Part1

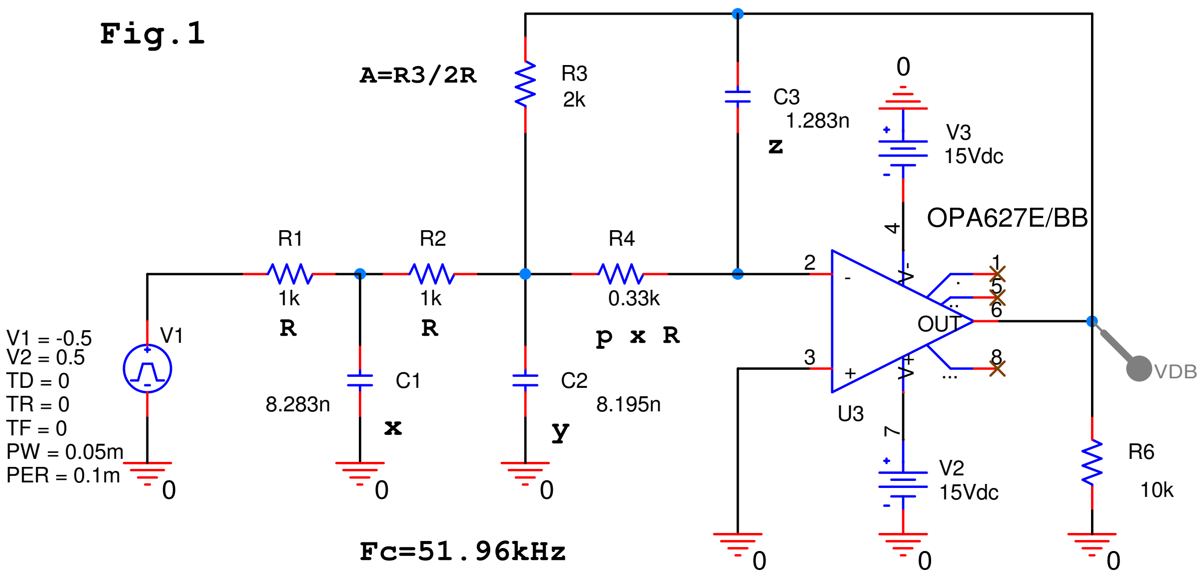
I had to split it in to parts because of the 100kb upload limit.

This is a simple procedure to calculate "by hand" a third order Bessel or Butterworth filter in a MFB topology. I couldn't find it in any book. (well…perhaps I could use some more reading) There is some work at the horizon but not as much as it may seem. You may want to use a pocket calculator able to solve 3rd order equations (and with a few 'memories' to store partial results) - like a Casio fx-991MS….

Two good advices: always run a simulation of the circuit to check for any catastrophic errors. (experience talks here ☹) and use a bridge to match the components values and check DF vs. freq. of capacitors.

I'll start with a single ended example and in Part2 I'll show how to go for a differential configuration.

Fig.1



Bessel	Butterworth
C1 = 8.283n	C1 = 6.865n
C2 = 8.195n	C2 = 12.527n
C3 = 1.283n	C3 = 1.012n

Some general formulas that apply to the 3rd order filter in Fig.1:

$$\mathbf{G} = -\frac{\mathbf{A}}{\mathbf{s}^3 + \mathbf{a} * \mathbf{s}^2 + \mathbf{b} * \mathbf{s} + 1}\text{; where } \mathbf{s} = \mathbf{j} * 2 * \boldsymbol{\pi} * \mathbf{f_c}$$

$$|G| = -\frac{A}{\sqrt{(1 - a * \mathbf{t}^2)^2 + \mathbf{t}^2 * (b - t^2)^2}}\text{; where } \mathbf{t} = \frac{\omega}{\omega_c} = \frac{\mathbf{f}}{\mathbf{f_c}}\text{; } \boldsymbol{\varphi} = \pi \mp \operatorname{arctg}\frac{t * (b - \mathbf{t}^2)}{1 - a * \mathbf{t}^2}\text{;}$$

Now comes some "heavy" stuff. The generalized form for Bessel polynomials is:

$$\mathbf{P_n} = \sum_{k=0}^{n} \frac{(\mathbf{n} + \mathbf{k})!}{(\mathbf{n} - \mathbf{k})! * \mathbf{k}!} * \left(\frac{\mathbf{x}}{2}\right)^{\mathbf{k}}$$

- don't ask me any other details please, my maths… RIP (☺). So the first polynomials of interest for implementing electronic filters are:

$$\mathbf{P_1} = \mathbf{x} + 1$$

$$\mathbf{P}_2 = 3*\mathbf{x}^2 + 3*\mathbf{x} + 1 \xrightarrow{\text{OR}} \mathbf{t}^2 + \sqrt{3}*\mathbf{t} + 1$$

$$\mathbf{P}_3 = 15*\mathbf{x}^3 + 15*\mathbf{x}^2 + 6*\mathbf{x} + 1 \xrightarrow{\text{OR}} \mathbf{t}^3 + \sqrt[3]{15}*\mathbf{t}^2 + \frac{6}{\sqrt[3]{15}}*\mathbf{t} + 1 \;\; ; \;\; \mathbf{x} \longrightarrow \frac{\mathbf{t}}{\sqrt[3]{15}}$$ to match the transfer function of the circuit.

$$\mathbf{P}_4 = 105*\mathbf{x}^4 + 105*\mathbf{x}^3 + 45*\mathbf{x}^2 + 10*\mathbf{x} + 1 \ldots \text{etc.}$$

The filter coefficients are **a** and **b.** For a Bessel response $a = \frac{6}{\sqrt[3]{15}}$ = 2.432881 and $b = \sqrt[3]{15}$ = 2.466212. For a Butterworth response a = b= 2.

For the Bessel the cut-off frequency (fc) is defined as the frequency at which the phase response reaches half of its maximum (or $-\frac{n*\boldsymbol{\pi}}{4}$, where n is the filter order). The –3dB point occurs earlier (for a third order $f_{-3dB} = 0.711887*f_c$).

$$f_c = \frac{1}{2*\boldsymbol{\pi}*\sqrt[3]{\mathbf{R}_1 \mathrm{II} \mathbf{R}_2 * \mathbf{R}_3 * \mathbf{R}_4 * \mathbf{C}_1 * \mathbf{C}_2 * \mathbf{C}_3}} = \frac{1}{2*\pi*R*\sqrt[3]{\mathbf{A}*p*C_1*C_2*C_3}} ;$$

For simplifying things: $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}$; $\mathbf{A} = \frac{\mathbf{R}_3}{\mathbf{R}_1 + \mathbf{R}_2} = \frac{\mathbf{R}_3}{2*\mathbf{R}}$; $\mathbf{p} = \frac{\mathbf{R}_4}{\mathbf{R}_1}$

Further on I made some notations: $K_1 = A*p$; $K_2 = 2*A + p$ and also for the unknowns (x, y, z) that we have to find:

$$C_1 = \frac{x}{2*\pi*f_c*R}; \;\; x = \frac{C_1}{\sqrt[3]{A*p*C_1*C_2*C_3}}$$

$$C_2 = \frac{y}{2*\pi*f_c*R}; \;\; y = \frac{C_2}{\sqrt[3]{A*p*C_1*C_2*C_3}}$$

$$C_3 = \frac{z}{2*\pi*f_c*R}; \;\; z = \frac{C_3}{\sqrt[3]{A*p*C_1*C_2*C_3}}$$

Tip: at the end if you want to check quickly for errors, you can use this relation: $x*y*z = \frac{1}{A*p}$ (don't use it as part of the solving 3 equations system indicated bellow!)
Actually after writing the circuit eqn. and using the notations I made until now the following three equations will give us the solutions. I avoided the boring calculations to reach down to this point. (Huh! - imagine a few pages of this stuff edited in Word) You can use either mesh method or Y to Δ (and back) transformations…so after solving (1) it is easy to obtain x and then y from two simple 1st order equations:

$$z^3 - \frac{2*b}{K_1+K_2}*z^2 + \left[\frac{a}{(K_1+K_2)*(K_2+2*K_1)} + \left(\frac{b}{K_1+K_2}\right)^2\right]*z - \frac{a*b-1}{(K_1+K_2)^2*(K_2+2*K_1)} = 0 \quad (1)$$

$$2*b = x + 2*(K_1+K_2)*z \quad (2)$$

$$2*a = 4*K_1*y*z + (K_2+2*K_1)*x*z \quad (3)$$

Here usually I choose "p" as number smaller than unity from the 1% resistor tolerance series. By choosing "p" we force the ratio between R1(R2) and R4. Just a few examples: if p= 0.475 then at the end you will have plenty of pairs to choose from: (1.21k, 0.576k), (1k, 0.475k), (931, 442), (820, 390) etc.

Then you may want to choose A=1 (0dB gain => R3= 2R) as it is often the case and we can finally solve for z. We retain only the real solution of course. There's a risk of getting negative values for x or y or even z. Anyway a 3rd order equation has always at least one real solution. The problem is to get a positive one. Usually for $\mathbf{p} \le 0.9$ and for A reasonably small you'll get valid solutions.

The same equations if A= 1 can be written:

$$z^3 - \frac{b}{p+1} * z^2 + \left[\frac{a}{2*(p+1)*(3*p+2)} + \frac{1}{4}*\left(\frac{b}{p+1}\right)^2\right] * z - \frac{a*b-1}{4*(3*p+2)*(p+1)^2} = 0$$

$$\mathbf{x} = 2*\mathbf{b} - 4*(\mathbf{p}+1)*\mathbf{z}$$

$$\mathbf{y} = \frac{2*\mathbf{a} - (3*\mathbf{p}+2)*\mathbf{z}*\mathbf{x}}{4*\mathbf{p}*\mathbf{z}}$$

You can tune the filter by choosing ($f_c, \mathbf{R}$) and then fine-tune it by choosing ($f_c, \mathbf{C_x}$). Then you can go down in tighter tolerances series and reiterate the calculations if you feel that you're getting close to some nice standard available values.