

Electrically Manifested Distortions of Condenser Microphones in Audio Frequency Circuits*

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Current approaches defining the electrically manifested distortions of condenser microphones yield results of varying accuracy. Based on the analytical solution of Ernsthausen, a calculation for the audio frequency circuit is presented which reveals the influence on distortion of the actual membrane movement. Results are shown for older and modern studio condenser microphones constructed of a single membrane and without center suspension.

0 INTRODUCTION

Most condenser microphones have a minimum distortion-free range of 128 dB. While this is sufficient for most studio or broadcast applications, there are a few sound sources, such as trumpets on axis and drums, which, if the microphone is not to cause distortion, require a greater range.

The nonlinearities of condenser microphones have two principal causes:

1) Acoustical-mechanical distortions, caused by the mechanical properties of the membrane, air damping, and the stiffness of the cavity.

2) Electrical distortions.

The main origin of electrical distortions is the amplifier, with a maximum input voltage of 2.2 V. Input voltages above 2.2 V are "clipped" by the field effect transistor (FET) or the vacuum tube. The clipping can, however, be reduced, either through circuit design or by means of the "10-dB switch," to reduce the polarization voltage (along with other effects). Other electrical distortions are caused by the capacitance of the capsule and the amplifier, as will be discussed in this engineering report.

1 PREVIOUS WORK

Distortions in the electric circuitry of condenser microphones were first investigated in the 1930s by Ernsthausen [1]. Apart from a small error (the integral), he developed a successful mathematical solution. In 1955 Brand [2] showed, however, that the simplified formula

for the second harmonic k_2 ,

$$k_2 = \frac{\ddot{u}}{U_0} \cdot 50\%$$

as often cited in older literature, is incorrect. Results of this formula lead to overcharges. His experimental setup consisted of two pistonphones—a large one for 50 Hz and a smaller one for frequencies up to 800 Hz. Although the arrangement was adequate for his purposes, two problems exist:

1) A pistonphone resting on a "Kundt tube" has harmonic eigenfrequencies. A better way to measure the distortions of the pressure microphone is shown in [3].

2) The acoustical-mechanical distortions of the capsule color the result.

For measuring difference-frequency distortions Hibbing and Griese showed a better arrangement [4].

In the present engineering report the analytical method of Ernsthausen is developed further.

2 ELECTRIC CIRCUIT

For an investigation of electrically manifested distortions the relevant components of the microphone are the power supply V_0 , the capacitance C of the capsule, and the input resistance R of the amplifier. I is the current and V is the voltage drop across the resistor (see Fig. 1).

In the absence of acoustical excitation a plate condenser of capacitance C_0 results,

$$C_0 = \frac{\epsilon A}{d_0} \quad (1a)$$

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where

ϵ = permittivity ϵ_0 + permittivity of air ϵ_1
 A = area
 d_0 = distance.

With sinusoidal sound pressure excitation Eq. (1a) is modified by including the displacement ξ of the membrane,

$$C = \frac{\epsilon A}{d_0 + \xi \sin \omega t} \quad (1b)$$

where

ω = angular frequency
 t = time.

The polarization voltage V_0 is given by

$$V_0 = IR + \frac{1}{C} \int I dt \quad (2)$$

which, for a charge q ,

$$q = \int I dt \quad (3)$$

yields the differential equation

$$\frac{dq}{dt} - \frac{V_0}{R} + \frac{q}{RC_0} \left(1 + \frac{\xi}{d_0} \sin \omega t \right) = 0. \quad (4)$$

The analytical solution of this differential equation is best obtained via a power series for the charge q .

3 SOLUTION OF THE DIFFERENTIAL EQUATION

Using a power series, q can be written

$$q = q_0 + \delta q_1 + \delta^2 q_2 + \dots + \delta^n q_n \quad (5a)$$

and the time derivative,

$$q' = q'_0 + \delta q'_1 + \delta^2 q'_2 + \dots + \delta^n q'_n \quad (5b)$$

where

$$\delta = \frac{\xi}{d_0}.$$

The complete equation of the electric circuit is then

$$\begin{aligned} & -\frac{V_0}{R} + (q'_0 + \delta q'_1 + \delta^2 q'_2 + \dots + \delta^n q'_n) \\ & + \frac{1}{RC_0} (q_0 + \delta q_1 + \delta^2 q_2 + \dots + \delta^n q_n) \\ & + \frac{\delta}{RC_0} \sin \omega t (q_0 + \delta q_1 + \delta^2 q_2 + \dots + \delta^n q_n) = 0. \end{aligned}$$

For $\delta = 0$ it follows,

$$q'_0 + \frac{1}{RC_0} q_0 - \frac{V_0}{R} = 0$$

and the static solution is $q_0 = V_0 C_0$. Then the dynamic solution of Eq. (4) becomes (neglecting the transient behavior),

$$Q_n = \frac{V_0}{2^{n-1}} \cdot \frac{C_0}{\sqrt{\prod_{m=1}^n [1 + (m^2 \omega R C_0)^2]}} \quad (6)$$

where

Q_n = magnitude of the static charge of the single harmonics n ($Q_n = |q_n(t)|$)

$n = 1, 2, 3, \dots$

$m = 1, 2, 3, \dots, n$

Π = product series.

The alternating voltage v on the input resistor of the amplifier is the product of the resistor and the time derivative of the total charge q ,

$$v = R \cdot I = R \frac{dq}{dt}. \quad (7)$$

This gives

$$v = \sum_{n=1}^{\infty} R n \delta^n \omega Q_n \cos(n\omega t + \varphi_n) \quad (8a)$$

or

$$v = \sum_{n=1}^{\infty} \left[\frac{n(\delta)^n \omega \dot{V}_0 R C_0}{2^{n-1} \sqrt{\prod_{m=1}^n [1 + (m\omega R C_0)^2]}} \cos(n\omega t + \varphi_n) \right]. \quad (8b)$$

With a constant phase the phase term is negligible. The sum has rapid convergence such that the calculation can be halted after only a few terms; the third harmonic is the 10^{-24} part of the first harmonic.

In conjunction with well-known equations for the total harmonic distortions (THD) (such as [5]) the nonlinearity can now be calculated.

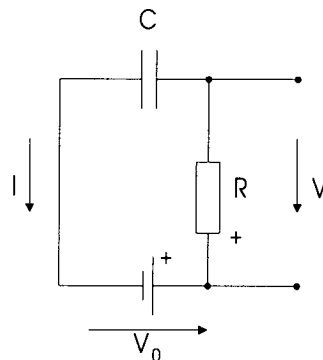


Fig. 1. Relevant circuit diagram.

4 PRECISION OF MOVEMENT

A limitation of the model described is that the condenser microphone is considered to be constructed of rigid surfaces. This assumption is inaccurate because the membrane of a microphone exhibits deformation. In [6] it was shown that the deformation of the membrane can, in a monofrequent (harmonic-free) sound field, be approximated from a superposition of the ground and the first mode of the bending membrane (higher order modes were not observed). For frequencies up to 300 Hz the ground mode is dominant, whereas for higher frequencies the influence of the bending mode increases with frequency. To include such membrane deformation, Eq. (8b) must be used in conjunction with the corresponding spatial functions. Thus the membrane can be described with a Bessel function of the first kind,

$$\xi = \sum_{n=0}^{\infty} \xi_n J_0 \left(x_n \frac{r}{b} \right) \quad (9)$$

where

- ξ = membrane displacement
- b = membrane radius
- x_n = zeros of Bessel function J_0 .

The first eigenfunction (ground mode) is shown in Fig. 2. With the Bessel function it is possible to calculate the distortion more precisely.

5 CALCULATIONS

To begin, the THD of an older studio microphone of 90-pF capacitance and 100-M Ω input resistance is compared with the THD of a modern (studio) miniature microphone of 37-pF capacitance and 3-G Ω input resistance (see Fig. 3). In the figure the THD is divided by δ so that it is possible to evaluate the THD for each distance. In this calculation the output level of the microphones is irrelevant since the "clipping" of the amplifier is not considered.

It is realized that the distance depends not only on the frequency but also on the sound pressure level. The THD is shown in Fig. 4 for a sound pressure level of approximately 140 dB. At 128 Hz the displacement at the center of the membrane was 1.4 μm . It increased to 2.3 μm at 300 Hz. For still higher frequencies the displacement decreased. The change in displacement with frequency cannot be illustrated since this is a difficult mechanical problem. Fig. 4 reveals that the variable displacement causes the THD to increase.

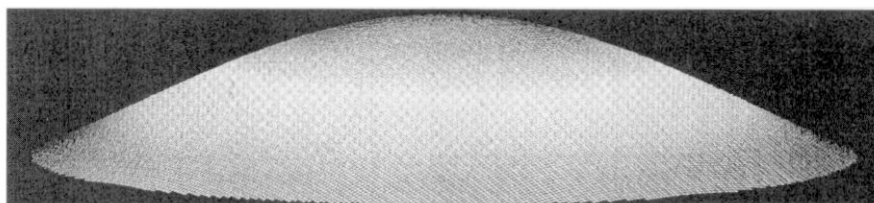


Fig. 2. Ground mode of membrane.

In microphone design the capacitance varies little. For studio microphones it varies between 28 and 100 pF and for measuring microphones between 3.5 pF [1/8 in (6-mm)] and 65 pF [1 in (25 mm)]. The improvement seen in the nonlinearity comes from the development of extremely high-valued resistors. For example, resistors of 3 G Ω are used in good studio microphones. Moreover, in measuring microphones an input resistance of approximately 20 G Ω is possible if the capsule is connected directly to the gate of the FET. Whether using a FET or a vacuum tube, there is an additional charge such that the capacitance increases by approximately 3 pF. The calculations are for an "ideal" FET or vacuum tube.

For selected frequencies (from 20 Hz to 500 Hz) and for capacitances typical of large-diaphragm and miniature studio microphones, Fig. 5 shows the influence the amplifier input resistance has on the THD.

6 CONCLUDING REMARKS

It has been shown analytically that modern studio microphones with an input resistance of 3 G Ω and a small capsule capacitance have less than 0.5% electrically manifested distortion at even the highest sound pressure levels found in studios. For the reasons given (the acoustical-mechanical distortions of the capsule color the result), verification by measurement is not possible. All other sources of distortions ("clipping" and acoustical-mechanical behavior) produce higher distortions.

The low resistance of a traditional tube circuit design (in the area of 100 M Ω) can create distortions. If a natural recording is desired, it is suggested that for levels higher than speech or moderate music, microphones with

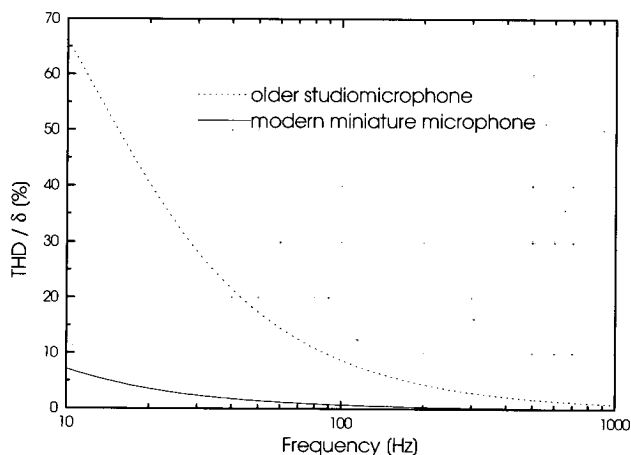


Fig. 3. Comparing microphones (normalized) with simplified membrane movement.

a FET or a modern tube circuit design be used rather than a traditional vacuum tube circuit. Measuring microphones with a non-tube amplifier (input resistance approximately $20\text{ G}\Omega$) are practically free from electrically manifested distortions, that is, in the worst case the THD of a $\frac{1}{2}$ -in (12.7-mm) microphone at 10 Hz is less than 0.003%.

It should, of course, be remembered that the electrically manifested distortions are only one contribution to the nonlinearities of a microphone. Other causes, such

as mechanical properties of the membrane, air damping, and the stiffness of the cavity, are of greater influence.

7 REFERENCES

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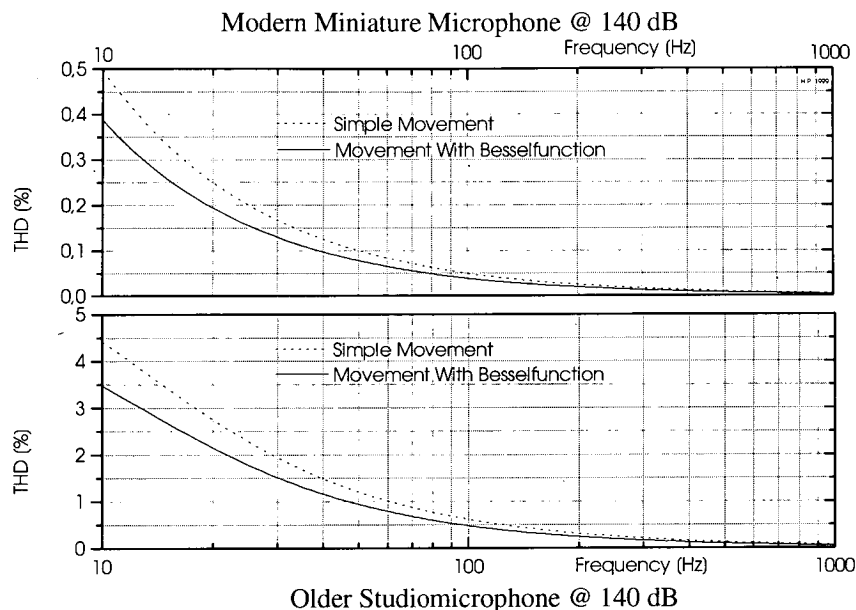


Fig. 4. Comparing membrane movements at 140 dB.

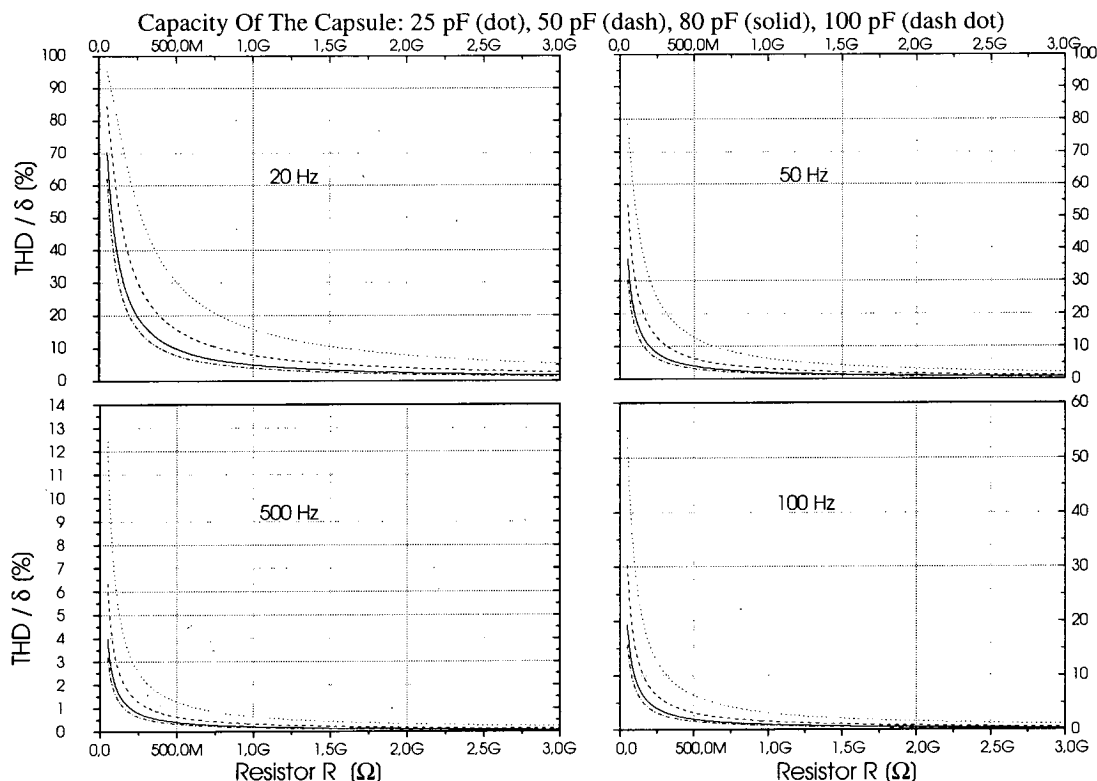


Fig. 5. Normalized nonlinearity versus resistance for four frequencies.

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Holger Pastillé was born in 1967 in Berlin, Germany. After an apprenticeship in audio engineering in television, he worked in this field until 1990. He received the M.A. degree in 1996 from the Technical University (TU) of Berlin where he studied the science of communi-

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