5.17. Horns.—A horn is an acoustical transducer consisting of a tube of varying sectional area. Horns have been used widely for centuries for increasing the radiation from a sound source. The principal virtue of a horn resides in the possibility of presenting practically any value of acoustical impedance to the sound generator. This feature is extremely valuable for obtaining maximum overall efficiency in the design of an acoustical system. As an example, in a horn loudspeaker high efficiency is obtained by designing the system so that the driving force works against resistance instead of inertia of the diaphragm. Employing suitable combination of horns, directional characteristics which are independent of frequency, as well as practically any type of directional pattern, may be obtained. The combination of high efficiency and the possibility of any directional pattern makes the horn loudspeaker particularly suitable for larger scale sound reproduction. It is the purpose of this section to consider some of the factors which influence the characteristics of a horn.

5.18. Fundamental Horn Equation.[20,21,22,23,24,25,26,27,28,29,30,31]—Consider a tube with a certain rate of flare and with the diameter small compared to the wavelength of the sound passing through it. Let the axis of the tube coincide with the x axis. Take an element of volume of the tube defined as

$$S\Delta x \tag{5.32}$$

where S = cross-sectional area of the tube at x, and

Δx = length of the element of volume.

The growth of matter in this volume is the difference between the influx and efflux of fluid through the faces and may be expressed as

$$\Delta x \frac{\partial(S\rho' u)}{\partial x} \tag{5.33}$$

where u = component of the particle velocity along the axis, and

ρ' = density of the medium.

[20] Webster, A. G., *Jour. Natl. Acad. Sci.*, Vol. 5, p. 275, 1919.

[21] Stewart, G. W., *Phys. Rev.*, Vol. 16, No. 4, p. 313, 1920.

[22] Goldsmith and Minton, *Proc. Inst. Rad. Eng.*, Vol. 12, No. 4, p. 423, 1924.

[23] Slepian and Hanna, *Jour. Amer. Inst. Elec. Eng.*, Vol. 43, p. 393, 1924.

[24] Ballantine, G., *Jour. Frank. Inst.*, Vol. 203, No. 1, p. 85, 1927.

[25] Crandall, " Vibrating Systems and Sound," D. Van Nostrand Company, Princeton, N.J., 1926.

[26] Stewart and Lindsay, " Acoustics," D. Van Nostrand Company, Princeton, N.J., 1930.

[27] Olson and Massa, " Applied Acoustics," P. Blakiston's Son and Company, Philadelphia, Pa., 1934.

[28] Mawardi, Osman K., *Jour. Acous. Soc. Amer.*, Vol. 21, No. 4, p. 323, 1949.

[29] Lambert, Robert F., *Jour. Acous. Soc. Amer.*, Vol. 26, No. 4, p. 1024, 1954.

[30] Jensen and Lambert, *Jour. Acous. Soc. Amer.*, Vol. 26, No. 4, p. 1029, 1954.

[31] Scibor-Marchoki, R. I., *Jour. Acous. Soc. Amer.*, Vol. 27, No. 5, p. 939, 1955.

The principle of continuity was expressed in Sec. 1.3. Applying the principle, the difference between the influx and efflux of the fluid into the element of volume must be equal to the time rate of growth of mass.

$$\frac{\partial \rho'}{\partial t} S \Delta x = -\Delta x \frac{\partial (S\rho' u)}{\partial x} \qquad 5.34$$

or

$$S\frac{\partial \rho'}{\partial t} + \frac{\partial (S\rho' u)}{\partial x} = 0 \qquad 5.35$$

From equations 1.19 and 1.6

$$-\rho \ddot{\phi} = c^2 \cdot \dot{\rho}' \qquad 5.36$$

From equation 1.11

$$u = \frac{\partial \phi}{\partial x} \qquad 5.37$$

Substituting equations 5.36 and 5.37 in 5.35 the result may be written as

$$\ddot{\phi} - c^2 \frac{\partial \phi}{\partial x} \frac{\partial}{\partial x}(\log S) - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \qquad 5.38$$

Equation 5.38 is the wave equation for the axial motion in a tube of varying section.

5.19. Infinite Cylindrical Horn (Infinite Pipe).—The equation expressing the cross-sectional area as a function of the distance along the axis is

$$S = S_1 \qquad 5.39$$

where S_1 = cross section of the pipe, in square centimeters.

The general horn equation for the infinite pipe from equations 5.38 and 5.39 is

$$\ddot{\phi} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \qquad 5.40$$

The velocity potential, pressure, and volume current are

$$\phi = A\epsilon^{jk(ct-x)} \qquad 5.41$$

$$p = kc\rho A\epsilon^{jk(ct-x)} \qquad 5.42$$

$$U = S_1 kA\epsilon^{jk(ct-x)} \qquad 5.43$$

where $k = 2\pi/\lambda$,

λ = wavelength, in centimeters, and

ρ = density of the medium, in grams per cubic centimeter.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat or input end of the pipe are

$$r_A = \frac{\rho c}{S_1} \qquad 5.44$$

$$x_A = 0 \qquad 5.45$$

5.20. Infinite Parabolic Horn.[32]—The equation expressing the cross-sectional area as a function of the distance along the axis is

$$S = S_1 x \tag{5.46}$$

The general horn equation for the parabolic horn is

$$\ddot{\phi} - \frac{c^2}{x}\frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \tag{5.47}$$

The velocity potential, pressure, and volume current are

$$\phi = A\,[J_0(kx) - jY_0(kx)]\,\epsilon^{j\omega t} \tag{5.48}$$

$$p = -j\omega\rho A\,[J_0(kx) - jY_0(kx)]\,\epsilon^{j\omega t} \tag{5.49}$$

$$U = ASk\,[-J_0'(kx) + jY_0'(kx)]\,\epsilon^{j\omega t} \tag{5.50}$$

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1}\frac{2}{\pi k x_1[J_1^2(kx_1) + Y_1^2(kx_1)]} \tag{5.51}$$

$$x_A = \frac{\rho c}{S_1}\frac{J_0(kx_1)J_1(kx_1) + Y_0(kx_1)Y_1(kx_1)}{J_1^2(kx_1) + Y_1^2(kx_1)} \tag{5.52}$$

where J_0, J_1 = Bessel functions of the first kind of order zero and one,
Y_0, Y_1 = Bessel functions[33] of the second kind of order zero and one,
ρ = density of the medium, in grams per cubic centimeter,
c = velocity of sound, in centimeters,
S_1 = area at x_1, in square centimeters,
x_1 = distance of the throat from $x = 0$, in centimeters,
$k = 2\pi/\lambda$, and
λ = wavelength, in centimeters.

5.21. Infinite Conical Horn.—The equation expressing the cross-sectional area as a function of the distance along the axis is,

$$S = S_1 x^2 \tag{5.53}$$

The general horn equation for the conical horn is

$$\ddot{\phi} - \frac{2c^2}{x}\frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \tag{5.54}$$

The velocity potential, pressure, and volume current are

$$\phi = \frac{A}{x}\epsilon^{j(\omega t - kx)} \tag{5.55}$$

$$p = -\frac{j\omega\rho A}{x}\epsilon^{j(\omega t - kx)} \tag{5.56}$$

$$U = -\frac{AS(1 + jkx)\epsilon^{j(\omega t - kx)}}{x^2} \tag{5.57}$$

[32] Olson and Wolff, *Jour. Acous. Soc. Amer.*, Vol. 1, No. 3, p. 410, 1930.
[33] Jahnke and Emde, "Tables of Functions," Tuebner, Berlin, 1928.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \frac{k^2 x_1{}^2}{1 + k^2 x_1{}^2} \qquad 5.58$$

$$x_A = \frac{\rho c}{S_1} \frac{k x_1}{1 + k^2 x_1{}^2} \qquad 5.59$$

where S_1 = area at x_1, in square centimeters,
x_1 = distance of throat from $x = 0$, in centimeters,
$k = 2\pi/\lambda$, and
λ = wavelength, in centimeters.

5.22. Infinite Exponential Horn.—The equation expressing the cross-sectional area as a function of the distance along the axis

$$S = S_1 \epsilon^{mx} \qquad 5.60$$

where S_1 = area at the throat, that is, at $x = 0$, and
m = flaring constant.

The general horn equation for the exponential horn is

$$\ddot{\phi} - c^2 m \frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \qquad 5.61$$

The velocity potential, pressure, and volume current are

$$\phi = \epsilon^{-(m/2)x} \left[A \epsilon^{-j \frac{\sqrt{4k^2 - m^2}}{2} x} \right] \epsilon^{j\omega t} \qquad 5.62$$

$$p = -j\omega\rho \epsilon^{-(m/2)x} \left[A \epsilon^{-j \frac{\sqrt{4k^2 - m^2}}{2} x} \right] \epsilon^{j\omega t} \qquad 5.63$$

$$U = -AS \left[\frac{m}{2} + j \frac{\sqrt{4k^2 - m^2}}{2} \right] \epsilon^{-\frac{m}{2}x - j\frac{\sqrt{4k^2 - m^2}}{2}x + j\omega t} \qquad 5.64$$

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \sqrt{1 - \frac{m^2}{4k^2}} \qquad 5.65$$

$$x_A = \frac{\rho c}{S_1} \frac{m}{2k} \qquad 5.66$$

When $m = 2k$ or $4\pi f = mc$ the acoustical resistance is zero. This is termed the cutoff frequency of the exponential horn.

Below the cutoff frequency the acoustical impedance is entirely reactive and

$$x_A = \frac{\rho c}{S_1} \left(\frac{m}{2k} - \sqrt{1 - \frac{m^2}{4k^2}} \right) \qquad 5.67$$

5.23. Infinite Hyperbolic Horn.[34]—The equation expressing the cross-sectional area along the axis is

$$S = S_1 (\cosh \alpha + T \sinh \alpha)^2 \qquad 5.68$$

where T = family parameter, in the hyperbolic horn $T < 1$,
$\alpha = x/x_0$, dimensionless axial distance,
x = axial distance from the throat, in centimeters,
x_0 = reference axial distance, in centimeters, and
S_1 = area at the throat, in square centimeters, that is, at $x = 0$.

The expressions for the velocity potential, pressure, and volume current are quite complex and will not be considered.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \frac{\sqrt{1 - \frac{1}{\mu}}}{1 - \frac{1 - T^2}{\mu^2}} \qquad 5.69$$

$$x_A = \frac{\rho c}{S_1} \frac{\frac{T}{\mu}}{1 - \frac{1 - T^2}{\mu^2}} \qquad 5.70$$

where $\mu = kx_0 = f/f_0$,
$k = 2\pi/\lambda$,
f_0 = cutoff frequency, and
f = frequency under consideration.

Below the cutoff frequency, $\mu = 1$, the acoustical impedance is entirely reactive and

$$x_A = \frac{\rho c}{S_1} \frac{\frac{T}{\mu} - \sqrt{\frac{1}{\mu^2} - 1}}{1 - \frac{1 - T^2}{\mu^2}} \qquad 5.71$$

5.24. Throat Acoustical Impedance Characteristic of Infinite Parabolic, Conical, Exponential, Hyperbolic, and Cylindrical Horns.—The throat acoustical impedance of infinite horns may be computed from the equations of Secs. 5.18. 5.19, 5.20, 5.21, 5.22, and 5.23. In order to compare the throat acoustical impedance characteristics of infinite parabolic, conical, exponential, hyperbolic, and cylindrical horns, a specific example has been selected in which the throat area is the same in all horns. In

[34] Salmon, V., *Jour. Acous. Soc. Amer.*, Vol. 17, No. 3, p. 212, 1946.

addition, the area at a distance of 100 centimeters from the throat is the same for the four horns with flare, as shown in Fig. 5.5. The value of T for the hyperbolic horn is .5. The acoustical resistance and acoustical reactance frequency characteristics for the five horns are shown in Fig. 5.5.

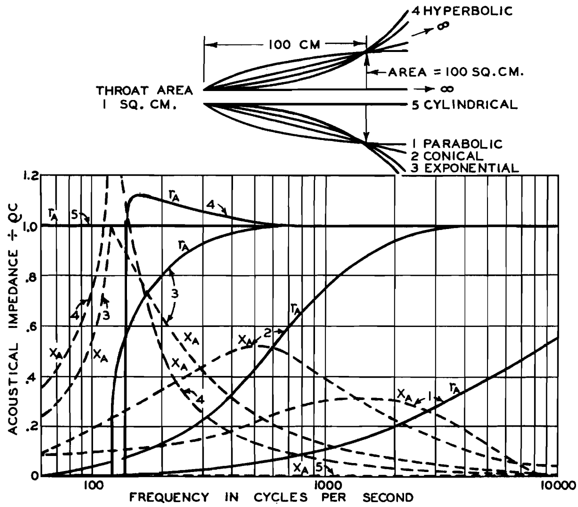

FIG. 5.5. Throat acoustical resistance r_A, and acoustical reactance x_A, frequency characteristics of infinite parabolic, conical, exponential, hyperbolic, and cylindrical horns having a throat area of 1 square centimeter. The cross-sectional area of the parabolic, conical, exponential, and hyperbolic horns is 100 square centimeters at a distance of 100 centimeters from the throat.

5.25. Finite Cylindrical Horn.—The acoustical impedance, in acoustical ohms, at the throat of the finite cylindrical horn of Fig. 5.6 is

$$z_{A1} = \frac{p_1}{U_1} \qquad 5.72$$

where p_1 = pressure at the throat, in dynes per square centimeter, and
U_1 = volume current, in cubic centimeters per second.

The acoustical impedance, in acoustical ohms, at the mouth of a cylindrical horn is

$$z_{A2} = \frac{p_2}{U_2} \qquad 5.73$$

where p_2 = pressure at the mouth, in dynes per square centimeter, and
U_2 = volume current, in cubic centimeters per second.

From equations 5.52 and 5.53 the expressions for the pressures and volume currents at the throat and mouth are given by

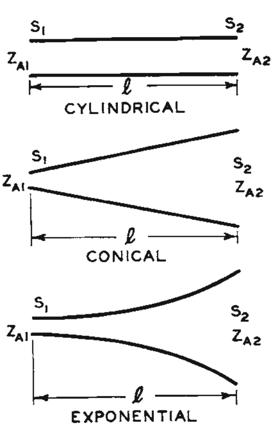

FIG. 5.6. Finite cylindrical, conical, and exponential horns. z_{A1} = input acoustical impedance at the throat. S_1 = cross-sectional area at the throat, in square centimeters. x_{A2} = terminating acoustical impedance at the throat. S_2 = cross-sectional area at the mouth, in square centimeters. l = length, in centimeters.

At $x = 0$,
$$p_1 = kc\rho A\epsilon^{jkct} \qquad 5.74$$
$$U_1 = S_1kA\epsilon^{jkct} \qquad 5.75$$
At $x = l$,
$$p_2 = kc\rho A\epsilon^{jk(ct-l)} \qquad 5.76$$
$$U_2 = S_1kA\epsilon^{jk(ct-l)} \qquad 5.77$$

From equations 5.72, 5.73, 5.74, 5.75, 5.76, and 5.77 the expression for the acoustical impedance, z_{A1}, at the throat in terms of the length and cross-sectional area of the horn and the acoustical impedance, z_{A2}, at the mouth is

$$z_{A1} = \frac{\rho c}{S_1}\left(\frac{S_1z_{A2}\cos(kl) + j\rho c\sin(kl)}{jS_1z_{A2}\sin(kl + \rho c\cos(kl)}\right) \qquad 5.78$$

where ρ = density of the medium, in grams per cubic centimeter,
$k = 2\pi/\lambda$,
λ = wavelength, in centimeters,
c = velocity of sound, in centimeters per second,
S_1 = cross-sectional area of the pipe, in square centimeters,
l = length of the pipe, in centimeters, and
z_{A2} = acoustical impedance at the mouth, in acoustical ohms.

The throat acoustical impedance characteristics of a finite cylindrical horn or pipe are shown in Fig. 5.7. The mouth acoustical impedance is assumed to be the same as that of a piston in an infinite baffle. In this case the mouth acoustical impedance, z_{A2}, is given by equation 5.12. It will be seen that the variations in the acoustical resistance and acoustical reactance components are quite large at the low frequencies where the mouth acoustical resistance is small.

5.26. Finite Conical Horn.—The acoustical impedance at the throat of a finite conical horn of Fig. 5.8 may be obtained in a manner similar to the procedure for the finite cylindrical horn in the preceding section by employing the equations for the pressure and velocity in an infinite conical horn and applying the proper boundary conditions. The expression for the acoustical impedance, z_{A1}, at the throat in terms of the dimensions of the horn and the acoustical impedance, z_{A2}, at the mouth is

$$z_{A1} = \frac{\rho c}{S_1}\left[\frac{jz_{A2}\dfrac{\sin k(l-\theta_2)}{\sin k\theta_2} + \dfrac{\rho c}{S_2}\sin kl}{z_{A2}\dfrac{\sin k(l+\theta_1-\theta_2)}{\sin k\theta_1 \sin k\theta_2} - \dfrac{j\rho c}{S_2}\dfrac{\sin k(l+\theta_1)}{\sin k\theta_1}}\right] \qquad 5.79$$

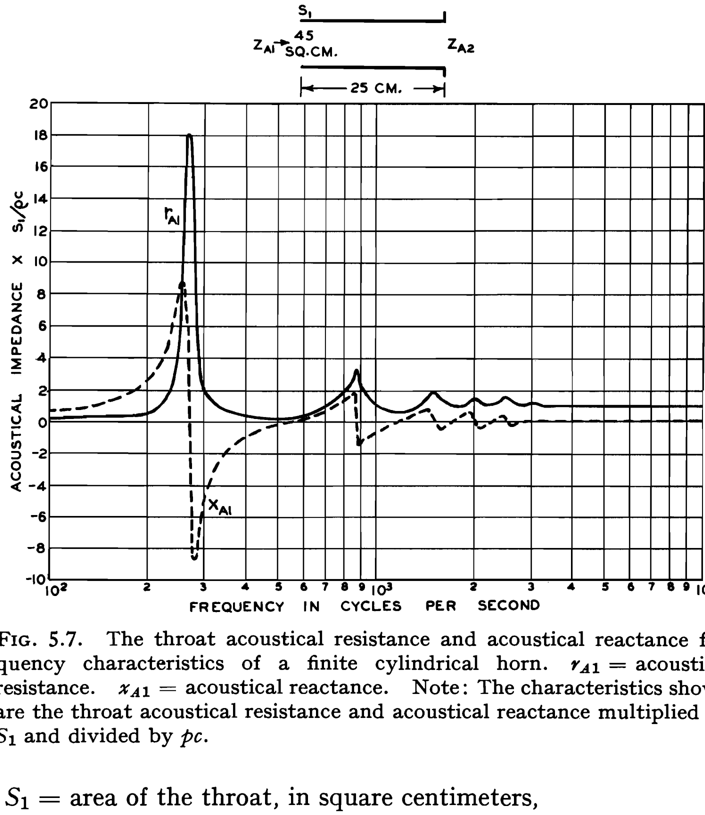

FIG. 5.7. The throat acoustical resistance and acoustical reactance frequency characteristics of a finite cylindrical horn. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance and acoustical reactance multiplied by S_1 and divided by ρc.

where S_1 = area of the throat, in square centimeters,

S_2 = area of the mouth, in square centimeters,

l = length of the horn, in centimeters,

$k\theta_1 = \tan^{-1} kx_1$,

$k\theta_2 = \tan^{-1} kx_2$

x_1 = distance from the apex to the throat, in centimeters,

x_2 = distance from the apex to the mouth, in centimeters,

$k = 2\pi/\lambda$,

λ = wavelength, in centimeters,

c = velocity of sound, in centimeters per second,

ρ = density of air in grams per cubic centimeter,

z_{A2} = acoustical impedance at the mouth, in acoustical ohms.

The throat acoustical impedance characteristics of a finite conical horn are shown in Fig. 5.8. The acoustical impedance at the mouth of the horn is usually assumed to be the same as that of a piston in an infinite baffle. In this case the mouth acoustical impedance, z_{A2}, is given by equation 5.12.

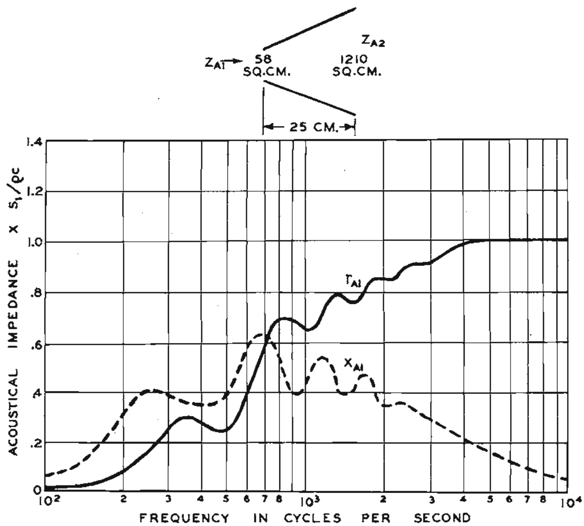

FIG. 5.8. The throat resistance and acoustical reactance frequency characteristics of a finite conical horn. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance and acoustical reactance multiplied by S_1 and divided by ρc.

5.27. Finite Exponential Horn.[35]—The acoustical impedance at the throat of a finite exponential horn of Fig. 5.6 may be obtained in a manner similar to the procedure for the finite cylindrical horn in the preceding section by employing the equations for the pressure and velocity in an infinite exponential horn and applying the proper boundary conditions. The expression for the acoustical impedance, z_{A1}, at the throat in terms of the length and flare constant of the horn and the acoustical impedance, z_{A2}, at the mouth is

$$z_{A1} = \frac{\rho c}{S_1}\left[\frac{S_2 z_{A2}\,[\cos\,(bl + \theta)] + j\rho c\,[\sin\,(bl)]}{jS_2 z_{A2}\,[\sin\,(bl)] + \rho c\,[\cos\,(bl - \theta)]}\right] \qquad 5.80$$

where S_1 = area of the throat, in square centimeters,

S_2 = area of the mouth, in square centimeters,

l = length of the horn, in centimeters,

z_{A2} = acoustical impedance of the mouth, in acoustical ohms,

$\theta = \tan^{-1} a/b$,

$a = m/2$, and

$b = \frac{1}{2}\sqrt{4k^2 - m^2}$.

[35] Olson, H. F., *RCA Review*, Vol. 1, p. 68, 1937.

For $b = 0$, equation 5.80 is indeterminate. To evaluate, take the derivative of the numerator and denominator with respect to b and set $b = 0$. Then the expression for the throat acoustical impedance becomes

$$z_{A1} = \frac{\rho c}{S_1}\left[\frac{z_{A2}\left(1 - \frac{ml}{2}\right) + j\frac{\rho c}{S_2}\frac{lm}{2}}{jz_{A2}\frac{lm}{2} + \frac{\rho c}{S_2}\left(1 + \frac{ml}{2}\right)}\right] \qquad 5.81$$

Below the frequency range corresponding to $b_1 = 0$, b_1 is imaginary. For evaluating this portion of the frequency range the following relations are useful:

$$\tan^{-1} jA = j \tanh^{-1} A = \tfrac{1}{2} j[\log_\epsilon (1 + A) - \log_\epsilon (1 - A)] \qquad 5.82$$

$$\log_\epsilon (-1) = \pm j\pi (2K + 1),\ K = \text{any integer} \qquad 5.83$$

$$\cos (A \pm jB) = \cos A \cosh B \mp j \sin A \sinh B \qquad 5.84$$

$$\sin jA = j \sinh A \qquad 5.85$$

The resistive and reactive components of the acoustical impedance of a finite exponential horn are shown in Fig. 5.9. The acoustical impedance, z_{A2}, at the mouth was assumed to be that of a piston in an infinite baffle as given by equation 5.12. An examination of the acoustical resistance characteristic of Fig. 5.9 shows that there is a sudden change in acoustical

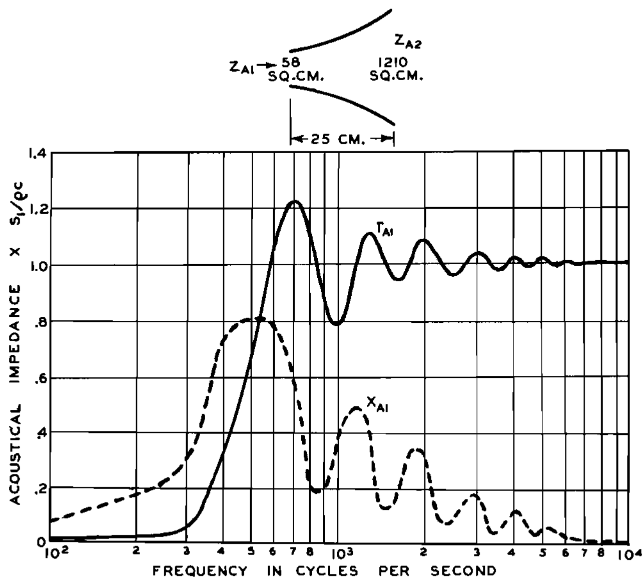

FIG. 5.9. The throat acoustical resistance and acoustical reactance frequency characteristics of a finite exponential horn. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance or acoustical reactance multiplied by S_1 and divided by ρc.

impedance in the frequency region, $f = mc/4\pi$. Above this frequency the acoustical resistance multiplied by $S_1/\rho c$ approaches unity, below this region the acoustical resistance is relatively small. In the finite exponential horn the acoustical resistance is not zero below the frequency, $f = mc/4\pi$, the flare cutoff frequency, which means that the horn will transmit below this frequency. In the case of the finite conical horn, Fig. 5.8, there is no sudden change in the acoustical resistance. On the other hand, the exponential horn shows a larger ratio of acoustical resistance to acoustical reactance. This, coupled with the more uniform acoustical resistance characteristic, makes the exponential horn more desirable and accounts for its almost universal use. In view of its widespread use it is interesting to examine some of the other characteristics of exponential horns.

5.28. Throat Acoustical Impedance Characteristics of Finite Exponential Horns.[36]—The throat acoustical impedance characteristic as a function of the mouth area, with the flare and throat kept constant, is of interest in determining the optimum dimensions for a particular application. The acoustical impedance characteristics of four finite horns having a cutoff of 100 cycles, throat diameter of 1 inch and mouth diameters of 10, 20, 30, and 40 inches and the corresponding infinite horn are shown in Fig. 5.10. These results may be applied to horns of a different flare by multiplying all the dimensions by the ratio of 100 to the new flare cutoff frequency (see Sec. 1.13). The flare cutoff frequency of an exponential horn is given by

$$2\omega = mc \tag{5.86}$$

where $\omega = 2\pi f$,

f = frequency, in cycles per second, and

c = velocity of sound, in centimeters per second.

The acoustical radiation resistance of a mouth 10 inches in diameter is relatively small below 500 cycles. The large change in acoustical impedance in passing from the mouth to the free atmosphere introduces reflections at the mouth and as a result wide variations in the acoustical impedance characteristic as shown in Fig. 5.10A. For example, the first maximum in the acoustical resistance characteristic is 150 times the acoustical resistance of the succeeding minimum.

By doubling the diameter of the mouth the maximum variation in the acoustical resistance characteristic is 7.5, Fig. 5.10B.

Fig. 5.10C shows the acoustical impedance characteristic of a horn with a mouth diameter of 30 inches. The maximum variation in the acoustical resistance characteristic of this horn is 2.

The acoustical impedance characteristic of a horn with a mouth diameter of 40 inches, Fig. 5.10D, shows a deviation in acoustical resistance of only a few per cent from that of the infinite horn of Fig. 5.10E.

These results show that as the change in acoustical impedance in passing from the mouth to the free atmosphere becomes smaller by employing a

[36] Olson, H. F., *RCA Review*, Vol. 1, No. 4, p. 68, 1937.

mouth diameter comparable to the wavelength, the reflection becomes correspondingly less and the variations in the acoustical impedance characteristic are reduced.

The throat acoustical impedance characteristic as a function of the

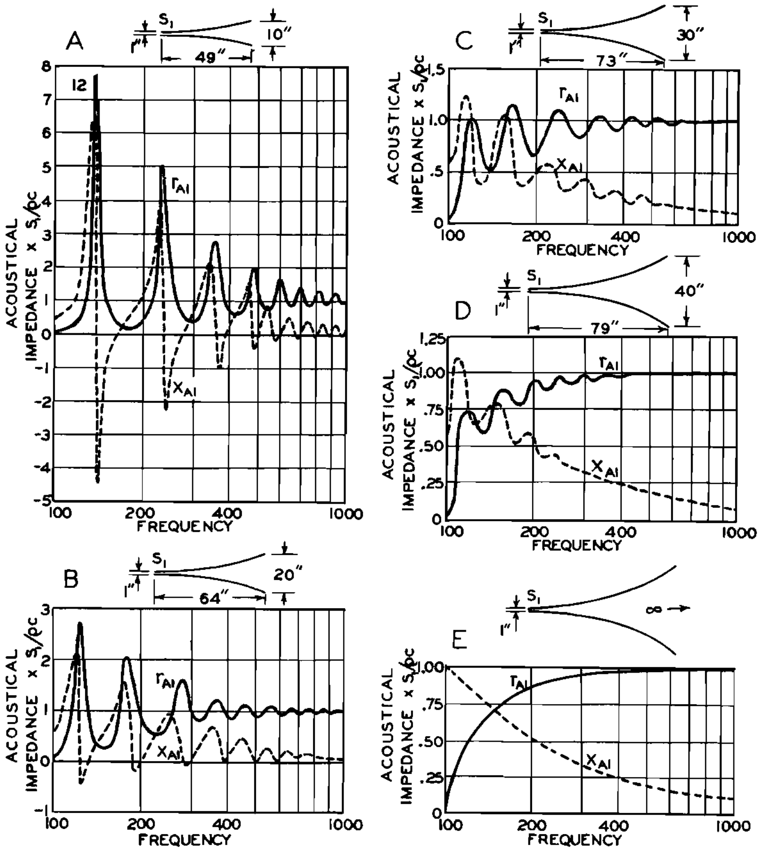

FIG. 5.10. The throat acoustical resistance and acoustical reactance frequency characteristics of a group of exponential horns, with a flare cutoff of 100 cycles and a throat diameter of 1 inch, as a function of the mouth diameter. S_1 = the throat diameter in square centimeters. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance or acoustical reactance multiplied by S_1 and divided by ρc.

throat size with the mouth and flare held constant is of interest in determining the optimum length and a suitable matching impedance for the driving mechanism. The acoustical impedance characteristics of four horns having a cutoff of 100 cycles, mouth diameter of 20 inches, and throat diameter of 1, 2, 4, and 8 inches are shown in Fig. 5.11. A consideration of these characteristics shows that the throat size has no appreciable effect upon the amplitude of the variations in the acoustical impedance characteristics. However, the separation in frequency between successive maxima is increased, as the throat becomes larger, due to the decreased length of the horn. The frequency at which the first maximum in the acoustical resistance characteristic occurs becomes progressively higher as the length is decreased.

The characteristics in Figs. 5.10 and 5.11 cover the range from 100 to 1000 cycles, the lower value being the flare cutoff frequency. The finite

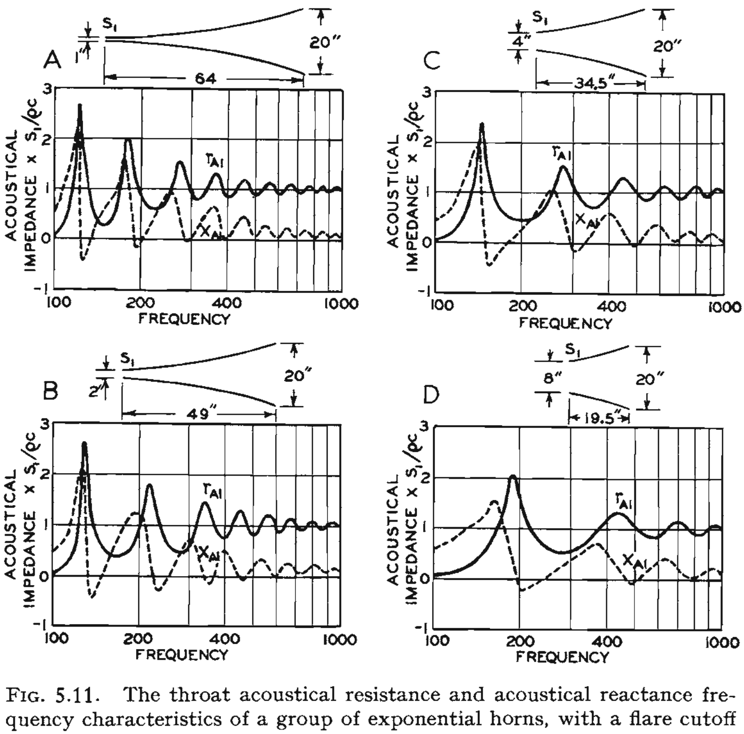

FIG. 5.11. The throat acoustical resistance and acoustical reactance frequency characteristics of a group of exponential horns, with a flare cutoff of 100 cycles and a mouth diameter of 20 inches, as a function of the throat mouth diameter. S_1 = the throat diameter, in square centimeters. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance or acoustical reactance multiplied by S_1 and divided by ρc.

horn, of course, transmits below this frequency because the acoustical resistance is not zero. However, save for the case where the throat is comparable to the mouth, as for example, Fig. 5.11D, the value of the acoustical resistance, at and below the flare cutoff frequency, is quite small.

5.29. Exponential Connectors.—A transformer is used in electrical circuits to transfer between two acoustical impedances of different values without appreciable reflection loss. In acoustical systems a horn may be used to transfer from one acoustical impedance to another. As a matter of fact a horn may be looked upon as an acoustical transformer, transforming large pressures and small volume currents to small pressures and large volume currents. It is the purpose of this section to show how an exponential horn or connector may be used to transfer from one acoustical impedance to another.

Fig. 5.10 shows an exponential horn coupled to an infinite tube. The acoustical impedance of an infinite tube is

$$\frac{\rho c}{S_2} \tag{5.87}$$

where ρ = density, in grams per cubic centimeter,
c = velocity of sound, in centimeters per second, and
S_2 = cross-sectional area of the infinite tube, in square centimeters.

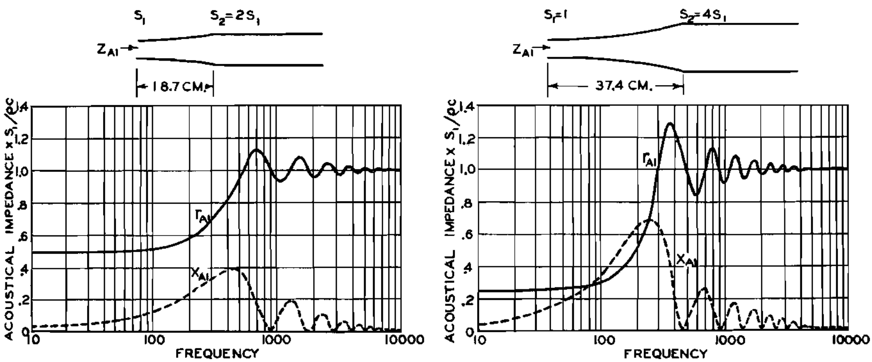

FIG. 5.12. The throat acoustical resistance and acoustical reactance frequency characteristics of two exponential connectors with a flare cutoff of 100 cycles. The mouth of the horn is connected to an infinite pipe. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the acoustical resistance or acoustical reactance multiplied by S_1 and divided by ρc.

Equation 5.87 is the mouth acoustical impedance of the exponential horn. Equation 5.80 then becomes

$$z_{A1} = \frac{\rho c}{S_1}\left[\frac{\cos(bl+\theta) + j\sin(bl)}{\cos(bl-\theta) + j\sin(bl)}\right] \tag{5.88}$$

For $b = 0$, equation 5.88 is indeterminate. To evaluate take the derivative of the numerator and denominator with respect to b and set $b = 0$. Then the expression for the throat acoustical impedance becomes

$$z_{A1} = \frac{\rho c}{S_1}\left[\frac{1 + j\dfrac{lm}{2} - \dfrac{lm}{2}}{1 + \dfrac{lm}{2} + j\dfrac{lm}{2}}\right] \tag{5.89}$$

Below the frequency corresponding to $b = 0$, b is imaginary. This portion of the range may be evaluated by employing equations 5.82, 5.83, 5.85, and 5.89.

The acoustical impedance characteristics of two exponential connectors with a flare cutoff of 100 cycles (that is $b = 0$ at 100 cycles) is shown in Fig. 5.12. Below 100 cycles the throat acoustical impedance is the same as that of the infinite pipe. However, at the high frequencies the throat acoustical impedance is the same as the surge acoustical impedance of a pipe of the diameter of the throat. In order to effect a constant transfer of acoustical impedance with respect to frequency over a certain frequency range the cutoff of the connector must be placed below the low-frequency limit of the frequency range.

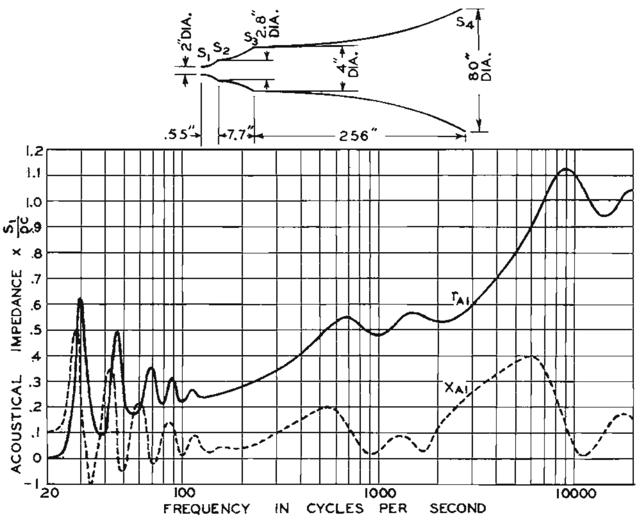

FIG. 5.13. The throat acoustical resistance and acoustical reactance frequency characteristics of a multiple flare exponential horn of three sections. The cutoffs due to flare of the three horns are 25, 100, and 1400 cycles. r_{A1} = acoustical resistance. x_{A1} = acoustical reactance. Note: The characteristics shown are the throat acoustical resistance or acoustical reactance multiplied by S_1 and divided by ρc. S_1 = area at the throat of the small horn in centimeters.

5.30. A Horn Consisting of Manifold Exponential Sections.[37]—The efficiency of a horn loudspeaker is governed, among many other factors, by the throat acoustical resistance. To obtain the maximum efficiency at any frequency the effective acoustical reactance of the entire vibrating system should be equal to the effective acoustical resistance. This, in general, means that to obtain maximum efficiency the throat acoustical resistance

[37] Olson, H. F., *Jour. Soc. Mot. Pic. Eng.*, Vol. 30, No. 5, p. 511, 1938.

of the horn should be proportional to the frequency, since the acoustical reactance is primarily an inertive reactance and, therefore, proportional to the frequency. Practically any throat acoustical impedance frequency characteristic may be obtained by employing a horn consisting of manifold exponential sections.

A horn consisting of three rates of flare is shown in Fig. 5.13. The acoustical impedance characteristic at the throat of the small horn is obtained in stages. First, the throat acoustical impedance characteristic for the large horn is obtained by using equation 5.80. The throat acoustical impedance obtained for the large horn now becomes the mouth acoustical impedance of the intermediate horn. The acoustical impedance of the throat of the intermediate horn is obtained by employing equation 5.80. For the frequency corresponding to $b = 0$ of the intermediate horn the acoustical impedance at the throat of the intermediate horn becomes indeterminate. The expression can be evaluated as shown in Sec. 5.27 on the finite exponential horn. Next, the throat acoustical impedance at the throat of the small horn is obtained by again employing equation 5.80. The mouth acoustical impedance of the small horn is the throat acoustical impedance just obtained for the intermediate horn. The acoustical impedance characteristic of Fig. 5.13 shows three distinct steps depicting the surge acoustical impedance of each section.