

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/228830905>

A Spherical Harmonic Approach to 3D Surround Sound Systems

Article · January 2005

CITATIONS

2

READS

1,206

1 author:



[Mark A Poletti](#)

82 PUBLICATIONS 2,324 CITATIONS

SEE PROFILE

A Spherical Harmonic Approach to 3D Surround Sound Systems

Mark Poletti

Industrial Research Limited, PO Box 31-310, Lower Hutt, New Zealand m.poletti@irl.cri.nz

This paper considers the recording and reproduction of three dimensional (3D) sound fields, based on spherical harmonic expansions of the field. It is shown that the plane wave description is insufficient for the description of fields with point sources which have wavefront curvature. The recording of a soundfield requires the measurement of the coefficients of the spherical harmonic expansion. The use of spherical and general arrays for recording the coefficients is discussed. The reproduction of the soundfield requires the resynthesis of the field using the spherical harmonic coefficients. It will be shown that there are two approaches to the determination of the speaker weights. The mode matching approach leads to a pseudo-inverse solution. The simple source approach is formally introduced, and it is shown that its application yields a matrix transpose approach. Computer simulations of soundfield synthesis are given to illustrate the two approaches.

Introduction

Surround sound systems provide the potential for reproducing soundfields over a substantial volume of space. Two dimensional systems can reproduce azimuthal sound fields [1,2], but the reproduction of 3D fields requires 3D recording microphone arrays and loudspeaker systems. A common framework for both recording and reproduction of 3D fields is spherical harmonics. The spherical harmonic description of sound fields is briefly reviewed here and two methods for recording sound fields are presented.

Two methods for 3D sound reproduction are the Kirchoff-Helmholtz and spherical harmonic approaches [3]. The Kirchoff-Helmholtz integral shows that a sound field inside a volume may be produced if the pressure and normal velocity on the surface of the volume are known. In practice, it has been shown that the velocity is sufficient, allowing the use of monopole speakers alone [4,5]. A second approach is spherical harmonics, in which the harmonics of a sound field are recorded and used to derive signals for an array of loudspeakers [3]. This approach is followed here. The simple source approach is introduced, and its spherical harmonic form derived. Comparisons are made with the mode matching approach [2]. Examples are given to illustrate the differences between these two methods.

1 Sound Field Description

1.1 Spherical harmonic expansion

For the interior case – where all sources lie outside the region of interest – the sound pressure at a single frequency can be expressed as [3]

$$p(r, \theta, \phi, k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n A_n^m(k) j_n(kr) Y_n^m(\theta, \phi) \quad 1$$

where k is the wavenumber, $j_n(\cdot)$ is the spherical Bessel function of the first kind and the spherical harmonics are defined as [6]

$$Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)(n-|m|)!}{4\pi(n+|m|)!}} P_n^{|m|}(\cos \theta) e^{im\phi} \quad 2$$

Each harmonic is the product of an elevation term and the azimuthal variation $\exp(im\theta)$ which appears in 2D ambisonics theory. In practice the sum is truncated to a maximum $n=N$, and there are $(N+1)^2$ terms in the expansion with $2n+1$ terms in m for each n .

For the exterior case the field outside a region containing sources can be expressed as

$$p(r, \theta, \phi, k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n B_n^m(k) h_n(kr) Y_n^m(\theta, \phi) \quad 3$$

where $h_n(kr)$ is the spherical Hankel function [3].

1.2 Plane and spherical wave expansions

The spherical harmonic expansion of the spatial variation of a plane wave arriving from angle of incidence (θ_i, ϕ_i) is

$$e^{i\vec{k}_i \cdot \vec{r}} = 4\pi \sum_{n=0}^{\infty} i^n j_n(kr) \sum_{m=-n}^n Y_n^m(\theta, \phi) Y_n^m(\theta_i, \phi_i)^* \quad 4$$

and that of a spherical source at $\vec{r}_s = (r_s, \theta_s, \phi_s)$ is

$$G(\vec{r}|\vec{r}_s) = \frac{e^{ik|\vec{r}-\vec{r}_s|}}{4\pi|\vec{r}-\vec{r}_s|} \quad 5$$

$$= ik \sum_{n=0}^{\infty} j_n(kr) h_n(kr_s) \sum_{m=-n}^n Y_n^m(\theta, \phi) Y_n^m(\theta_s, \phi_s)^*$$

We assume a harmonic dependency of $\exp(-i\alpha)$ so that equation 5 represents propagation outwards from r_s .

1.3 Fourier description

For a distribution of plane waves with complex amplitudes $Q(\theta, \phi)$, the Fourier transform of the field is only non-zero at a single spatial frequency k_0 . Q can be written in terms of its spherical harmonic expansion

$$Q(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^n q_n^m(k) Y_n^m(\theta, \phi) \quad 6$$

and the inverse Fourier transform in spherical coordinates becomes

$$p(r, \theta, \phi, k_0) = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^\pi Q(\theta_k, \phi_k) \sin(\theta_k) e^{ik_0 \vec{r} \cdot \vec{\theta}_k} d\theta_k d\phi_k \quad 7$$

Substituting equations 4 and 6 into 7 yields the same form as equation 1 with $A_n^m(k_0) = 4\pi i^n q_n^m(k_0)$, i.e. the coefficients of the plane wave field are simply related to those of the far field distribution. This is the 3D form of the expansion in [2].

If the soundfield includes point sources, the Fourier transform is nonzero for all k and the plane wave description is invalid. For example, the sound field produced by a single point source at \vec{r}_s generating a frequency f_0 is the solution to the Helmholtz equation

$$\nabla^2 p(\vec{r}) + k_0^2 p(\vec{r}) = -\delta(\vec{r} - \vec{r}_s) \quad 8$$

yielding

$$P(\vec{k}) = \frac{e^{-ik \cdot \vec{r}_s}}{k^2 - k_0^2} \quad 9$$

and the inverse Fourier transform yields equation 5. A general sound field requires an integral over all sources and all spatial frequencies are required to produce the divergence due to localized sources [3].

2 Soundfield Recording

A soundfield may be recorded by determining its spherical harmonic coefficients. This produces a set of spherical harmonic responses which include the first order ambisonics responses. Two approaches are reviewed here.

2.1 Sphere Decompositions

Multiplying equation 1 by $Y_l^q(\theta, \phi)^*$ and integrating over the sphere yields the (l, q) th response [7]

$$A_l^q(k) = \frac{1}{j_l(kr)} \int_0^{2\pi} \int_0^\pi p(r, \theta, \phi, k) Y_l^q(\theta, \phi)^* \sin(\theta) d\theta d\phi \quad 10$$

In a similar manner to the 2D case [2], this produces zeros where $j_l(kr) = 0$. One solution is to use outward-facing first order microphones (a weighted sum of pressure and acoustic-impedance-scaled radial velocity responses) which has the ideal form

$$s_\alpha(r, \theta, \phi, k) = \alpha p(r, \theta, \phi, k) - (1 - \alpha) \rho c v_R(r, \theta, \phi, k) \quad 11$$

producing the first order decomposition

$$s_\alpha(r, \theta, \phi, k) = \sum_{n=0}^{\infty} [\alpha j_n(kr) - i(1 - \alpha) j_n'(kr)] \sum_{m=-n}^n A_n^m(k) Y_n^m(\theta, \phi) \quad 12$$

in which case the coefficients are

$$A_l^q(k) = \frac{1}{D_l(kr)} \int_0^{2\pi} \int_0^\pi s_\alpha(r, \theta, \phi, k) Y_l^q(\theta, \phi)^* \sin(\theta) d\theta d\phi \quad 13$$

where the ‘mode’ amplitude is $D_l(kr) = \alpha j_l(kr) - i(1 - \alpha) j_l'(kr)$ which produces no zeros.

An alternative approach is to use pressure microphones mounted in a solid sphere [8]. The sound pressure produced by a solid sphere of radius a is the sum of the incident field (equation 1) and a scattered field described by equation 3. The sum of the two fields has the form [3]

$$p_i(r, \theta, \phi, k) = \sum_{n=0}^{\infty} \left[j_n(kr) - \frac{j_n'(ka)}{h_n'(ka)} h_n(kr) \right] \sum_{m=-n}^n A_n^m(k) Y_n^m(\theta, \phi) \quad 14$$

On the sphere surface $r=a$ and the term in square brackets can be simplified as

$$j_n(ka) - \frac{j_n'(ka)}{h_n'(ka)} h_n(ka) = \frac{1}{h_n'(ka)} \left[j_n(ka) h_n'(ka) - j_n'(ka) h_n(ka) \right] \\ = \frac{i}{(ka)^2} \frac{1}{h_n'(ka)} \quad 15$$

using the Wronskian expression [3]. Hence

$$A_l^q(k) = -i(ka)^2 h_l'(ka) \int_0^{2\pi} \int_0^\pi p_i(a, \theta, \phi, k) Y_l^q(\theta, \phi)^* \sin(\theta) d\theta d\phi \quad 16$$

The responses have no zeros, avoiding the need for first order microphones [8]. Any practical microphone array approximates equations 13 or 16 with a discrete sum, which produces aliasing spherical harmonics.

2.2 General sampling

A more general approach to the above is to use a sampling of the sound pressure field at L arbitrary positions. The field can be recreated at the same relative positions in a room using L speakers if the matrix of transfer functions from the speakers to those positions is known [9]. Alternatively, the method can be applied to the measurement of spherical harmonics. Expressing the pressure at L positions $p(r_b, \theta_b, \phi_b)$ as in equation 1, the vector of pressures, \mathbf{p} , can be written

$$\mathbf{p}(k) = \Lambda(k) \mathbf{A}(k) \quad 17$$

where \mathbf{A} is the vector of $K=(N+1)^2$ spherical harmonic coefficients to be determined and Λ is the matrix with elements $j_n(kr_l) Y_n^m(\theta_l, \phi_l)$ with row indexed by l and column indexed by n and m . Typically, $L > K$, the system is overdetermined, and the vector of coefficients can then be obtained as the regularized least squares inverse of equation 17 [10]

$$\mathbf{A} = [\Lambda^\dagger \Lambda + \lambda \mathbf{I}]^{-1} \Lambda^\dagger \mathbf{p} \quad 18$$

When the sample points are at the same radius, r_0 , then equation 17 can be written

$$\mathbf{p} = \mathbf{Y} \mathbf{J} \mathbf{A} \quad 19$$

where \mathbf{Y} is the matrix of spherical harmonic terms and \mathbf{J} a diagonal matrix with elements $j_n(kr_0)$. If the positions are those of a regular polyhedron and $L=K$, then \mathbf{Y} is a unitary matrix, and

$$\mathbf{A} = \mathbf{J}^{-1} \mathbf{Y}^\dagger \mathbf{p} \quad 20$$

which is the discrete matrix form of equation 10. Hence the general approach includes the open sphere approach as a special case. It can also accommodate directional element responses [10].

3 Soundfield Synthesis and Reproduction

To reconstruct a sound field, a spherical array of L loudspeakers at positions (R, θ_l, ϕ_l) in a free field is assumed. Each speaker produces a field which has a spherical harmonic expansion, and the sum of these must equal the expansion of the desired field in a region of space near the origin. We describe two approaches.

3.1 Mode matching approach

Consider first the generation of a single plane wave [11,12]. If it is assumed that R is large then each speaker produces a plane wave near the origin. If the amplitude of each speaker signal is w_l , the synthesised field is a sum of L plane waves with expansions given by equation 4. This must equal the expansion of the desired plane wave arriving from (θ_s, ϕ_s) , yielding

$$\sum_{l=1}^L w_l(\theta_s, \phi_s) Y_n^m(\theta_l, \phi_l)^* = Y_n^m(\theta_s, \phi_s)^* \quad 21$$

This must be solved for the required weights w_l . At low frequencies the weights are the panning functions for plane wave synthesis for the particular speaker layout [1].

If the plane wave approximation for the speaker fields is replaced by a spherical point source approximation then matching the L spherical wave expansions (equation 5) to that of the plane wave yields [12]

$$\sum_{l=1}^L w_l(\theta_s, \phi_s) Y_n^m(\theta_l, \phi_l)^* = \frac{4\pi i^n}{ik h_n(kR)} Y_n^m(\theta_s, \phi_s)^* \quad 22$$

The panning function solutions include the Hankel function for the speaker sources.

If the desired field is due to a point source at (r_s, θ_s, ϕ_s) , then the mode matching equation is [13]

$$\sum_{l=1}^L w_l(r_s, \theta_s, \phi_s) Y_n^m(\theta_l, \phi_l)^* = \frac{h_n(kr_s)}{h_n(kR)} Y_n^m(\theta_s, \phi_s)^* \quad 23$$

In this case the panning functions are different for each source radius. Finally, for an arbitrary field with coefficients $A_n^m(k)$ the mode matching equation is

$$\sum_{l=1}^L w_l Y_n^m(\theta_l, \phi_l)^* = \frac{A_n^m(k)}{ik h_n(kR)} \quad 24$$

For a finite N th order expansion this can be written

$$\Psi \mathbf{w} = \mathbf{d} \quad 25$$

where Ψ is a $K=(N+1)^2$ by L matrix of spherical harmonics and \mathbf{d} is the vector containing the terms on the right hand side of equation 24

$$\mathbf{d} = \mathbf{H}^{-1} \mathbf{A} \quad 26$$

where \mathbf{H} is a diagonal matrix with entries $ik h_n(kR)$ appearing $2n+1$ times. Ψ is the reproduction equivalent of the general sampling matrix Λ in equation 17.

If $K < L$ the minimum energy solution is [12]

$$\mathbf{w} = \Psi^\dagger [\Psi \Psi^\dagger]^{-1} \mathbf{H}^{-1} \mathbf{A} \quad 27$$

If $K=L$ then

$$\mathbf{w} = \Psi^{-1} \mathbf{H}^{-1} \mathbf{A} \quad 28$$

and if $K \geq L$ the regularised least squared error solution is

$$\mathbf{w} = [\Psi^\dagger \Psi + \lambda \mathbf{I}]^{-1} \Psi^\dagger \mathbf{H}^{-1} \mathbf{A} \quad 29$$

In practice the lowest errors were found when $K=L$. Equation 29 was used as it allowed control of the error. (In practical systems, however, there may be less recorded modes than speakers.)

3.2 Simple Source Approach

In [3] it is shown that there are alternatives to the Kirchhoff-Helmholtz integral which do not require both pressure and normal velocity on the surface to be known. One of these is the simple source approach, for which the field is obtained from a distribution of monopole sources over a surface S as

$$p(r, \theta, \phi, k) = \int_0^{2\pi} \int_0^\pi \mu(\vec{r}_s) \frac{e^{ik|\vec{r}-\vec{r}_s|}}{4\pi|\vec{r}-\vec{r}_s|} \sin(\theta) d\theta d\phi \quad 30$$

The required simple source distribution is given by [3]

$$\mu(\vec{r}_s) = \frac{\delta p_o(\vec{r})}{\delta n} - \frac{\delta p_i(\vec{r})}{\delta n} \quad 31$$

where n is the inward facing normal, $p_o(\vec{r})$ is the exterior field produced by a source distribution confined in S and $p_i(\vec{r})$ is the interior field produced by a source distribution outside S , with the condition that the two fields are equal on the surface. Equating the general interior and exterior expansions on a sphere of radius R yields

$$B_n^m(k) = \frac{j_n(kR)}{h_n(kR)} A_n^m(k) \quad 32$$

Substituting the two expansions into 31, and employing the Wronskian relation [3] yields the simple source solution

$$\mu(R, \theta_s, \phi_s) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{A_n^m(k)}{ikR^2 h_n(kR)} Y_n^m(\theta_s, \phi_s) \quad 33$$

In practice L speakers and a finite order expansion is used, and the l th speaker weighting is

$$w_l = \frac{g_l}{ik} \sum_{n=0}^L \sum_{m=-n}^n \frac{A_n^m(k)}{h_n(kR)} Y_n^m(\theta_l, \phi_l) \quad 34$$

where g_l is a weighting term associated with the discrete approximation to the integral (which depends on the speaker geometry – see for example [14]). The solution can be written in matrix form as

$$\mathbf{w} = \mathbf{G} \Psi^\dagger \mathbf{H}^{-1} \mathbf{A} \quad 35$$

where \mathbf{G} is the diagonal matrix of weights g_l . Equation 35 contains the transpose of the mode matrix, as opposed to the inverse forms in equations 27-29 and is

therefore robust to small singular values. Equations 28 and 35 are identical when $K=L$ and the loudspeaker geometry regular. The mode matrix is then unitary and $\mathbf{G}=\mathbf{I}$ [4]. However the solution in equation 35 is also valid for non-regular geometries.

Errors in the simple source solution can be controlled by applying a windowing function $\Omega_n^m(k)$ to the spherical harmonics, as in the 2D case [2]. Letting Ω be the diagonal matrix of weights the matrix solution is

$$\mathbf{w} = \mathbf{G} \Psi^\dagger \Omega \mathbf{H}^{-1} \mathbf{A} \quad 36$$

A simple window is $\Omega_n^m = W_1(n) W_2(m)$ where W_1 is one-sided and W_2 is a standard window. An exponential window $W_1 = \exp(-\mathcal{M}/N)$ and Kaiser window in m will be used in the example below.

3.3 Reproduction error

A useful measure of the error in the reproduced field is the normalised radial error, which is the magnitude of the difference between the actual and ideal fields at radius r integrated over all angles [1,12];

$$\bar{\varepsilon}(r) = \frac{\int_0^{2\pi} \int_0^\pi |p(r, \theta, \phi) - \hat{p}(r, \theta, \phi)|^2 \sin(\theta) d\theta d\phi}{\int_0^{2\pi} \int_0^\pi |p(r, \theta, \phi)|^2 \sin(\theta) d\theta d\phi} \quad 37$$

A useful lower bound to the error is the truncation error due to limiting the spherical harmonic expansion of the field to a maximum of $n=N$. The plane wave truncation error is given in [12]. For a spherical point source the truncation error is

$$\bar{\varepsilon}_T(r) = 1 - \frac{\sum_{n=0}^N (2n+1) j_n^2(kr) |h_n(kr_s)|^2}{\sum_{n=0}^{\infty} (2n+1) j_n^2(kr) |h_n(kr_s)|^2} \quad 38$$

This assumes that only spherical harmonics up to order N are created by the speaker array and ignores alias terms.

4 Examples

We present a simulation example for both mode matched and simple source cases. The number of speakers required for plane wave synthesis at a given frequency and radius is approximately $L \geq ([kr] + 1)^2$ where $[.]$ denotes the next highest integer [12]. For example for a reproduction area of radius 0.1m and a frequency of 4.5 kHz L must exceed 100 speakers. We assume $L=100$ speakers at a radius of 2m, which allows the use of spherical harmonics up

to order 9. The speaker angles and weights were obtained from [14].

The radial errors are shown in figure 1 for a point source at radius 2.5m on the x axis.

For small kr , the mode match error reduces to zero, since the number of speakers is sufficient to represent all significant spherical harmonics. For kr above 10 the mode match approach cannot match the higher modes which are required. Regularisation reduces this error at the expense of increased error for small kr .

The simple source error is higher than the mode match error at small kr , but is lower at large kr . However, the windowed simple source solution produces the lowest error at large kr and is smaller than the mode match error at small kr . Reducing the mode match regularization to equal the simple source performance at low kr increases the error further at large kr . Typically, the simple source solution is better performing than the regularized mode match solution.

The field reproduction errors for the regularized mode match and windowed simple source cases are shown in figures 2 and 3. The upper plots show the speaker weight magnitudes, sorted by azimuthal angle. The lower plots show the real part of the field in x .

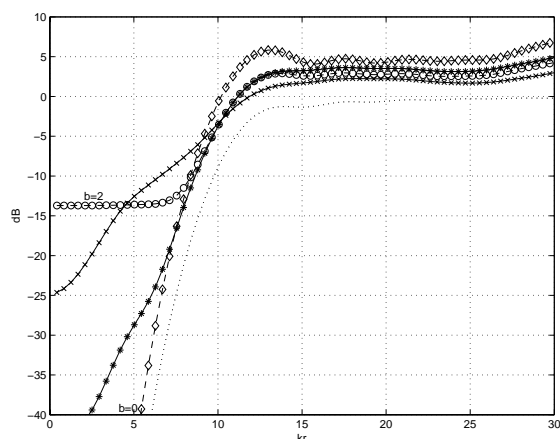


Figure 1: Simple source (*), windowed simple source (x), mode match (\diamond) and regularized mode match (o) errors for an array of 100 loudspeakers at radius 2 m, source at 2.5 metres. The truncation error (dotted) is also shown

Both solutions produce the largest speaker amplitudes for angles near the desired source direction. The effect of regularization on the mode match solution is to decrease the magnitudes of speaker amplitudes at other angles, which reduces interference effects.

The windowed simple source solution produces lower speaker magnitudes than the mode match solution at angles away from the desired direction, and maintains a more accurate field at the center of the array.

Finally, as has been demonstrated in the 2D case [4], sources can be synthesized within the array of speakers. Figure 4 shows the field produced in the (x,y) plane using the simple source approach for a spherical source on the x axis at 1.2m, using an array with 256 speakers. The pressure for radii less than 1.2m is correctly reproduced, but the error is large for radii between 1.2 and the speaker radius. Figure 5 shows that a greater number of speakers must be activated to generate the field, and the error is larger for radii greater than 1.2m. Since the field amplitudes can become very large near the speakers high power amplifier and speaker dissipations would be required to generate internal sources.

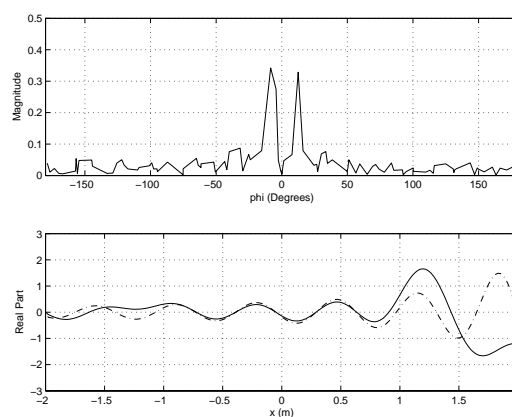


Figure 2: Mode match solution with regularisation, Upper: speaker weight magnitudes, lower: Actual (--) and synthesised (-) field on x axis

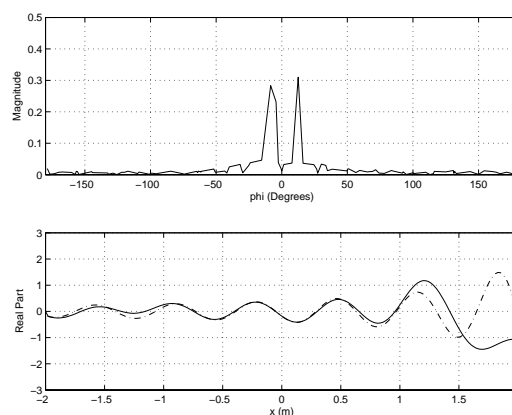


Figure 3: Simple source solution with windowing Upper: speaker weight magnitudes, lower: Actual (--) and synthesised (-) field on x axis

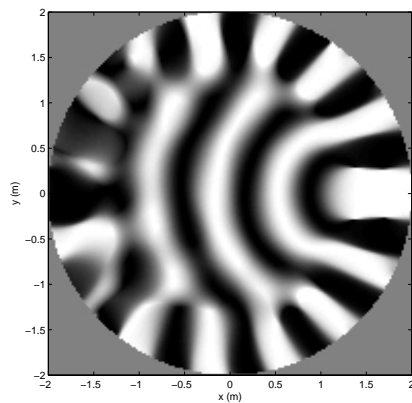


Figure 4: Simple source synthesis of a source inside the array at 1.2 m

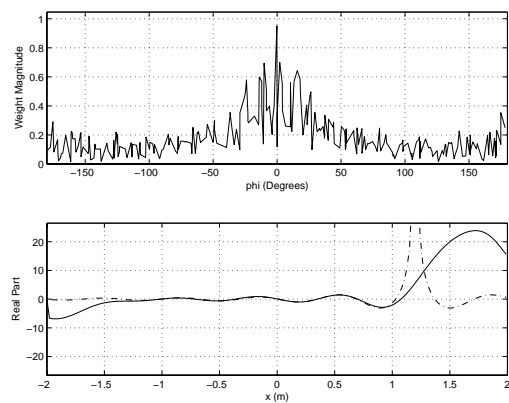


Figure 5: Speaker magnitudes and field for a source at 1.2 m, Upper: speaker weight magnitudes, lower: Actual(--) and synthesised (-) field on x axis

5 Summary

The spherical harmonic theory of sound recording and reproduction has been examined. Two methods for sound recording have been reviewed and two approaches to sound synthesis and reproduction considered in more detail, with synthesis simulations given. It has been shown that the spherical harmonic approach provides an alternative general foundation to sound reproduction to the Kirchoff-Helmholtz approach. The simple source method is a useful alternative to mode matching, and windowing of the spherical harmonic components allows for control of interference in the field while maintaining accuracy at the center of the array.

Both recording and reproduction of 3D sound fields require large numbers of transducers, making practical feasibility an issue, although simplifications are possible for particular reproduction systems [15]. Furthermore, the systems described here do not compensate for the semi-reverberant fields encountered in most listening rooms, which requires sounding and calibration for the desired listening positions.

References

- [1] M. A. Poletti, "The design of encoding functions for stereophonic and polyphonic sound systems," *Journal of the Audio Engineering Society*, vol. 44, no. 11, pp 948-963, November 1996
- [2] M. A. Poletti, "A Unified Theory of Horizontal Holographic Sound Systems," *Journal of the Audio Engineering Society*, vol. 48, no. 12, pp 1155-1182, December 2000
- [3] E. G. Williams, *Fourier Acoustics*, Academic Press, 1999
- [4] J. Daniel, R. Nicol and S. Moreau, "Further investigations of high order ambisonics and wavefield synthesis for holophonic sound imaging," *114th AES convention*, preprint 5788
- [5] R. Nicol and M. Emerit, "3D-sound reproduction over an extensive listening area: A hybrid method derived from Holophony and Ambisonic," *AES 16th International Conference*, Finland 1999
- [6] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Applied Mathematical Sciences Vol. 93, 2nd Edition, Springer, 1998
- [7] T. D. Abhayapala and D. B. Ward, "Theory and design of high order sound field microphones using spherical microphone array," *IEEE International Conference on Acoustics, Speech, and Signal Processing*, 2002, volume 2, 2002, pages 1949-1952
- [8] J. Meyer, "Beamforming for a circular microphone array mounted on spherically shaped objects," *J.*

Acoust. Soc. Am., vol. 109, no. 1, pp 185-193, 2001

- [9] P. A. Nelson, F. Orduna-Bustamente and H. Hamada, "Multichannel signal processing techniques in the reproduction of sound," *J. Audio Eng. Soc.* Vol. 44, No. 11, pp 973-989, 1996
- [10] A. Laborie, R. Bruno and S. Montoya, "A new comprehensive approach of surround sound recording," *114th AES convention*, preprint 5717, Amsterdam, March 2003
- [11] O. Kirkeby and P. A. Nelson, "Reproduction of plane wave sound fields," *J. Acoust. Soc. Am.*, vol. 94, no. 5, pp 2992 – 3000, 1993
- [12] D. B. Ward and T. D. Abhayapala, "Reproduction of a plane-wave sound field using an array of loudspeakers," *IEEE Trans. Speech and Audio Processing*, vol. 9, no. 6, pp 697 – 707, 2001
- [13] J. Daniel, "Spatial sound encoding including near field effect: Introducing distance coding filters and a viable new ambisonics format," *AES 23rd Int. Convention*, Copenhagen, May 2003.
- [14] J. Fliege, Integration Nodes for the sphere, www.mathematik.uni-dortmund.de/lx/research/projects/fliege/nodes/nodes.html
- [15] A. Laborie, R. Bruno and S. Montoya, "Designing high spatial resolution microphones," *117th AES convention*, preprint 6231, San Francisco, October 2004