MEASUREMENTS OF THE DEVIATION FROM OHM'S LAW IN METALS AT HIGH CURRENT DENSITIES

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Any picture of the mechanism of current conduction in metals which takes account of the part played by the electrons would lead to the expectation of departures from Ohm's law at high current densities. On the classical free electron basis J. J. Thomson has shown that at currents of the order of 10^9 amp./cm^2. the current would be expected to increase as the square root of the applied E. M. F., and hence that the resistance will increase indefinitely. Many attempts have been made to detect the existence of this effect experimentally, but without success. The chief source of difficulty has been the necessity for separating the change of resistance due to the great temperature rise under the heavy current from the change due to a departure from Ohm's law. The best known attempt in this direction is perhaps that of Maxwell.[1] Assuming that the departure from Ohm's law must be proportional to the square of the current, which is plausible on grounds of symmetry, he showed that at a density of 1 amp./cm^2. the resistance of platinum, iron, and German silver does not differ by more than 1 part in 10^{12} from the resistance at infinitely small currents. His maximum density was about 5×10^4 amp./cm^2.

By the application of a new method I have been able to eliminate the source of error due to temperature rise, to detect the existence of the effect, and to measure it with a fair degree of accuracy. The specimen is made one of the arms of a bridge, and is traversed simultaneously by a heavy direct current and a small superposed alternating current of acoustical frequency. The resistance of the specimen to the direct current is measured with an ordinary galvanometer, and the resistance to the alternating current is measured at the same time with a telephone. If there is a departure from Ohm's law under the heavy current, that is if the relation between current and E. M. F. is not linear, the two resistances will not be equal, and from their difference the departure from Ohm's law may be calculated. The reason for

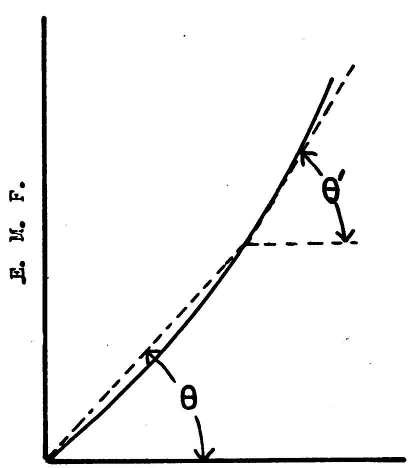

FIG. 1

this will be evident from an inspection of figure 1. It is to be remembered that a bridge is an instrument for balancing potentials in different parts of a net-work. At D. C. balance the potential drop corresponding to $\tan\theta$ in figure 1 is measured and at A. C. balance the potential drop corresponding to $\tan\theta'$.

It is evident that the direct effect of the unknown rise of temperature is eliminated by this method, because the temperature of the conductor is the same to both the direct and alternating current. There is, however, an indirect effect which is important, and which must be eliminated. Under the joint action of the direct and alternating current the wire receives a comparatively large supply of heat steadily, with a small heating and cooling effect superposed. Under this superposed heating and cooling the wire experiences alternations of temperature which give rise to fluctuations of resistance. The large direct current flowing through the fluctuating resistance gives rise to an alternating difference of potential in one of the bridge arms, which affects the A. C. balance. This action is like that of a microphone. It becomes vanishingly small at high frequencies of the alternating current, because the fluctuations of temperature become vanishingly small under these conditions.

The "microphone" effect was eliminated by making readings at a number of frequencies, plotting against the reciprocal of frequency and extrapolating to zero (that is, infinite frequency). The range of frequencies employed was from 320 to 3750 cycles per second. The extrapolation is therefore over a range only one-tenth of the observed readings. As further adding to the certainty of the extrapolation, it may be shown by a dimensional argument that the curve extrapolates to zero as a straight line. The extrapolated difference between D. C. and A. C. resistance gives the sought for departure from Ohm's law.

Measurements were made on gold and silver. These metals were in the form of thin leaf, cemented to a glass backing, cut into the shape of a narrow isthmus at the part intended to carry the high current density, and cooled by a stream of distilled water flowing over the glass. Two thicknesses of gold were used, 8×10^{-6} and 1.67×10^{-5}, and one thickness of silver, 2×10^{-5} cm. Greater thicknesses of gold were tried, but good results could not be obtained. It was possible to reach current densities up to about 5×10^{-6} amp./cm^2.

Further details of the experiment and of the electrical arrangements, which were sufficiently simple and obvious, will be described elsewhere, probably in the *Proc. Amer. Acad. Arts. and Sci.*

All of the experimental results obtained on eleven different samples of 8×10^{-6} gold (three of these were films formed by cathode deposit) are collected in figure 2. The ordinates are the extrapolated difference between D. C. and A C. resistance in terms of the initial resistance, and the abscissae are current densities in 10^6 amp./cm^2. The specimens varied in breadth from 0.06 to 0.22 mm. There was no correlation between the results and the breadth, although the microphone effect has a very

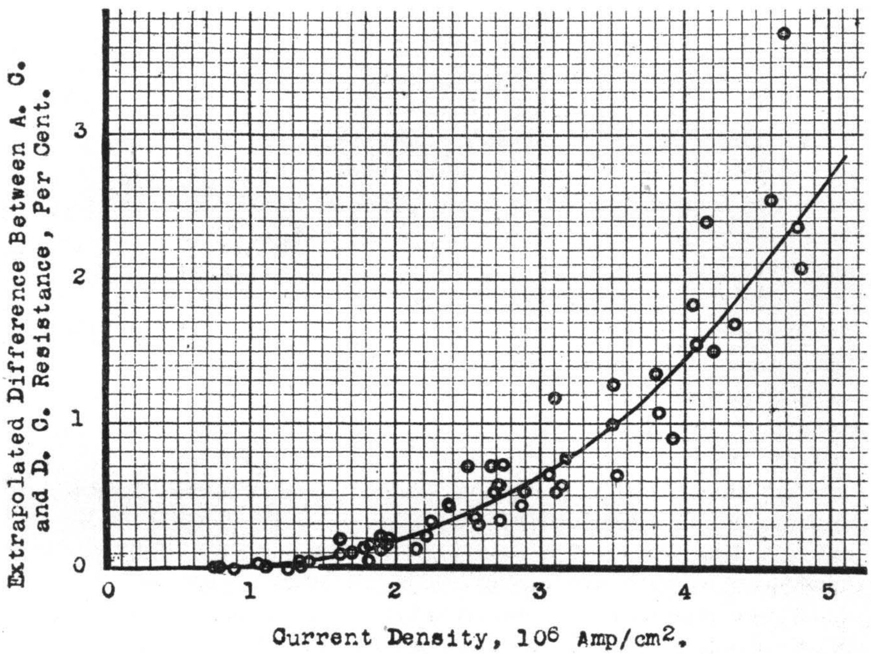

FIG. 2

strong connection. The results are scattering, but perhaps not more than would be expected when the magnitude of the current densities and the fact that different samples of gold leaf may differ in specific resistance by a factor of 2 or more is considered.

The curve drawn through the observed points is taken as the best mean of the experimental results. Let us denote the equation of this curve by $\varphi(x)$. Then the departure from Ohm's law is given by the expression $\int \varphi(x)dx/x$. This will be proved in the detailed paper. This integral may be calculated graphically from the observed points.

In figure 3 is given the departure from Ohm's law, calculated in this way, for gold of two thicknesses and for silver. The departure is positive, that is, the resistance is greater at high density. For thin gold and silver the departure rises to something of the order of 1% at a density of 5×10^{-6} amp./cm^2; for the thicker gold it is greater. It was not possible to reach as high current densities in the thick as the thinner gold. The accuracy is greatest for thin and least for thick gold.

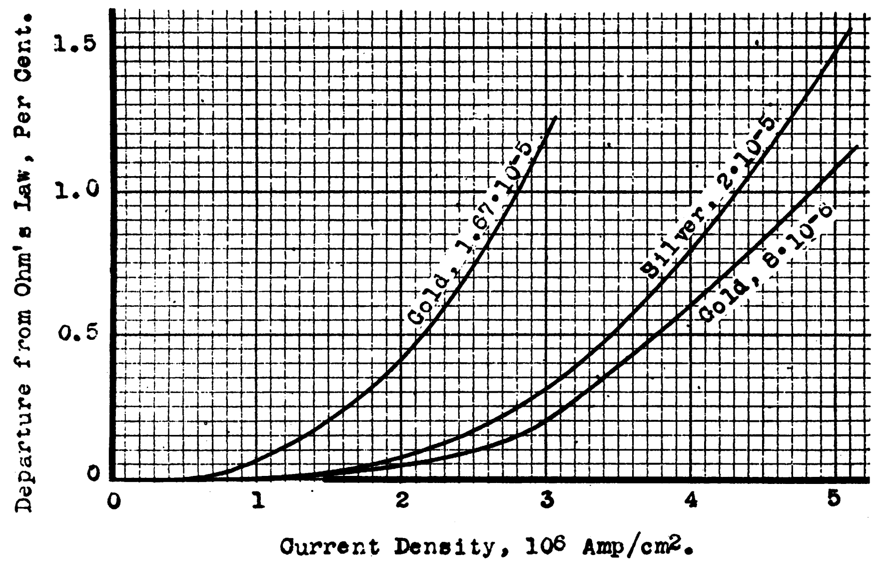

FIG. 3

The form of the curve is not that supposed as a first approximation by Maxwell, but the departure rises more rapidly than the square of the current. In fact the form of the curves seems to suggest infinitely high order of contact with axis at the origin. If however, for the purpose of numerical comparison we assume that below current densities of 10^6 amp./cm^2. the departure from Ohm's law is proportional to the square of the current within the limits of error, than the curve for thin gold shows that for it the resistance at 1 amp./cm^2. cannot differ by more than 1 part in 10^{16} from the resistance at infinitely small density (against 1 part in 10^{12} of Maxwell).

Theoretically the existence of this effect at currents of the order of 10^6 amp./cm^2. means that, granted a free path mechanism of conduction at all, the free path must be much longer than supposed on the classical basis, and accordingly the number of free electrons much less. Now on other grounds I have been led to the belief that conduction is by means of a free path mechanism,[2] of a different sort than that supposed in the classical theory, and that the number of electrons is much less and their paths longer than according to that theory. These new results are now in accord with this point of view. I have not been able as yet to carry through a more exact discussion on the free path basis and to obtain the numerical value for the length of the free path which these results would involve.

The difference in results for gold of different thickness is also what would be expected. The results for thick gold are not nearly as accurate as for the thinner, but there can seem no question but that the departure is really greater for thick than for thin gold. If the free path is of the order of 10^{-5} cm., that is, comparable with the thickness of the leaf, then the average path in the thicker metal will be longer than in the thinner, and a greater departure would be expected in the thicker metal, as found.

[I am much indebted to my assistant, Mr. J. C. Slater, for his skill in making the readings.]

[1] Maxwell, C., Everett, J. D. and Schuster, A., *B. A. Rep.* 1876 (36–63).

[2] Bridgman, P. W., *Physic. Rev., Ithaca,* (2), **17,** 1921 (161–194).

AN INTEGRAL EQUALITY AND ITS APPLICATIONS

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The purpose of this note[1] is to deduce an integral equality adjoined to a linear homogeneous differential equation of the second order and to show some applications of such equalities to the question of the distribution of zeros of solutions of such differential equations in the complex plane.

Let $G(z)$ and $K(z)$ be two single-valued and analytic functions of z throughout the region under consideration and take the differential equation

$$\frac{d}{dz}\left[K(z)\frac{dw}{dz}\right] + G(z)w = 0, \tag{1}$$

or the equivalent system

$$\left\{\begin{aligned} \frac{dw_1}{dz} &= \frac{w_2}{K(z)}, \\ \frac{dw_2}{dz} &= -G(z)w_1, \end{aligned}\right. \qquad \text{with} \qquad \left\{\begin{aligned} w_1 &= w, \\ w_2 &= K(z)\frac{dw}{dz}. \end{aligned}\right. \tag{2}$$

From this system we easily deduce the relation

$$\left[\bar{w}_1 w_2\right]_{z_1}^{z_2} - \int_{z_1}^{z_2} |w_2|^2 \overline{\left[\frac{dz}{K(z)}\right]} + \int_{z_1}^{z_2} |w_1|^2 G(z)dz = 0, \tag{3}$$

where $\bar{u}$ denotes the conjugate of u. If we put

$$dK = \frac{dz}{K(z)}, \quad d\Gamma = G(z)dz, \tag{4}$$

relation (3) becomes

$$\left[\bar{w}_1 w_2\right]_{z_1}^{z_2} - \int_{z_1}^{z_2} |w_2|^2\, d\bar{K} + \int_{z_1}^{z_2} |w_1|^2 d\Gamma = 0, \tag{5}$$

or, split up into real and imaginary parts,