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Acoustic metasurface-based perfect absorber with deep subwavelength thickness

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Conventional acoustic absorbers are used to have a structure with a thickness comparable to the working wavelength, resulting in major obstacles in real applications in low frequency range. We present a metasurface-based perfect absorber capable of achieving the total absorption of acoustic wave in an extremely low frequency region. The metasurface possessing a deep subwavelength thickness down to a feature size of $\sim \lambda/223$ is composed of a perforated plate and a coiled coplanar air chamber. Simulations based on fully coupled acoustic with thermodynamic equations and theoretical impedance analysis are utilized to reveal the underlying physics and the acoustic performances, showing an excellent agreement. Our realization should have an high impact on amount of applications due to the extremely thin thickness, easy fabrication, and high efficiency of the proposed structure. © 2016 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4941338>]

An acoustic perfect absorber in deep subwavelength is always a challenge due to the fact that the frictional power is linearly proportional to the elastic deformation energy in linear dissipative systems.¹ To enhance the coherent dissipation, the intuition and common way is to increase the energy density, for example, introducing resonant structures. Recently, an acoustic metasurface (with thickness $\sim 1/133$ of the working wavelength) coupled a decorated membrane resonator (DMR)²⁻⁴ with thin air chamber was constructed to form hybrid resonances for the realization of the total absorption in the low frequency range.⁵⁻⁷ However, it is not flexible to tune the physical parameters of the system, such as the prescribed tension of the membrane. Meanwhile, total absorption could be realized by the acoustic coherent perfect absorber (CPA),^{8,9} in which the scattering waves at the resonant frequency can be cancelled through interference with another coherent incident acoustic wave with a suitable phase and amplitude. As another classical while stable absorbing structure, perforated systems consisting of a perforated plate and a back air cavity could also realize the total absorption taking into account the viscosity in the perforated holes.¹⁰ However, the geometries, especially the thickness of the air back cavity, are usually comparable to the working wavelength, which inevitably hinders the applications in low frequency.

The concept of the coiling up space, based on which artificial structures could exhibit extreme acoustic properties, such as high refractive index, double negativity, near-zero index, etc., has been investigated intensively recently due to the fascinating underlying physics and diverse potential applications.¹¹⁻²⁴ One of the most important functions is the ability to shrink bulky structures into deep subwavelength scale. However, the concept is rarely introduced to an acoustic absorption system before. It is innovative to extend the

concept of coiling up space into the perforated system to significantly reduce the thickness of the system. Furthermore, it is apparent that the perfect absorber with deep subwavelength thickness, if can be successfully realized, would have deep implications for acoustic device, applications, and in the field of acoustics in general.

To this end, we propose a distinct while more direct way to construct an acoustic metasurface-based perfect absorber composed of a perforated plate and a coiled coplanar air chamber. Total sound absorption could be achieved in an extremely low frequency range with the total thickness in the deep subwavelength scale.

In general, an acoustic absorbing system could be regarded as an artificial boundary with normalized acoustic special impedance $z_s = x_s + iy_s$, where the x_s and y_s are acoustic special resistance and reactance normalized to the impedance of air $z_a = \rho_0 c_0 / S$ with ρ_0 and c_0 referring to the mass density and sound speed of air, and S being the cross section area of the unit cell in the incident side. The absorption coefficient for this effective boundary could be expressed as

$$\alpha = \frac{4x_s}{(1 + x_s)^2 + (y_s)^2}. \quad (1)$$

Equation (1) implies that total absorption could be realized when two conditions are satisfied simultaneously: (i) $y_s = 0$ revealing a resonant state and (ii) $x_s = 1$ indicating the impedance matching. As a result, sound energy is totally dissipated by the resistance, x_s .

Imagine that an incident acoustic wave along z direction normally impinges the perforated system with periodic holes (period a) [cf. Fig. 1(a)] and then penetrates the holes (diameter d and thickness t) to the back cavity (length l) with hard objects. The sound energy could be highly absorbed at the resonant frequency resulting from the energy dissipation. The contribution of the energy dissipation comes from the

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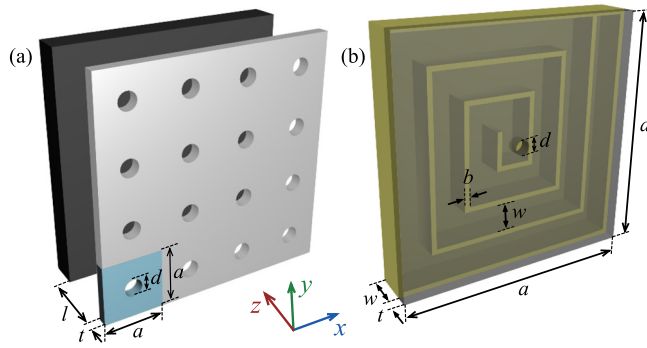


FIG. 1. (a) Conventional perforated system with a perforated plate (gray area) (thickness t) with centered holes (diameter d and period a) placing in front of a hard object (dark area) to form a straight back air cavity (length l). (b) The metasurface (width a and whole thickness $t + w$) composed of a perforated plate (transparent gray region) (thickness t) with a hole (diameter d) and a coiled air chamber (width and thickness w) (yellow region). Several solid beams (width b and thickness w) are used to form the coplanar air chamber. The perforated plate is combined together with the coplanar coiled air chamber. The normal incident wave propagating along the z direction penetrates into the coiled chamber through the perforated hole. For clarification, only one unit cell of the metasurface (cross area $a \times a$) is illustrated. Transparent effect is imposed on the perforated plate to display the details of the back coplanar air chamber.

viscous friction on the wall of the perforated holes since its geometrical size is comparable to the thickness of the viscous boundary layer, $d_v = \sqrt{2\mu/\rho_0\omega}$, with ω and μ referring to angular frequency and the coefficient of dynamic viscosity.²⁵ This energy dissipation can be represented by a normalized acoustic special resistance and reactance of the hole as¹⁰

$$x_h = \frac{32\mu t}{pc d^2} \left(\sqrt{1 + \frac{K^2}{32}} + \frac{\sqrt{2}Kd}{8t} \right), \quad (2a)$$

$$y_h = \frac{\omega t}{pc} \left(1 + \frac{1}{\sqrt{9 + \frac{K^2}{2}}} + 0.85 \frac{d}{t} \right), \quad (2b)$$

with

$$K = \frac{d}{2} \sqrt{\frac{\omega}{\mu}}, \quad (3)$$

where the $p = \pi d^2/4S$ is the porosity of the perforated plate. The rigid back cavity with length l and cross section area of the unit cell S' can be regarded as a pure special reactance (normalized to the impedance of air in the incident side, z_a)

$$y_c = -iS/S' \cot \frac{\omega l}{c_0}. \quad (4)$$

S' referring to the cross section of the back cavity/chamber is a different geometric parameter and can be different from S representing the cross section of the unit cell in the incident side. In the case of the conventional absorbers consisting of a perforated plate and a straight air back cavity, the value of S' is identical with that of S . Then, the normalized acoustic special impedance for the whole system is $z_s = x_s + iy_s = x_h + i(y_h + y_c)$ and corresponding absorption coefficient can be expressed as

$$\alpha = \frac{4x_h}{(1 + x_h)^2 + (y_h + y_c)^2}. \quad (5)$$

From Eq. (5), it is readily to obtain that the total absorption condition as

$$x_h = 1, \quad (6a)$$

$$y_h + y_c = 0. \quad (6b)$$

To achieve these two conditions, the length of the cavity, l , yields

$$l = \frac{c_0 S'}{\omega S} \cot^{-1} y_h. \quad (7)$$

In classic perforated systems [cf. Fig. 1(a)], the cross-section area of the unit cell, S , and the cross section of the straight back cavity, S' , is identical, $S = S'$. From Eq. (7), the total thickness, $t + l$, of the classic absorber generally should be comparable to the working wavelength to ensure a resonant state,¹⁰ resulting in the major obstacles for applications and designing related devices in low frequency range. From the Eq. (4), it could be found that the length l in the cotangent function rather than the cross section area S' plays dominant roles in determining the resonant frequency and the absorbing peak. This inspired us to shrink the cross section area to be S' in order to obtain extra place to construct a coiled coplanar air chamber instead of the straight cavity. Then, the metasurface is formed combining the perforated plate with the coplanar air chamber ($S = a^2$ and $S' = w^2$) [cf. Fig. 1(b)]. By this approach, the effective length, l_{eff} , of the coplanar channels could be tuned hundreds greater than the real thickness w , and then, the total thickness, $t + w$, of the structure should be significantly decreased.

To obtain the absorption spectrum of the metasurface, simulations are conducted with a commercial software, COMSOL MultiphysicsTM Version 5.2 with preset Acoustic-Thermoacoustic interaction module. The effect of the viscous friction and the heat transfer is concluded in the linearized compressible Navier-Stokes equation, the continuity equation, and the energy equation if the diameter of the holes, d , is comparable to the thickness of the viscous boundary layer, d_v .²⁶ The surrounding medium is air with its density $\rho = 1.21 \text{ kg/m}^3$, sound speed $c = 343 \text{ m/s}$, and dynamic viscosity $\mu = 1.56 \times 10^{-5} \text{ Pa s}$. A normally incident plane wave with unit amplitude impinges along $+z$ direction. Hard boundaries are imposed on the interfaces between air and solid due to the huge impedance mismatch between air and solid materials.²⁵ The absorption coefficient, α , can be expressed as $\alpha = 1 - |r|^2$ with r representing the complex reflection coefficient.

Figure 2(a) shows the absorption coefficient, α , of the presented metasurface with an absorption peak at 125.8 Hz. The total thickness of the system, $t + w = 12.2 \text{ mm}$, is only 1/223 of the working wavelength, revealing the fascinating capability of the presented perfect absorber in deep subwavelength scale. For comparison, the absorption coefficient derived from the theoretical impedance analysis is also illustrated in Fig. 2(a). Here, the effective length of the coplanar air channel is retrieved from the reflected phase obtained from simulations

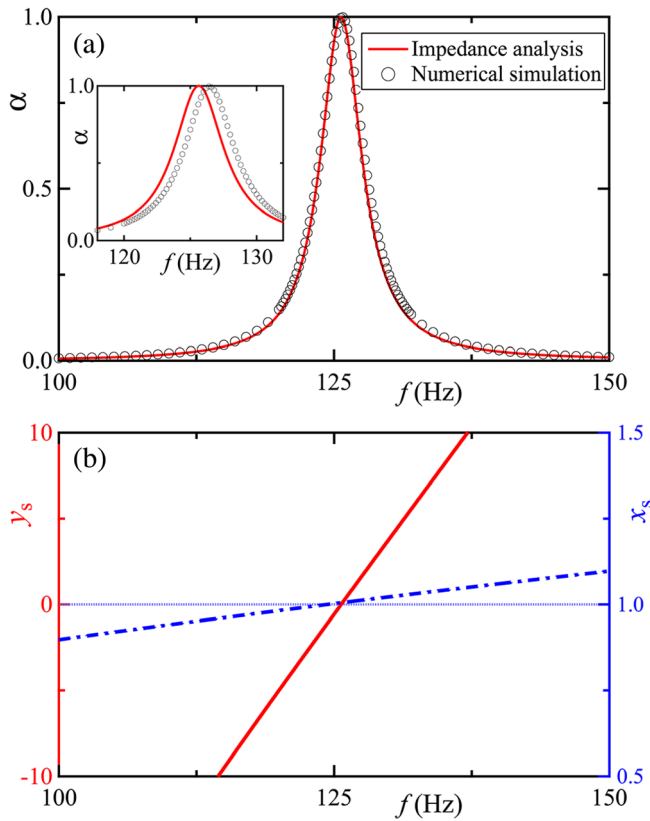


FIG. 2. (a) The absorption coefficient, α , of the presented metasurface (black circles) with geometrical parameters ($d = 3.3$ mm, $t = 0.2$ mm, $a = 100$ mm, $w = 12$ mm, and $b = 1$ mm). Total absorption is realized at 125.8 Hz. The absorption coefficient obtained from the theoretical impedance analysis (red lines) is also illustrated with the same parameters while $d = 3.5$ mm. The inset shows the simulated and theoretical absorption coefficient with the same diameter $d = 3.5$ mm. (b) The normalized specific acoustic reactance y_s (red line) and resistance x_s (blue dotted line) of the whole metasurface system. The curve of y_s crosses zero at the frequency, 125.8 Hz, where the $x_s = 1$.

$$l_{eff} = \frac{c_0}{\omega} \cot^{-1} \left(-\text{Im} \left(\frac{1+r}{1-r} \right) \right), \quad (8)$$

considering the existing radiation impedance at the cross of coiled channel.^{14,20} Then, we can obtain the effective length $l_{eff} = 6.35a$. By substituting l_{eff} into Eq. (4) replacing l , the absorption spectrum based on the impedance analysis technique then can be obtained according to the Eq. (5). We found that the absorption coefficient derived from the impedance analysis with diameter $d = 3.5$ mm presents an excellent agreement with the simulated one with the hole diameter of $d = 3.3$ mm. The absorption coefficient with the same diameter of $d = 3.5$ mm is also illustrated as an inset figure in Fig. 2(a) with a slightly deviated absorbing peak. The slightly difference of the hole diameters or the absorption peaks convincingly stem from the fact that the thermal conduction equation is considered in the simulations while omitted in the impedance analysis for simplifying the derivation of the special acoustic impedance.

To reveal the underlying physics of the total absorption of the presented metasurface, Fig. 2(b) illustrates the normalized acoustic special impedance for the perforated hole and the coiled coplanar air chamber. The curve of the $y_s = y_h + y_c$ crosses 0 at 125.8 Hz indicating the existing resonant

state. Actually, the functionality of the coiled coplanar air chamber is to provide an extra reactance, y_c , to compensate the acoustic reactance provided by the hole, y_h , which is the essential prerequisite to realize perfect absorption. At the same frequency, the x_s , stemming from the viscous effect in the perforated hole, reaches to 1, revealing the impedance matching between the metasurface and the air. With these two conditions simultaneously achieved, it can be convincingly concluded from Eq. (5) that the total absorption could be realized at 125.8 Hz with our proposed metasurface. Incident sound energy could be total dissipated within the perforated hole at the resonant state. The total absorption is realized due to the optimized balance between the scattering wave and the dissipation.

We have also further investigated the balance by tuning the geometrical parameters and the corresponding absorption spectra are illustrated in Fig. 3. Figure 3(a) shows the absorption spectrum as a function of the diameter of the hole, d , with other parameters fixed. It is found that with the increase in d , the absorption coefficient, α , has a peak at 125.8 Hz with $d = 3.3$ mm, which is consistent with the results in Fig. 2. The resonant frequency becomes greater when increasing the d according to the solution of the equation $y_s = y_h + y_c = 0$. Even though there are some absorption peaks with other parameters, the total absorption cannot be realized due to the impedance mismatch. Figure 3(b) illustrates the absorption spectrum as a function of the thickness of the perforated holes, t , while fixed $d = 3.3$ mm. It is not surprising to find a total absorption peak at 125.8 Hz at $t = 0.2$ mm. The absorption peak goes down to lower frequency and the maximum value

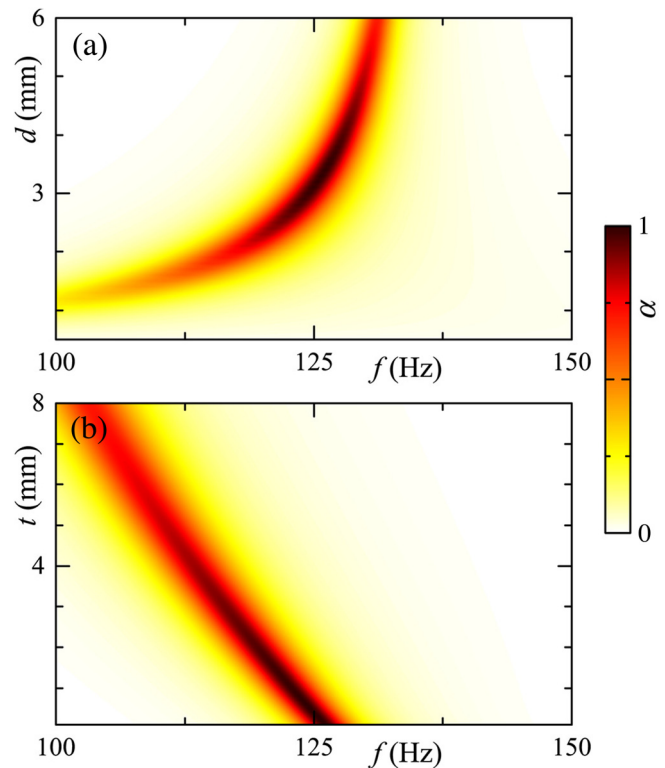


FIG. 3. The absorption spectrum of the metasurface as a function of (a) the diameter d with fixed parameters ($a = 100$ mm, $b = 1$ mm, $w = 12$ mm, and $t = 0.2$ mm) and (b) the thickness t with fixed parameters ($a = 100$ mm, $b = 1$ mm, $w = 12$ mm, and $d = 3.3$ mm).

of the peak decreases when $t > 0.2$ mm. Nearly perfect absorption can be realized within a certain range of t because of the insensitivity with respect to t . The flexural modes of the plate also need to be considered when t becomes thinner. In general, by tuning the geometrical size of the holes, a balance between the scattering and the dissipation could be optimized so that the perfect absorber can be constructed. One promising perspective we can find from Fig. 3 is that the possibility to realize broadband absorption making use of super-cells composed of several unit cells with different d or t . The width and thickness of the coiled air chamber, w , plays dominant roles in the effective length and the position of the absorption peak. Perfect absorption will take place at high frequencies if increasing w . Meanwhile, if the w becomes small enough, the viscous effect in the coplanar chamber also needs to be taken into account for the absorption.

Here we coiled air back cavity to compensate the acoustic reactance provided by the perforated holes, one can also employ a coiled narrow channel while keeping a straight air chamber to realize the total absorption.²⁷ However, it is not readily to derive an analytical description because the real length of the coiled channel is relatively long so that the impedance analysis is not suitable while taking into consideration the viscous effect in the channels. Coiled cavity is more direct way to shrink the thickness of the system, considering that the length of the cavity plays dominant roles in the position of the absorption peak in Eq. (7).

In summary, we have designed a metasurface-based structure which can totally absorb the incident acoustic energy at an extremely low frequency range around 125 Hz. The perfect absorber is composed of the combination of a perforated plate and a coiled coplanar air chamber. The latter can compensate the acoustic reactance caused by the viscosity in the perforated holes to realize the impedance matching and the perfect absorption. Significantly, the total thickness of the whole system is as small as $1/223$ of sound wavelength which is much smaller than the conventional absorber¹⁰ as well as the previous metamaterial-based absorbers.^{5,27–29} Our engineered metasurface actually presents the thinnest perfect absorber, overcoming the major obstacles for acoustic perfect absorbers in the low frequency domain.

Constructing a perfect and stable acoustic absorber with deep subwavelength thickness is a challenge due to the intrinsically physical dissipation mechanism. By solving this hard challenge, our designed metasurface-based perfect absorber has intriguing applications and paves the way towards the related devices. Perfect absorption under oblique incidence can be realized after optimizations since the geometries of the metasurface are much smaller than the working wavelength, indicating the validity of the impedance analysis. Our proposed structure can be easily fabricated

with 3D printing technique and takes advantages of the compact size, stable structure, and high efficiency.

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- ¹L. D. Landau and E. M. Lifshitz, *Theory of Elasticity (Course of Theoretical Physics)*, 3rd ed. (Butterworth-Heinemann, 1986), Vol. 7.
- ²Z. Yang, J. Mei, M. Yang, N. H. Chan, and P. Sheng, *Phys. Rev. Lett.* **101**, 204301 (2008).
- ³S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, *Phys. Rev. Lett.* **104**, 054301 (2010).
- ⁴C. M. Park, J. J. Park, S. H. Lee, Y. M. Seo, C. K. Kim, and S. H. Lee, *Phys. Rev. Lett.* **107**, 194301 (2011).
- ⁵G. Ma, M. Yang, S. Xiao, Z. Yang, and P. Sheng, *Nat. Mater.* **13**, 873 (2014).
- ⁶M. Yang, C. Meng, C. Fu, Y. Li, Z. Yang, and P. Sheng, *Appl. Phys. Lett.* **107**, 104104 (2015).
- ⁷M. Yang, Y. Li, C. Meng, C. Fu, J. Mei, Z. Yang, and P. Sheng, *C. R. Méc.* **343**, 635 (2015).
- ⁸J. Z. Song, P. Bai, Z. H. Hang, and Y. Lai, *New J. Phys.* **16**, 033026 (2014).
- ⁹P. Wei, C. Croënne, S. T. Chu, and J. Li, *Appl. Phys. Lett.* **104**, 121902 (2014).
- ¹⁰D. Y. Maa, *J. Acoust. Soc. Am.* **104**, 2861 (1998).
- ¹¹Z. Liang and J. Li, *Phys. Rev. Lett.* **108**, 114301 (2012).
- ¹²Z. Liang, T. Feng, S. Lok, F. Liu, K. B. Ng, C. H. Chan, J. Wang, S. Han, S. Lee, and J. Li, *Sci. Rep.* **3**, 1614 (2013).
- ¹³Y. Li, B. Liang, X. Tao, X. F. Zhu, X. Y. Zou, and J. C. Cheng, *Appl. Phys. Lett.* **101**, 233508 (2012).
- ¹⁴Y. Li, G. K. Yu, B. Liang, X. Y. Zou, G. Y. Li, S. Cheng, and J. C. Cheng, *Sci. Rep.* **4**, 6830 (2014).
- ¹⁵Y. Xie, B. I. Popa, L. Zigoneanu, and S. A. Cummer, *Phys. Rev. Lett.* **110**, 175501 (2013).
- ¹⁶Y. Li, B. Liang, Z. M. Gu, X. Y. Zou, and J. C. Cheng, *Appl. Phys. Lett.* **103**, 053505 (2013).
- ¹⁷Y. Li, B. Liang, X. Y. Zou, and J. C. Cheng, *Appl. Phys. Lett.* **103**, 063509 (2013).
- ¹⁸T. Frenzel, J. D. Brehm, T. Buckmann, R. Schittny, M. Kadic, and M. Wegener, *Appl. Phys. Lett.* **103**, 061907 (2013).
- ¹⁹Y. Li, B. Liang, Z. M. Gu, X. Y. Zou, and J. C. Cheng, *Sci. Rep.* **3**, 2546 (2013).
- ²⁰Y. Li, X. Jiang, R. Q. Li, B. Liang, X. Y. Zou, L. L. Yin, and J. C. Cheng, *Phys. Rev. Appl.* **2**, 064002 (2014).
- ²¹Y. Xie, W. Wang, H. Chen, A. Konneker, B. I. Popa, and S. A. Cummer, *Nat. Commun.* **5**, 5553 (2014).
- ²²K. Tang, C. Qiu, M. Ke, J. Lu, Y. Ye, and Z. Liu, *Sci. Rep.* **4**, 6517 (2014).
- ²³Y. Li, X. Jiang, B. Liang, C. Jian-Chun, and L. K. Zhang, *Phys. Rev. Appl.* **4**, 024003 (2015).
- ²⁴Y. Li and M. B. Assouar, *Sci. Rep.* **5**, 17612 (2015).
- ²⁵P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (Princeton University Press, Princeton, 1987).
- ²⁶In our design, $d \approx 3.5$ cm approximates $17.7d_v$ at 125.8 Hz so that the viscous effect should be considered in the perforated hole.
- ²⁷X. Cai, Q. Guo, G. Hu, and J. Yang, *Appl. Phys. Lett.* **105**, 121901 (2014).
- ²⁸J. Mei, G. Ma, M. Yang, Z. Yang, W. Wen, and P. Sheng, *Nat. Commun.* **3**, 756 (2012).
- ²⁹J. Christensen, V. Romero-García, R. Picó, A. Cebrecos, F. J. García de Abajo, N. A. Mortensen, M. Willatzen, and V. J. Sánchez-Morcillo, *Sci. Rep.* **4**, 4674 (2014).