MFB LOW-PASS FILTER DESIGN PROGRAM

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Although low-pass filters are vital in modern electronics, their design and verification can be tedious and time consuming. The Burr-Brown FilterPro™ program, FILTER2, makes it easy to design low-pass active filters. The program is intended to aid in the design of low-pass filters implemented with the Multiple Feedback (MFB) topology. Because there are instances where the Sallen-Key filter topology is a better choice, the program also supports Sallen-Key low-pass filter design.

An ideal low-pass filter would completely eliminate signals above the cutoff frequency, and perfectly pass signals below cutoff (in the pass-band). In real filters, various trade-offs are made in an attempt to approximate the ideal. Some filter types are optimized for gain flatness in the pass-band, some trade-off gain variation (ripple) in the pass-band for steeper roll-off, still others trade-off both flatness and rate of roll-off in favor of pulse-response fidelity. FILTER2 supports the three most commonly used all-pole filter types: Butterworth, Chebyshev, and Bessel.

Butterworth (maximally flat magnitude). This filter has the flattest possible pass-band magnitude response. Attenuation is –3dB at the design cutoff frequency. Attenuation above the cutoff frequency is a moderately steep –20dB/decade/ pole. The pulse response of the Butterworth filter has moderate overshoot and ringing.

Chebyshev (equal ripple magnitude). (Also transliterated Tschebychev, Tschebyscheff or Tchevysheff.) This filter type has steeper attenuation above the cutoff frequency than Butterworth. This advantage comes at the penalty of amplitude variation (ripple) in the pass-band. Unlike Butterworth and Bessel responses, which have 3dB attenuation at the cutoff frequency, Chebyshev cutoff frequency is defined as the frequency at which the response falls below the ripple band. For even-order filters, all ripple is above the 0dB-gain DC response, so cutoff is at 0dB—see Figure 1A. For odd-order filters, all ripple is below the 0dB-gain DC response, so cutoff is at –(ripple) dB—see Figure 1B. For a given number of poles, a steeper cutoff can be achieved by allowing more pass-band ripple. The Chebyshev has even more ringing in its pulse response than the Butterworth.

Bessel (maximally flat time delay). (Also called Thomson.) Due to its linear phase response, this filter has excellent pulse response (minimal overshoot and ringing). For a given number of poles, its magnitude response is not as flat, nor is its attenuation beyond the –3dB cutoff frequency as steep as the Butterworth. It takes a higher-order Bessel filter to give a magnitude response which approaches that of a given Butterworth filter, but the pulse response fidelity of the Bessel filter may make the added complexity worthwhile.
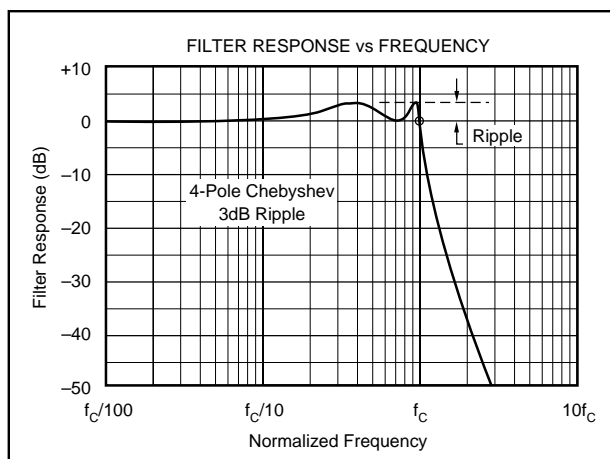

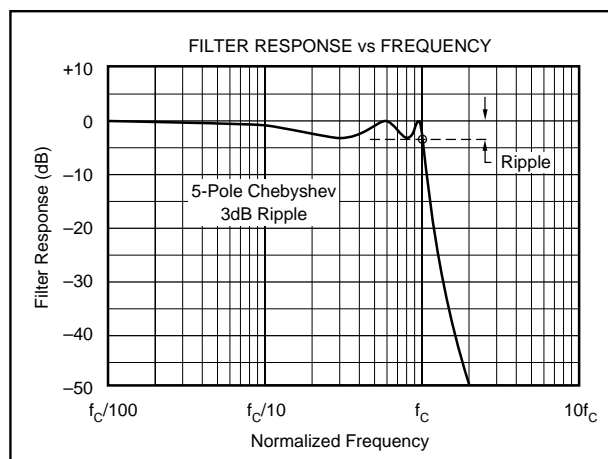FIGURE 1A. Response vs Frequency of Even-Order (4-pole), 3dB Ripple Chebyshev Filter Showing Cutoff at 0dB.

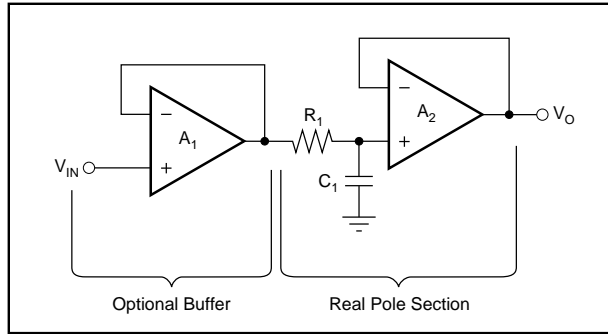FIGURE 1B. Response vs Frequency of Odd-Order (5-pole), 3dB Ripple Chebyshev Filter Showing Cutoff at –3dB.

FIGURE 2. Real Pole Section (Unity-Gain, First-Order Butterworth; $f_{-3dB} = 1/2 \cdot \pi \cdot R_1 \cdot C_1$).
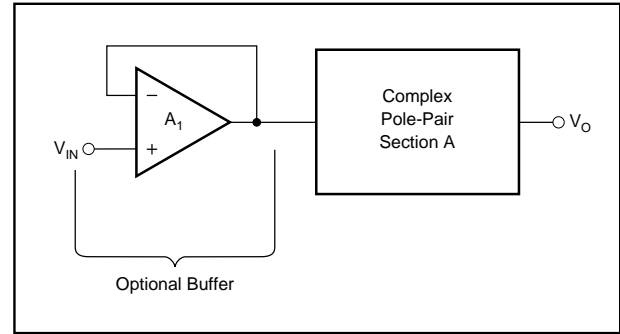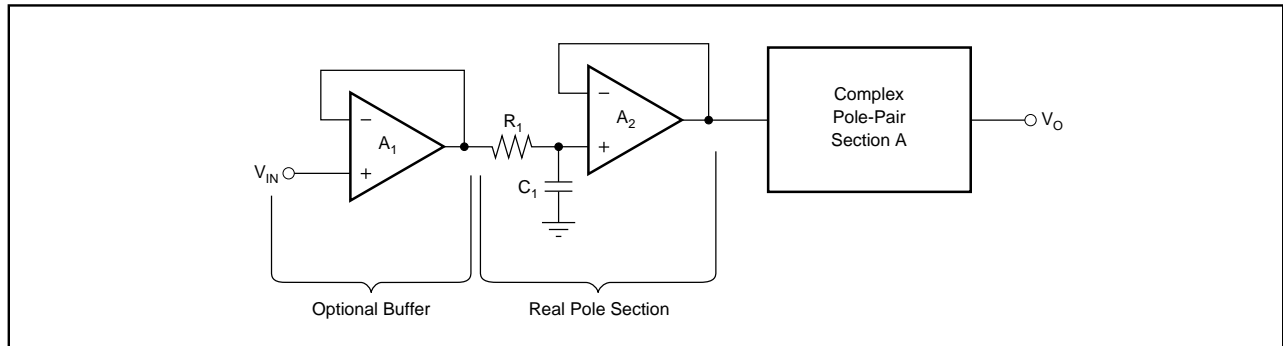

FIGURE 3. Second-Order Low-Pass Filter.

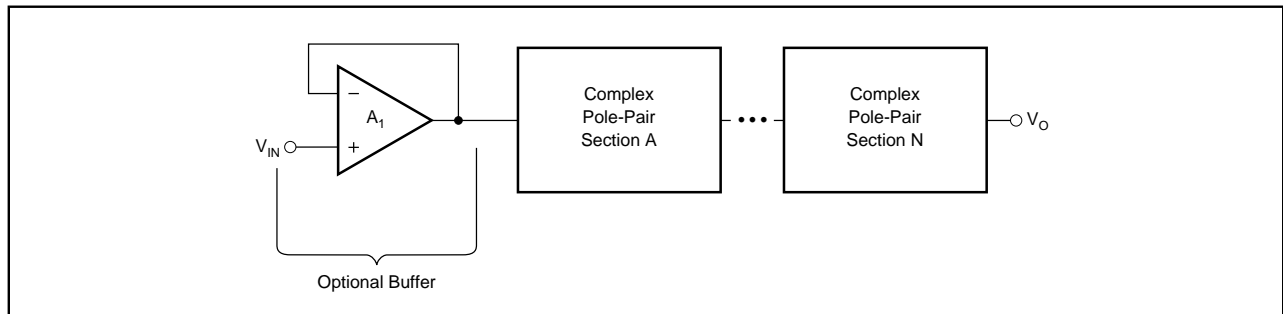FIGURE 4. Third-Order Low-Pass Filter.

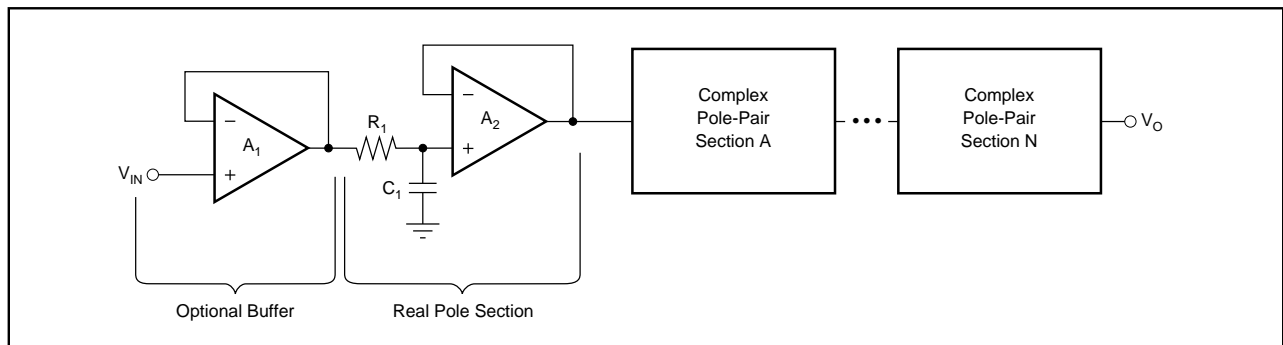FIGURE 5. Even-Order Low-Pass Filter Using Cascaded Complex Pole-Pair Sections.

FIGURE 6. Odd-Order Low-Pass Filter Using Cascaded Complex Pole-Pair Sections Plus One Real-Pole Section.

SUMMARY

Butterworth

Advantages   Maximally flat magnitude response in the pass-band. Good all-around performance. Pulse response better than Chebyshev. Rate of attenuation better than Bessel.

Disadvantages   Some overshoot and ringing in step response.

Chebyshev

Advantages   Better attenuation beyond the pass-band than Butterworth.

Disadvantages   Ripple in pass-band. Considerable ringing in step response.

Bessel

Advantages   Best step response—very little overshoot or ringing.

Disadvantages   Slower rate of attenuation beyond the pass-band than Butterworth.

CIRCUIT IMPLEMENTATION

Even-order filters designed with this program consist of cascaded sections of complex pole-pairs. Odd-order filters contain an additional real-pole section. Figures 2 through 6 show the recommended cascading arrangement. The program automatically places lower Q stages ahead of higher Q stages to prevent op amp output saturation due to gain peaking. The program can be used to design filters up to 8th order.

FILTER ORDER	FIGURE
1 pole	Figure 2
2 poles	Figure 3
3 poles	Figure 4
4 or more poles (even order)	Figure 5
5 or more poles (odd order)	Figure 6

TABLE I. Filter Circuit vs Filter Order.

COMPLEX POLE-PAIR CIRCUIT

The choice of a complex pole-pair circuit depends on performance requirements. FILTER2 supports the two most commonly used op amp pole-pair circuit topologies:

- Multiple Feedback (MFB)—shown in Figure 7.
- Sallen-Key—shown in Figures 8 and 9.

The MFB topology (sometimes called Infinite Gain or Rauch) is often preferred due to assured low sensitivity to component variations—see sensitivity section. There are instances, however, where the Sallen-Key topology is a better choice.

As a rule of thumb, the Sallen-Key topology is better if:

    1) Gain accuracy is important, *AND*
    2) A unity-gain filter is used, *AND*
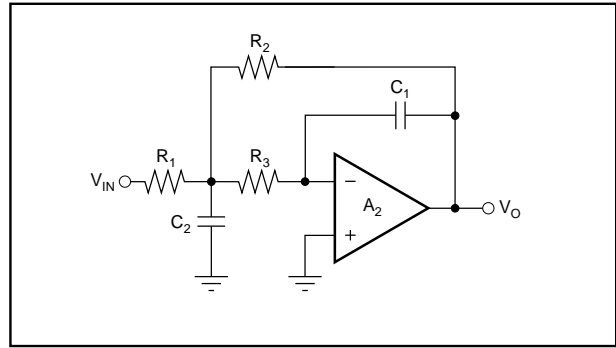   3) Pole-pair Q is low (e.g. Q < 3)

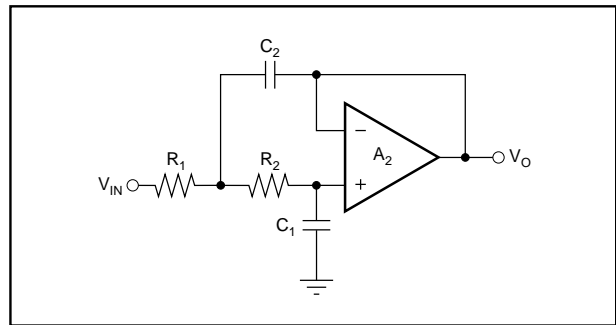FIGURE 7. MFB Complex Pole-Pair Section. (Gain = $-R_2/R_1$)

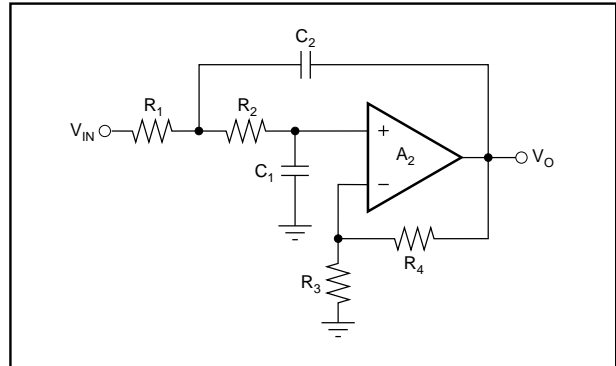FIGURE 8. Unity-Gain Sallen-Key Complex Pole-Pair Section. (Gain = 1)

FIGURE 9. Sallen-Key Complex Pole-Pair Section. (Gain = $1 + R_4/R_3$)

At unity-gain, the Sallen-Key topology inherently has excellent gain accuracy. This is because the op amp is used as a unity-gain buffer. With the MFB topology, gain is determined by the R_2/R_1 resistor ratio. The unity-gain Sallen-Key topology also requires fewer components—two resistors vs three for MFB.

The Sallen-Key topology may also be preferable for high-Q high frequency filter sections. In these sections the value required for C_1 in a MFB design can be quite low for reasonable resistor values. Low capacitor values can result in significant errors due to parasitic capacitances.

The best filter design may be a combination of MFB and Sallen-Key sections.

Use the appropriate circuits along with component values generated by the FILTER2 program to build your filter design.

USING THE FILTERPRO™ PROGRAM

With each data entry, the program automatically calculates filter performance and values for all filter components. This allows you to use a "what if" spreadsheet-type design approach. For example, you can quickly determine, by trial and error, how many poles are needed for a given roll-off.

GETTING STARTED

The first time you use the program, you may want to follow these steps.

Type **FILTER2 <ENTER>** to start the program.

At this point, you have already designed a 3-pole, 1kHz Butterworth filter. Component values are shown on the display screen. For a different filter design, use the arrow keys to move the cursor to the menu selection desired.

Start at the top of the menu and work your way down. On-screen prompts, to the left of the menu selections, will guide you in program use. Refer to this bulletin for more detail, if needed.

1) Choose pole-pair **Circuit**: MFB or Sallen-Key

2) Select **Filter type**: Butterworth
                          Chebyshev
                          Bessel

3) For Chebyshev filter type, enter **Ripple**: >0dB to 4dB

4) Enter **Number of poles**: 1 to 8

5) Enter filter **Cutoff frequency**: (Hz)

The following steps are optional:

6) If you want to view the gain/phase response of the current filter design at a particular frequency, enter the frequency of interest on the **Response @ fx** line. The gain/phase information can be viewed on the **fn, Q, Response** display window—see step 11.

7) If you want to change the resistor scaling, enter a value on the **Scale Resistors** line.

8) If you want to change the gain of a section, press **<ENTER>** on the **Gain Entry** line. Default value for gain is 1.0V/V in each section.

9) If you want to enter your own capacitor values, press **<ENTER>** on the **Capacitor Menu** line.

10) If you want to design with standard 1% resistors instead of exact resistors, press **<ENTER>** on the **Resistors** line.

11) To change the display screen press **<ENTER>** on the **Display** line. Available display screens are: Component values; fn, Q, Response; Sensitivities.

TO RETURN TO DOS

To exit the program and return to DOS, press **<F1>**.

USING THE PLOT FEATURE

A Plot feature allows you to view graphical results of filter gain and phase vs frequency. This feature is useful for comparing filter types.

To view a plot of the current filter design, press **<F2>**.

GRAPHIC DISPLAY COMMANDS

While viewing the graphic display, several commands can be used to compare filter responses:

S—**S**aves the plot of the current design for future recall.

R—**R**ecalls the **S**aved plot and plots it along with the current design.

P—Recalls the **P**revious plot of the last design plotted from the main program (by pressing **<F2>**) and plots it along with the current design. You can recall the **P**revious design and the **S**aved design to plot all three together.

C—**C**lears the display and replots only the current design.

GRAPHIC DISPLAY CURSOR CONTROL

While viewing the graphics display you can also use the left/right *arrow* keys to move a cursor and view gain and phase for plotted filter responses. The gain/phase of the current design is always displayed. In addition, the gain/phase of the **R**ecalled or **P**revious design can be viewed by pressing **R** or **P**.

TO PRINT RESULTS

To print results press **<F3>**. All three display screens will automatically be printed.

SENSITIVITY

Sensitivity is the measure of the vulnerability of a filter's performance to changes in component values. The important filter parameters to consider are natural frequency (f_n) and Q.

f_n SENSITIVITY FOR BOTH MFB AND SALLEN-KEY

Sensitivity of f_n to resistor, capacitor, and amplifier gain variations is always low for both the Sallen-Key and MFB filter topologies.

$$S_R^f = S_C^f = \pm 0.5\%/\%$$

$$S_K^f = 0$$

Where:

S_R^f, S_C^f, S_K^f = Sensitivity of f_n to resistor, capacitor, and gain variations (%/%)

Q SENSITIVITY

For the MFB topology, sensitivities to Q are also always low, but sensitivities for the Sallen-Key topology can be quite high—exceeding $2 \bullet K \bullet Q^2$. At unity gain, the Sallen-Key Q sensitivity to resistor and capacitor variations will always be low. Unfortunately, however, the sensitivity of the unity-gain Sallen-Key pole-pair to op amp gain can be high.

Q Sensitivity for MFB Pole-Pair

$$S_C^Q = \pm 0.5\%/\%$$

$$S_R^Q = \pm \frac{R_2 - R_3 - K \bullet R_3}{2(R_2 + R_3 + K \bullet R_3)} \quad \text{(MFB complex pole-pair)}$$

$$S_K^Q = \frac{K \bullet R_3}{R_2 + R_3 + K \bullet R_3} \quad \text{(MFB complex pole-pair)}$$

Notice, by inspection: S_R^Q is always less than $\pm 0.5\%/\%$, and S_K^Q is always less than $1.0\%/\%$.

Q Sensitivity for Gain = 1 Sallen-Key Pole-Pair

$$S_C^Q = \pm 0.5\%/\%$$

$$S_R^Q = \pm \frac{R_1 - R_2}{2(R_1 + R_2)} \quad \text{(Sallen-Key complex pole-pair)}$$

So, S_R^Q is always less than $0.5\%/\%$.

$$Q^2 < S_K^Q < 2 \bullet Q^2 \text{ (Sallen-Key complex pole-pair)}$$

Where:

S_R^Q, S_C^Q, S_K^Q = Sensitivity of f and Q to resistor, capacitor, and gain variations (%/%)

K = Op amp gain (V/V)

Figure 7 circuit, $K = R_2/R_1$
Figure 8 circuit, $K = 1.0$
Figure 9 circuit, $K = 1 + R_4/R_3$

NOTE: FilterPro™ always selects component values so unity-gain Sallen-Key S_K^Q will be closer to Q^2 than to $2 \bullet Q^2$.

However, FILTER2 will allow you to design Sallen-Key pole-pairs with high sensitivities (high Qs and GAIN >> 1). You must make sure that sensitivities to component variations do not make these designs impractical. A feature in the **Display** menu allows you to view the f_n and Q sensitivity of filter sections to resistor and capacitor variations.

USING THE SENSITIVITY DISPLAY FEATURE

To use the **Sensitivity** display option, move the cursor to the **Display** menu, press **<ENTER>**, move the cursor to the **Sensitivity** selection, and press **<ENTER>** again. The display shows sensitivity of f_n and Q to each component for each filter section. The format is S^f; S^Q.

Rather than displaying the derivative with respect to component variations, the program calculates f_n and Q change for a 1% change in component values. This gives a more realistic sensitivity value for real-world variations.

USING THE SCALE RESISTORS MENU OPTION

The **Scale Resistors** option allows you to scale the computer-selected resistor values to match the application. Move the cursor to the **Scale Resistors** menu selection and enter your *seed* resistor value. The default value of 10kΩ is suggested for most applications.

Higher resistor values, e.g. 100kΩ, can be used with FET-input op amps. At temperatures below about 70°C, DC errors and excess noise due to op amp input bias current will be small. Remember, however, that noise due to the resistors will be increased by $\sqrt{n}$ where *n* is the resistor increase multiplier.

Lower resistor values, e.g. 500Ω, are a better match for high-frequency filters using the OPA620 or OPA621 op amps.

CAPACITOR VALUES

Compared to resistors, capacitors with tight tolerances are more difficult to obtain and can be much more expensive. The **Capacitor** menu option allows you to enter actual measured capacitor values. In this way, an accurate filter response can be achieved with relatively inexpensive components.

USING THE CAPACITOR MENU OPTION

To use the **Capacitor** menu option, move the cursor to the **Capacitor** menu selection and press **<ENTER>**. Move the cursor to any capacitor and enter your value. Prompts on the left of the screen advise min/max capacitor entry limits. With each capacitor entry, the program will select exact or closest standard 1% resistor values as before.

COMPENSATE FOR OP AMP INPUT CAPACITANCE—SALLEN-KEY ONLY

If the common-mode input capacitance of the op amp used in a Sallen-Key filter section is more than approximately $C_1/400$ (0.25% of C_1), it must be considered for accurate filter response. You can use the **Capacitor** menu option to compensate for op amp input capacitance by simply adding the value of the op amp common-mode input capacitance to the actual value of C_1. The program then automatically

recalculates the exact or closest 1% resistor values for accurate filter response. No compensation for op amp input capacitance is required with MFB designs.

CAPACITOR SELECTION

Capacitor selection is very important for a high-performance filter. Capacitor behavior can vary significantly from ideal, introducing series resistance and inductance which limit Q. Also, nonlinearity of capacitance vs voltage causes distortion.

Common ceramic capacitors with high dielectric constants, such as "high-K" types, can cause errors in filter circuits. Recommended capacitor types are: NPO ceramic, silver mica, metallized polycarbonate; and, for temperatures up to 85°C, polypropylene or polystyrene.

OP AMP SELECTION

It is important to choose an op amp that can provide the necessary DC precision, noise, distortion, and speed.

OP AMP BANDWIDTH

In a low-pass filter section, maximum gain peaking is very nearly equal to Q at f_n (the section's natural frequency). So, as a rule of thumb:

For an MFB section: Op amp bandwidth should be at least $100 \cdot \text{GAIN} \cdot f_n$.
High-Q Sallen-Key sections require higher op amp bandwidth.
For a Sallen-Key section: For $Q > 1$, op amp gain-bandwidth should be at least $100 \cdot \text{GAIN} \cdot Q^3 \cdot f_n$. For $Q \leq 1$, op amp gain-bandwidth should be at least $100 \cdot \text{GAIN} \cdot f_n$.
For a real-pole section: Op amp bandwidth should be at least $50 \cdot f_n$.
Although Q is formally defined only for complex poles, it is convenient to use a Q of 0.5 for calculating the op amp gain required in a real-pole section.

For example, a unity-gain 20kHz 5-pole, 3dB ripple Chebyshev MFB filter with a 2nd pole-pair f_n of 19.35kHz and a Q of 8.82 needs an op amp with unity gain bandwidth of at least 17MHz. On the other hand, a 5-pole Butterworth MFB filter, with a worst case Q of 1.62 needs only a 3.2MHz op amp. The same 5-pole Butterworth filter implemented with a Sallen-Key topology would require a 8.5MHz op amp in the high-Q section.

USING THE f_n AND Q DISPLAY OPTION

To aid in selection of the op amp, a feature in the **Display** menu section allows you to view pole-pair section f_n and Q.

To use this feature move the cursor to the **Display** menu,

press **<ENTER>**, move the cursor to the **f_n & Q** selection, and press **<ENTER>** again. The f_n and Q information is also useful when trouble-shooting filters by comparing expected to actual response of individual filter sections.

OP AMP SLEW RATE

For adequate full-power response, the slew rate of the op amp must be greater than $\pi \cdot V_Op\text{-}p \cdot$ FILTER BANDWIDTH. For example, a 100kHz filter with 20Vp-p output requires an op amp slew rate of at least 6.3V/μs. Burr-Brown offers an excel-lent selection of op amps which can be used for high performance active filters. The guide on P-7 lists some good choices.

THE UAF42 UNIVERSAL ACTIVE FILTER

For other filter designs, consider the Burr-Brown UAF42 Universal Active Filter. It can easily be configured for a wide variety of low-pass, high-pass, band-pass, or band-reject (notch) filters. It uses the classical state-variable architecture with an inverting amplifier and two integrators to form a pole-pair. The integrators include on-chip 1000pF, ±0.5% capacitors. This solves one of the most difficult problems in active filter implementation—obtaining tight tolerance, low-loss capacitors at reasonable cost.

Simple design procedures for the UAF42 allow implementation of Butterworth, Chebyshev, Bessel, and other types of filters. An extra FET-input op amp in the UAF42 can be used to form additional stages or special filter types such as Inverse Chebyshev. The UAF42 is available in a standard 14-pin DIP. For more information, request the Burr-Brown Product Data Sheet PDS-1070 and Application Bulletin AB-035**.**

EXAMPLES OF MEASURED
MFB FILTER RESPONSE

Figures 10 and 11 show actual measured magnitude response plots for 5th-order 20kHz Butterworth, 3dB Chebyshev and Bessel filters designed with the program. The op amp used in all filters was the OPA627. As can be seen in Figure 10, the initial roll-off of the Chebyshev filter is fastest and the roll-off of the Bessel filter is the slowest. However, each of the 5th-order filters ultimately rolls off at $-N \cdot 20$dB/decade, where N is the filter order (-100dB/decade for a 5-pole filter).

The oscilloscope photographs (Figures 12-14) show the step response for each filter. As expected, the Chebyshev filter has the most ringing, while the Bessel has the least. Figure 15 shows distortion plots vs frequency for the three filters.

See Application Bulletin AB-017 for measured Sallen-Key filter performance of the same three designs.

OP AMP SELECTION GUIDE (In Order of Increasing Slew Rate)

$T_A = 25°C$, $V_S = ±15V$, specifications typ, unless otherwise noted, min/max specifications are for high-grade model.

OP AMP MODEL	BW typ (MHz)	FPR [1] typ (kHz)	SR typ (V/μs)	V_{OS} max (μV)	V_{OS}/dT max (μV/°C)	NOISE at 10kHz (nV/√Hz)	C_{CM} [3] (pF)
OPA177	0.6	3	0.2	10	±0.1	8	1
OPA27	8	30	1.9	25	±0.6	2.7	1
OPA2107 dual [2]	4.5	280	18	500	±5	8	4
OPA602 [2]	6	500	35	250	±2	12	3
OPA404 quad [2]	6	500	35	1000	±3 typ	12	3
OPA627 [2]	16	875	55	100	±0.8	4.5	7
OPA620 ($V_S = ±5V$)	300	16MHz(5Vp-p)	250	500	±8 typ	2.3 @ 1MHz	1

NOTES: (1) Unless otherwise noted, FPR is full power response at 20Vp-p as calculated from slew rate. (2) These op amps have FET inputs. (3) Common-mode input capacitance.

FIGURE 10. Gain vs Frequency for Fifth-Order 20kHz Butterworth, Chebyshev, and Bessel Unity-Gain MFB Low-Pass Filters, Showing Overall Filter Response.

FIGURE 11. Gain vs Frequency for Fifth-Order 20kHz Butterworth, Chebyshev, and Bessel Unity-Gain MFB Low-Pass Filters, Showing Transition-Band Detail.
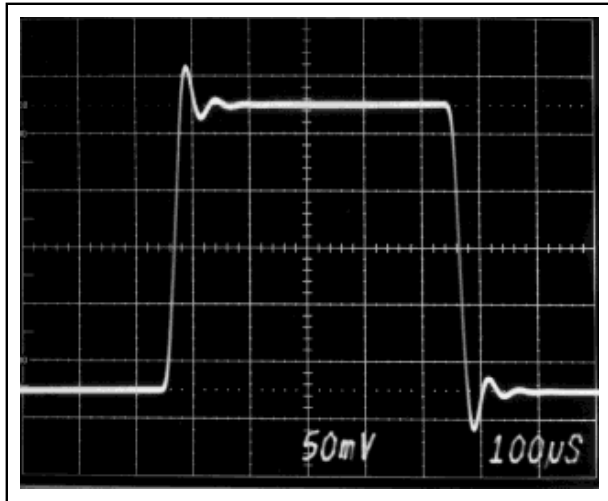
FIGURE 12. Step Response of Fifth-Order 20kHz Butterworth Low-Pass MFB Filter.
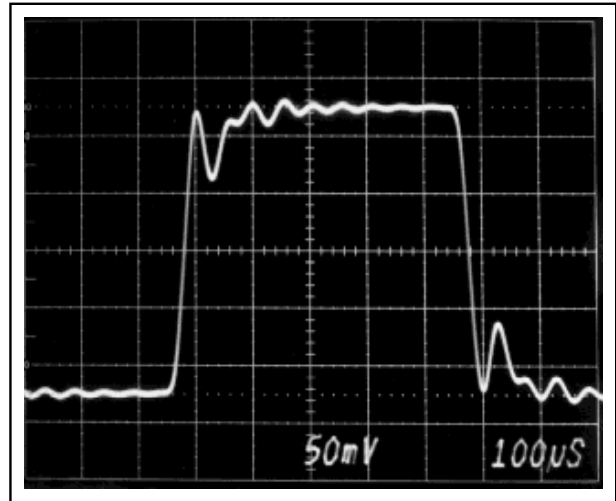

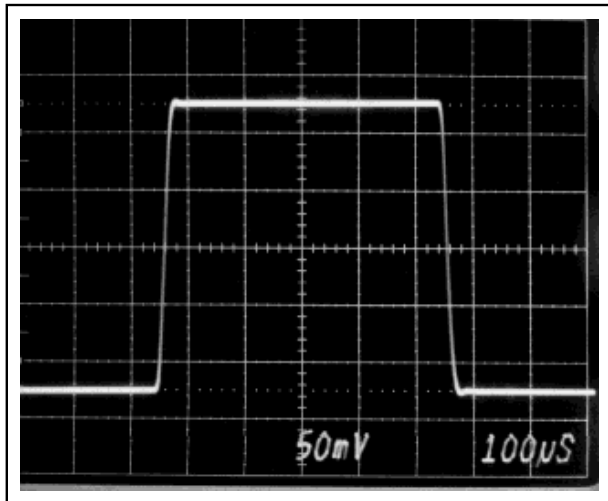FIGURE 13. Step Response of Fifth-Order 20kHz Chebyshev Low-Pass MFB Filter.

FIGURE 14. Step Response of Fifth-Order 20kHz Bessel Low-Pass MFB Filter.
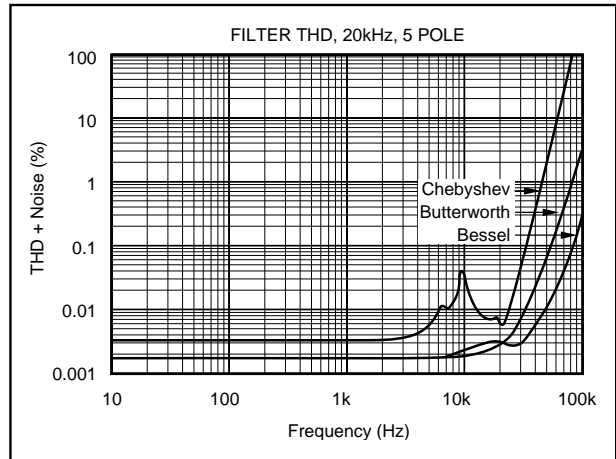

FIGURE 15. Measured Distortion for the Three 20kHz MFB Low-Pass Filters.