon the measured horn response. It was found that the effect of temperature was only a relative one, the entire response curve being shifted up or down by approximately 1 db for normal temperature variations about 25°C. The effects of wind and background noise were somewhat bothersome since these variants may not affect all portions of the response curve to the same degree. Measurements made under conditions of wind velocity below about 3 miles per hour and minimum background noise revealed that signal-to-noise ratios of 30 db could be

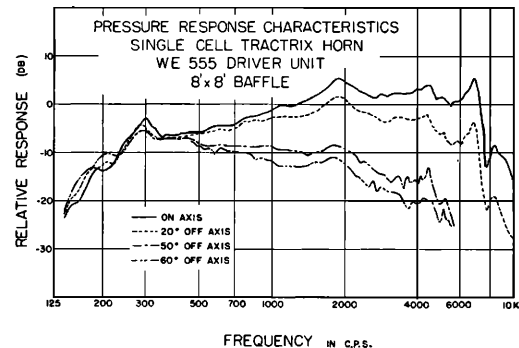


FIG. 3. Axial and off-axis response of single cell tractrix horn in 8-foot by 8-foot baffle.

achieved, with data being reproducible to within ± 0.5 db.

In order to determine to what degree the presence of the measuring equipment in the sound field disturbed the results, runs of axial response *versus* frequency were made for the microphone located at various distances from the horn mouth. Comparison of data from these investigations showed that any effects due to reflection or diffraction were insignificant. The only appreciable difference found was a general lowering of the response level as the axial distance increased, the shape of the curves being preserved.

For convenience in obtaining polar response and throat impedance measurements, investigations were continued on the roof of the Electrical Engineering building when weather permitted. At this location the horn was mounted in an 8-foot by 8-foot, 1-inch-thick, plywood baffle board reinforced by 2-inch by 4-inch cross braces. Response measurements on this structure, Fig. 1, indicated that any baffle resonances present had been lowered beneath the frequency range of interest and thus did not give rise to erroneous information about the radiation characteristics of the horn.

An effort was made to separate the horn response from the over-all response including horn and driver, Fig. 2, by feeding the driver into a $\frac{1}{8}$ in. wall, $1\frac{3}{8}$ in. i.d., 5 foot long steel tube terminated with a 32 in. long, flat taper, fiber glass wedge. Except possibly at frequencies below 600 cps, this served as an excellent termination with standing wave ratios below 1.15:1. The response was measured with a W.E. 640 AA condenser microphone having a 0.06-in. diameter probe tube attached, the probe being inserted in the terminating tube. The probe response was then removed from the combination probe and microphone response. The calibration data also showed that the driver response dropped off sharply above 7000 cps. Axial response data corrected for driver, cable, amplifier, and microphone response appear in Fig. 2. Since our main interest was in the relative response characteristics this is the only set of corrected data that appears in this paper. Several different drivers of the same type were tested and the results achieved were the same within experimental error.

The polar response characteristics of the single cell horn, Fig. 4, were obtained by mounting a rotary boom on the 8-foot by 8-foot baffle and attaching the W.E. 640 AA microphone with 19-in. long, 0.06-in. diameter probe tube to a cross member on this boom. This arrangement allowed a semicircular path of radius 85 inches to be traversed by the pickup probe and per-

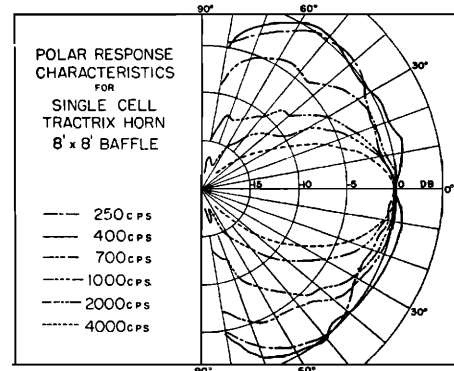


FIG. 4. Polar response of single cell tractrix horn in 8-foot by 8-foot baffle.

mitted measurements over 180° arc. Comparisons of off-axis response runs, Fig. 3, and the polar response runs, Fig. 4, for the single cell structure reveal data correspondence within about ± 10 percent between these two independent sets of measurements at 50 degrees off the axis. The axial and off-axis runs were made without a probe tube attached to the microphone.

Measurements were undertaken to determine the input acoustical impedance to the horn over the low-frequency range,[§] Fig. 5. The method⁵ employed was that due to Flanders. This technique consists essentially of determining an unknown acoustic impedance in

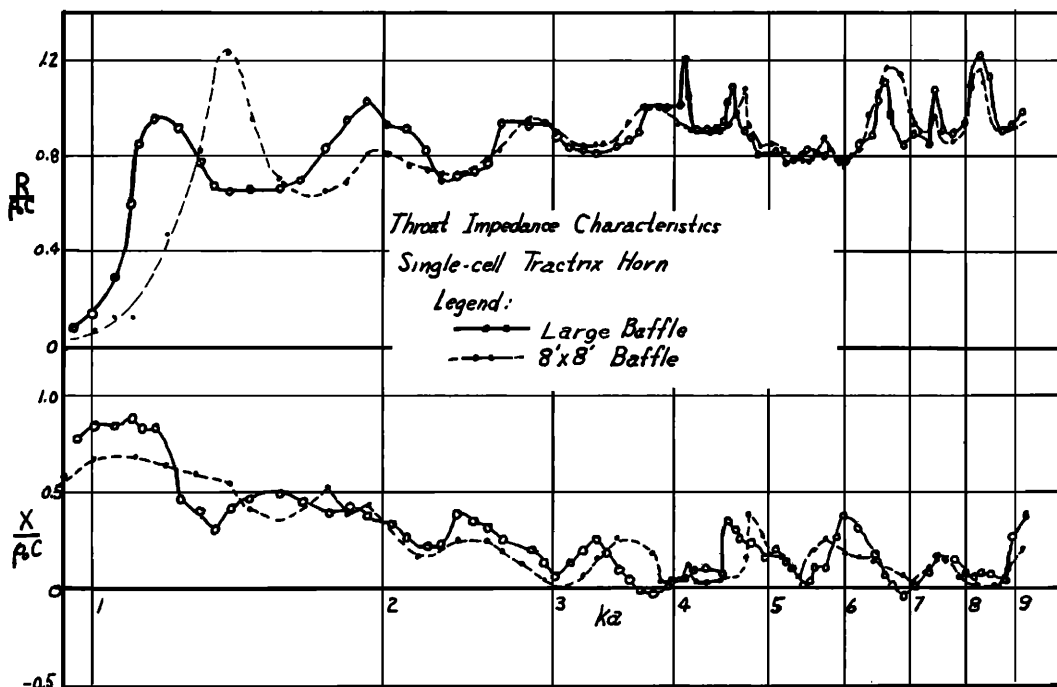


FIG. 5. Throat impedance characteristic (see footnote §) of single cell tractrix horn in an 8-foot by 8-foot baffle and in a large baffle.

[§] In Fig. 5, the real and imaginary part of the throat impedance is plotted *versus* the frequency-geometry parameter ka where k is the wave number and a is the radius of the horn mouth. The actual frequency range covered extends from 200 to 2000 cps.

⁵ P. B. Flanders, Bell System Tech. J. 11, 402-410 (1932).

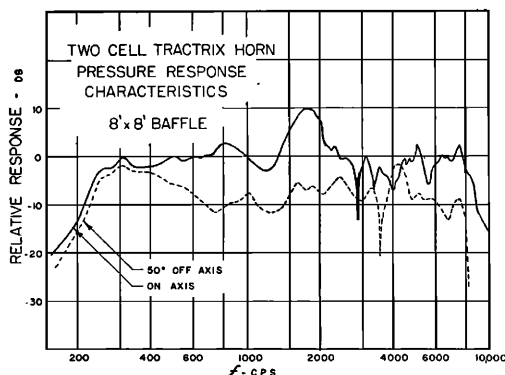


FIG. 6. Axial and off-axis response of a two cell tractrix horn in an 8-foot by 8-foot baffle.

terms of balanced bridge reading obtained from two known and the unknown acoustic impedances. The known impedances are an infinite impedance termination consisting of a rigid plug, and an eighth-wavelength closed end tube. In this particular investigation it was found necessary to include an acoustic attenuator in the source tube to reduce loading effects on the University type P.A. 30 driver. This attenuator consisted of a hollow cylindrical Styrofoam plug inserted in the throat of the source tube. To insure reproducible results considerable attention was accorded to the rigidity of the entire source tube, driver, and pickup assembly. It was found that better results were obtained with driver, source tube, and W.E. 640 A microphone and 0.06-in. diameter probe tube mounted as an integral unit. The microphone output voltage wave form was continuously monitored to surmise when nonlinearities were giving rise to meaningless data. A variable band-pass filter, Spencer Kennedy model No. 302, was found to be indispensable in obtaining reproducible results. In the vicinity of peaks in the impedance characteristic even slight drifts in frequency of the order of several cps produced noticeable changes in bridge measurements. Consequently, frequency was held fixed to within 1 part in 1000 in all bridge measurements. It was also found necessary to compensate for temperature variations when calculating sound velocity in order to gain the necessary precision for adjusting the eighth-wavelength tube to the correct length. In all of these measurements the tube was set to an eighth wavelength ± 0.1 mm over the frequency range 200 to 2000 cps. Wind gusts were found to invalidate any data taken during these tests. Winds of even moderate velocity evidently produce such great turbulence in the horn throat that acoustic impedance data have little meaning under such conditions.

An examination of the polar response characteristic of the single cell horn structure, Fig. 4, revealed a marked columniation process occurring for frequencies above 500 cps. In an effort to produce a more uniform distribution of the acoustic energy at frequencies higher than 500 cps, a two cell tractrix horn

was constructed with the inner cell contour located along theoretical velocity flow lines. The throat of the inner cell extended to within three inches of the throat end of the outer contour. The mouth end of the inner cell was cut off at a point such that its rim would lie on a hemisphere of radius 10 inches. It was hoped that this cellular arrangement would cause a more nearly spherical wave front to emanate from the horn mouth. Measurements of axial, off-axis, and polar response were made on this structure, Figs. 6 and 7.

RESULTS

The principal results obtained in the study will be presented in graphical form with few comments since the results shown are for the most part self-explanatory.

Figure 1 shows the axial response of the single cell tractrix horn structure for two different baffle mountings. These data are for an axial distance to horn mouth radius ratio of 8.5:1. The large baffle corresponds to the building wall baffle arrangement mentioned earlier while the 8-foot by 8-foot baffle measurements refer to those taken on the roof of the Electrical Engineering Building. These data were found to be reproducible to within 3 percent.

The curve of Fig. 2 shows the axial response of the single cell horn-driver combination corrected for driver, microphone, cable, and amplifier response. Consequently this response characteristic is due to horn propagation alone.

The data plotted in Fig. 3 show the off-axis performance of the single cell tractrix horn structure mounted in the 8-foot by 8-foot baffle. These curves reveal the energy columniation process taking place above 500 cps, a phenomenon which is also verified by the polar response characteristics of the same structure, Fig. 4. The data of Fig. 3 have not been corrected for driver response since this reduction is not warranted in comparative measurements. These data, however, have been corrected for microphone, cable, and amplifier response. Again, results were found to be reproducible to within 3 percent.

Polar response data for the single cell unit are pre-

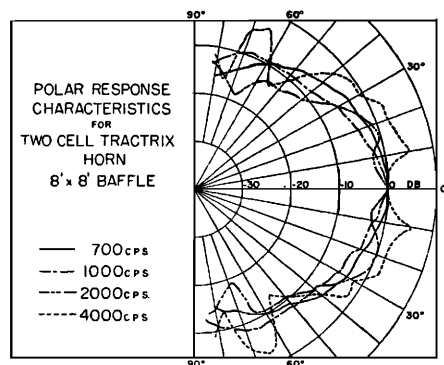


FIG. 7. Polar response of a two cell tractrix horn in an 8-foot by 8 foot baffle.

sented for several selected frequencies in Fig. 4, measurements being taken at a radial distance of 85 inches from the horn mouth center. As mentioned previously, a 19-in. long, 0.06-in. diameter, probe tube on the microphone was used for probing the sound field. These data, Fig. 4, correspond to those obtained in off-axis response runs, Fig. 3, to within approximately ± 10 percent and are reproducible to within 3 percent.

Throat impedance measurements on the single cell structure as obtained by the Flanders bridge method are plotted in Fig. 5. It is estimated that these data are accurate to within 15 percent. The largest uncertainty in the data occurred in the vicinity of peaks in the characteristics. Effects of finite baffle size on the horn input impedance are very evident here.

Any virtues of a multicellular horn structure in so far as uniformity of energy distribution is concerned are evident from the axial, off-axis, and polar response curves, Figs. 6 and 7. Also in evidence, Fig. 6, is the band rejection property of the multicellular unit. The throat impedance characteristic of the multicellular horn was not investigated.

SUMMARY

Experimental data on the "free-field" radiation patterns of a single cell tractrix horn for two baffle sizes have been presented. It was found that these data are reproducible to within 3 percent for axial and off-axis response and to within 10 percent for polar response using standard techniques. These results show that the performance of the tractrix compares favorably with

that of more familiar horn structures of roughly the same size. The polar response data do show a pronounced columniation of the radiated energy along the principal axis of the horn, a result characteristic of horn radiation in general.

The uniformity of the angular distribution may be improved somewhat by employing multicellular structures. However, there is some sacrifice in the smoothness of the frequency response of the horn. The axis and off-axis response do show pronounced frequency rejection bands, a phenomenon characteristic of radiation from multicellular horns.

A Flanders bridge method for measuring acoustic impedance was developed for precise measurements. It was found necessary to incorporate an acoustical attenuator between the driver unit and the measuring chamber in order to achieve good precision. This attenuator insured that the driver was loaded properly for all terminations and suppressed resonances within the measuring tube itself.

Throat impedance data taken over a range of frequencies imply some impedance mismatch at the horn mouth. The mismatch is more pronounced in the region of cutoff and is accentuated by smaller baffle size. These data are reproducible to well within 15 percent, any deviations being attributed largely to variable climatic conditions and slight errors in setting the eighth-wavelength tube, particularly at the higher frequencies.

In conclusion the writers express sincere thanks to Professor H. E. Hartig for his encouragement and valuable advice during the experimental investigation.