A faster method for establishing AC-equivalent diagrams for Triode Circuits.

Preface.

When making calculations in triode circuits most of them are performed after replacing the triodes used by its standard ac-equivalent circuit as drawn in Figure1.

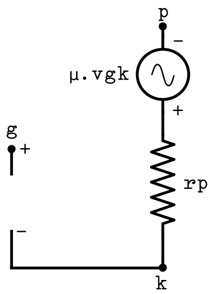


Fig.1

After having done that, applying Kirchoff's Laws provides us the equations necessary to obtain the mathematical expressions for the actual currents and voltages flowing in that particular diagram. And, although in most cases solving these equations is not a difficult job, it frequently will take a substantial amount of time to solve them. Below I will describe a faster method to draw the ac-equivalent circuits from the actual circuit when one wants to calculate its parameters thereby circumventing the above described standard method. I do not pretend that this fast method can be applied to all sorts of triode circuits but a lot of them can be analysed in this way.

1.
In Figure2, a triode configuration is given which frequently appears in triode circuit designs together with its usual ac-equivalent circuit diagram Figure3 derived from Figure2:

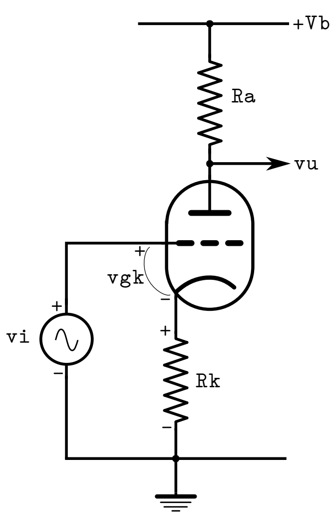


Fig.2

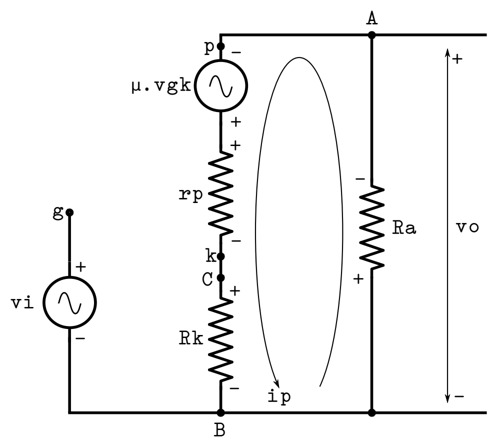


Fig.3

In Figure3 for loop current ip in mesh network BACB according to Kirchoff's 2nd Law:

$$-i_p \cdot R_a + \mu \cdot v_{gk} - i_p \cdot r_p - i_p \cdot R_k = 0$$

$$\mu \cdot v_{gk} = i_p \cdot R_a + i_p \cdot r_p + i_p \cdot R_k \qquad (1)$$

Also for the input circuit : $v_i - v_{gk} - i_p \cdot R_k = 0 \quad v_{gk} = v_i - i_p \cdot R_k$
This substituting in (1) results in:

$$\mu(v_i - i_p \cdot R_k) = i_p \cdot R_a + i_p \cdot r_p + i_p \cdot R_k$$

$$\mu v_i = i_p \{R_a + r_p + (\mu+1) R_k\} \qquad (2)$$

And $v_o = -i_p \cdot R_a$, $i_p = -\dfrac{v_o}{R_a}$. This in turn substituting in (2) for ip gives:

$$\mu v_i = -\frac{v_o}{R_a}\{R_a + r_p + (\mu+1) R_k\}$$

$$\mu v_i \cdot R_a = -v_o \{R_a + r_p + (\mu+1) R_k\}$$

$$\boxed{v_o = -\frac{R_a}{R_a + r_p + (\mu+1) R_k} \cdot \mu \cdot v_i} \qquad (3)$$

We wrote expression (3) in de form

$$v_o = \frac{R_a}{R_a + R_{out}} \cdot \mu \cdot v_i \qquad (4)$$

and this expression shows that Rout is the effective output resistance of the effective voltage source μ.vi and we can draw its circuit diagram in Figure4:

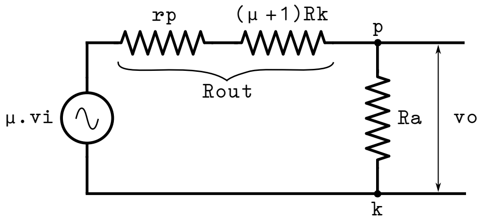


Fig.4

Rules:

When drawing an ac-equivalent circuit for the plate-loop network, then we multiply the voltage source vi at the grid input by a factor (μ).
Every voltage connected to the grid has to be multiplied by the factor (μ) to get its correct equivalent for the plate-loop network.
The cathode resistor Rk must be multiplied by a factor (μ +1) to get its correct equivalent for the plate-loop network.
Every voltage and impedance connected to the cathode has to be multiplied by that factor (μ +1) for their correct equivalent for the plate-loop network.
The plate resistance rp remains unaltered because it is already included in the plate-loop network configuration.
To easily recall these rules, I have drawn Figure5 with the correct multiplying factors.

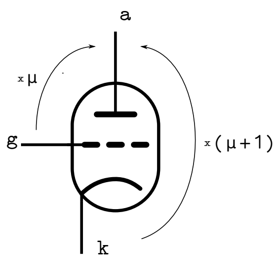


Fig.5

2.

In Figure6 the same schematic diagram is drawn as in Figure2 but now the output signal is taken from the cathode resistor and in Figure7 shows its ac-equivalent circuit diagram:

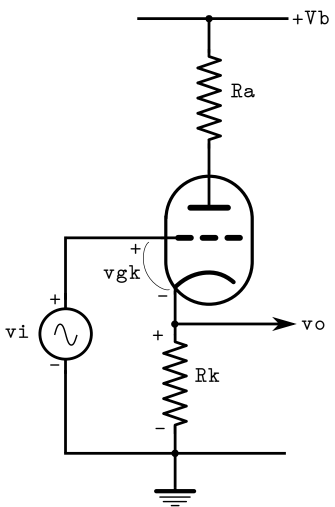


Fig.6

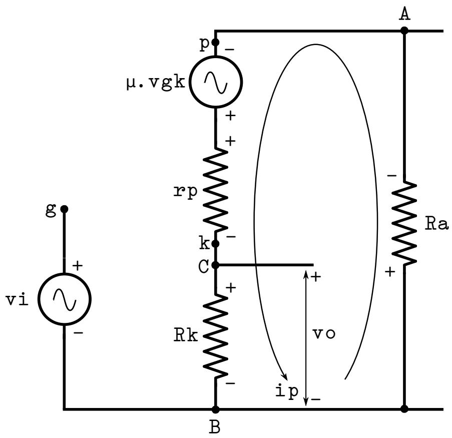


Fig.7

Again, for loop current ip in mesh network BACB according to the 2nd Kirchoff Law:

$$-i_p \cdot R_a + \mu \cdot v_{gk} - i_p \cdot r_p - i_p \cdot R_k = 0$$

Then:

$$v_{gk}=v_i-i_p\cdot R_k \qquad (5)$$

$$v_o=i_p\cdot R_k \quad \text{and} \quad i_p=\frac{v_o}{R_k}$$

$$v_{gk}=v_i-v_o$$

$$\mu\cdot v_{gk}=i_p\cdot R_a+i_p\cdot r_p+i_p\cdot R_k \qquad (1)$$

$$\mu(v_i-i_p\cdot R_k)=i_p\cdot R_a+i_p\cdot r_p+i_p\cdot R_k$$

$$\mu(v_i-i_p\cdot R_k)=i_p(R_a+r_p)+i_p\cdot R_k$$

We substitute $i_p\cdot R_k=v_o$ and for the factor $i_p(R_a+r_p)$ substitute $i_p=\frac{v_o}{R_k}$ resulting in:

$$\mu(v_i-v_o)=\frac{v_o}{R_k}(R_a+r_p)+v_o$$

After some re-arranging we get :

$$\mu\cdot v_i=v_o\{\frac{r_p+R_a}{R_k}+(\mu+1)\}$$

So the expression for vo becomes:

$$v_o=\frac{\mu\cdot v_i}{\frac{r_p+R_a}{R_k}+(\mu+1)}=\frac{\mu\cdot v_i}{\frac{r_p+R_a+(\mu+1)R_k}{R_k}}=\frac{R_k}{r_p+R_a+(\mu+1)R_k}\cdot\mu\cdot v_i$$

$$v_o=\frac{R_k}{(\mu+1)\{\frac{r_p+R_a}{\mu+1}+R_k\}}\cdot\mu\cdot v_i$$

$$v_o=\frac{R_k}{\frac{r_p+R_a}{\mu+1}+R_k}\cdot\frac{\mu}{\mu+1}\cdot v_i$$

I deliberately have reworked de above expression for vo to this form:

$$\boxed{v_o=\frac{R_k}{R_k+\frac{R_i}{\mu+1}+\frac{R_a}{\mu+1}}\cdot\frac{\mu}{\mu+1}\cdot v_i} \qquad (6)$$

And similar to (4), (6) has been written in the form

$$v_o = \frac{R_k}{R_k + R_{out}} \cdot \frac{\mu}{\mu+1} \cdot v_i \qquad (7)$$

Where the effective output resistance is the Rout of the effective voltage source $\frac{\mu}{\mu+1} \cdot v_i$,and we can draw its circuit diagram in Figure8:

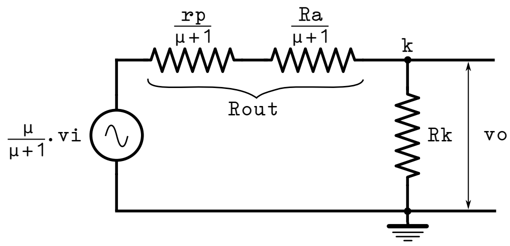


Fig.8

Rules:

When drawing an ac-equivalent circuit for the cathode-loop network configuration, then we multiply a voltage source vi at the grid input with a factor $\frac{\mu}{\mu+1}$ and both the plate resistor rp and the anode resistor Ra with a factor $\frac{1}{\mu+1}$.

Every voltage or impedance (also the plate resistance!) connected to the anode-loop network has to be multiplied with that factor $\frac{1}{\mu+1}$ to get its correct equivalent configuration to the cathode-loop network.

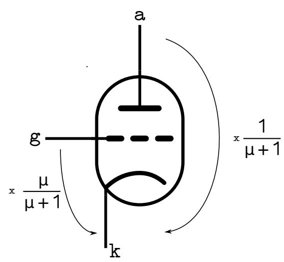


Fig.9

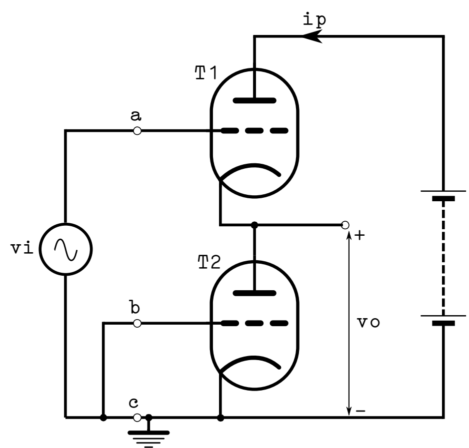
To easily recall this rule, I have drawn Figure9 with its correct multiplying factors.

Some examples.

Example1.

Given:

rp1; rp2; μ1; gm2.

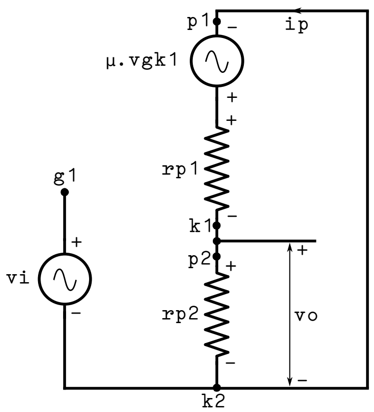


1. Derive the expression for vo when voltage source vi is connected between a and c, and b is connected to ground.
2. Ibidem, when voltage source vi is connected between b and c while a is connected to ground.

<u>Solution:</u>
Using the Barkhausen Formula for calculating μ2 gives

$$\mu_2 = g_{m2} \cdot r_{p2}$$

1.
Starting using the triode ac-equivalent diagram we can calculate the output voltage vo



$$v_o = \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1 \cdot v_{gk1}$$

$$v_o = \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1 (v_i - v_o)$$

$$v_o = \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1 \cdot v_i - \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1 \cdot v_o$$

$$v_o \left(1 + \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1\right) = \frac{r_{p2}}{r_{p1} + r_{p2}} \cdot \mu_1 \cdot v_i$$

$$v_o = -\frac{r_{p2}}{r_{p1} + (\mu_1 + 1) r_{p2}} \cdot \mu_1 \cdot v_i$$

$$v_o = \frac{r_{p2}}{(\mu_1 + 1)\left\{\frac{r_{p1}}{(\mu_1 + 1)} + r_{p2}\right\}} \cdot \mu_1 \cdot v_i$$

$$\boxed{v_o = \frac{r_{p2}}{r_{p2} + \frac{r_{p1}}{\mu_1 + 1}} \cdot \frac{\mu_1}{\mu_1 + 1} \cdot v_i} \qquad (1)$$

We expressed equation (1)

in the form $\quad v_o = \frac{R}{R + R_{out}} \cdot v_i$

<u>which we could have written instantaneously with the help of Figure9</u> and then drawing the ac-equivalent circuit diagram Figure10 for the cathode-loop network configuration of T1:

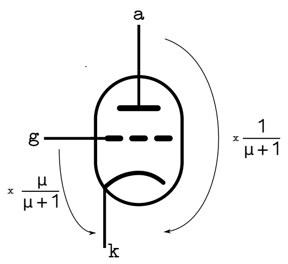


Fig.9

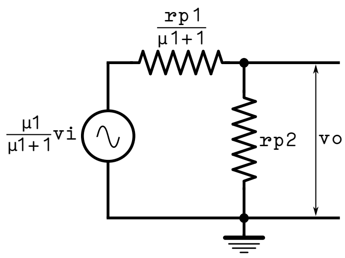


Fig.10

With the use of Figure10 it is not difficult to find the expression (1) for the output voltage vo.

2.
Looking at the circuit diagram below we immediately can draw the ac-equivalent circuit diagram Figure11 for the anode-loop network of T2 with the help of Figure5 and Figure9. Realizing that T2 "sees" the plate resistance of T1 multiplied by it proper factor as its plate load.

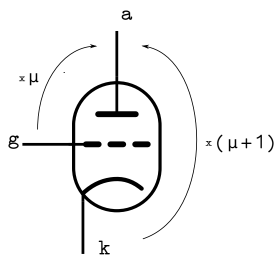
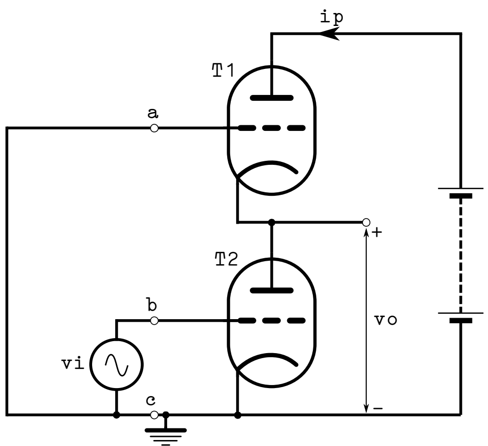


Fig.5



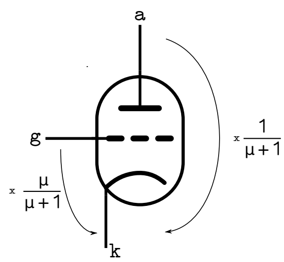


Fig.9

Looking at Figure11 it is easy to write an expression for the output voltage vo (which we will trust the reader to do this).

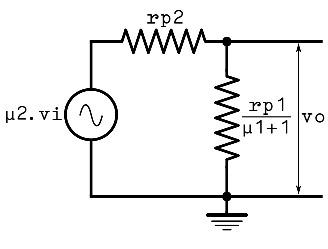
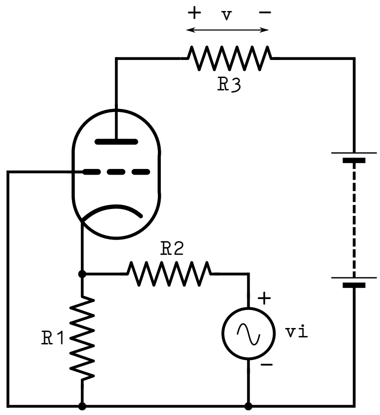


Fig.11

Example 2.



Given:

gm=2mA/V; μ=10; R1=R2=R3=500Ω;
$vi = 1\,Vrms$.

Calculate the voltage v across R3.

Solution:

Open-circuit the cathode and R1/R2, and calculating the transform voltage vk and the transform impedance Rv, using Thevenin's theorem:

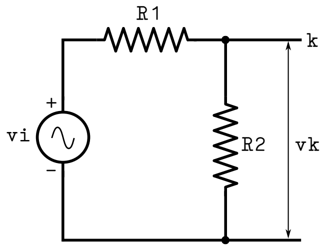


Fig.12

$$vk = \frac{R_2}{R_1 + R_2} \cdot v_i$$

$$Rv = \frac{R_1 \cdot R_2}{R_1 + R_2} = 500\,\Omega$$

Figure13 shows the "calculating" diagram and Figure14 its ac-equivalent circuit diagram when one wants to start deriving the necessary voltages and currents using Kirchoff's Laws:

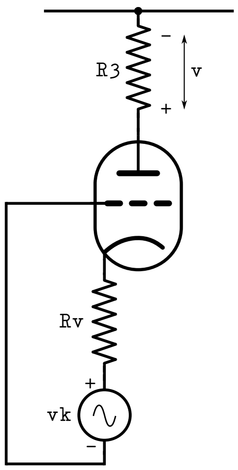


Fig.13

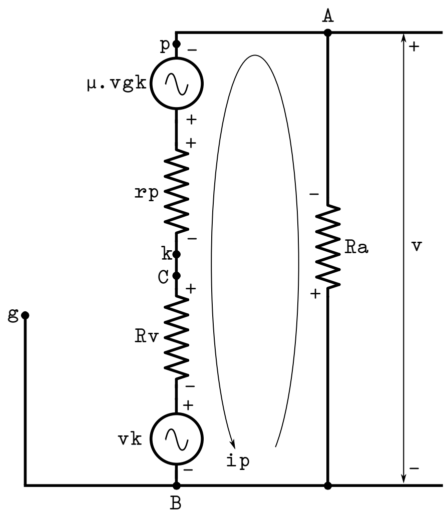


Fig.14

But instead, we will use the method described above drawing the ac-equivalent configuration for the anode-loop network using Figure5 resulting in the diagram of Figure15

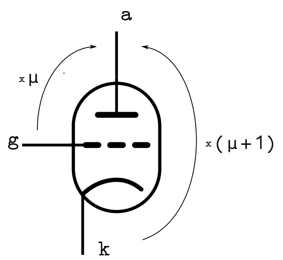


Fig.5

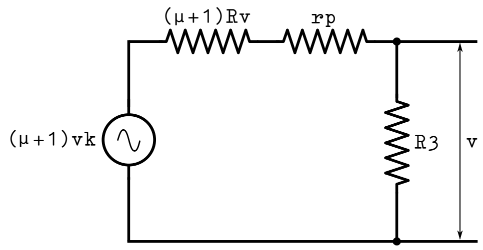


Fig.15

As we can see both Thevenin's transform- voltage and -impedance are multiplied by a factor (μ +1) to obtain its correct value for the anode-loop network. The plate resistor value remains the same.(Because no voltage source is at the input in the "calculating" diagram Figure14, we did not use the multiplication factor μ.).
So the expression for the voltage v is (with the expressions for Rv and vk directly substituted into the equation):

$$v=\frac{R_3}{r_p+R_3+\frac{(\mu+1)R_1\cdot R_2}{R_1+R_2}}\cdot\frac{(\mu+1)R_2}{R_1+R_2}\cdot v_i$$

$$v=\frac{(\mu+1)R_2\cdot R_3}{(\mu+1)R_1\cdot R_2+(R_1+R_2)r_p+R_3(R_1+R_2)}\cdot v_i$$

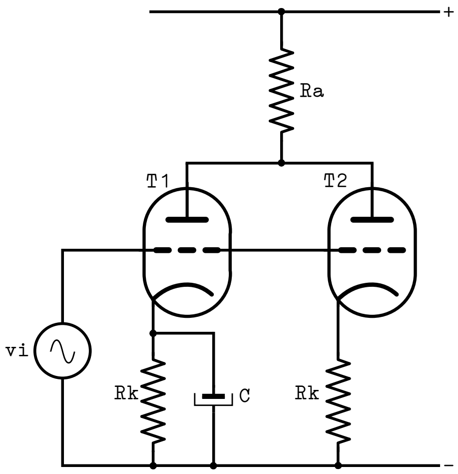
$$\boxed{v=\frac{(\mu+1)R_2\cdot R_3}{(\mu+1)R_1\cdot R_2+(R_1+R_2)(r_p+R_3)}\cdot v_i}$$

Using the values for: μ=10; $r_p=\frac{\mu}{gm}=5\,k\Omega$; R1=R2=R3=0,5kΩ;
$v_i=1\,Vrms$ we have

$$v=\frac{11\cdot 0{,}5\cdot 0{,}5}{11\cdot 0{,}5\cdot 0{,}5+(0{,}5+0{,}5)(5+0{,}5)}\cdot 1$$

$$\boxed{v=0{,}333\,Vrms}$$

Example 3.



Given:

For both tubes gm=5mA/V and μ=50. Ra=2kΩ en Rk=1kΩ. Rk is shorted for all frequencies by C. $v_i = 0{,}1\,V_{(RMS)}$.

Calculate the RMS-value across the anode resistance Ra.

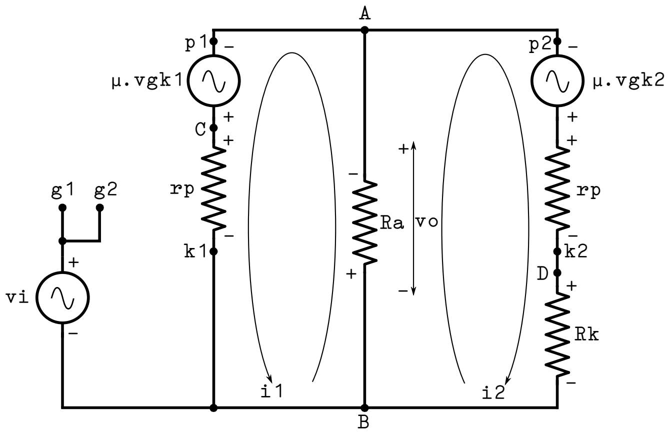
Solution:

From the data given, we immediately calculate the plate resistance rp of both tubes:

$$r_p = \frac{\mu}{gm} = \frac{50}{5} = 10\,k\,\Omega$$

Drawing the triode ac-equivalent diagrams of both triodes in the circuit diagram gives Figure16. Applying Kirchoff's 2nd Law subsequently for mesh network BACB and BADB we arrive at the equations for i1,i2 and vo.

Fig.16



Far easier is it to use Figure5 and apply its multiplication factors (but in the Appendix, I will give the standard way of solving this problem using Figure16):

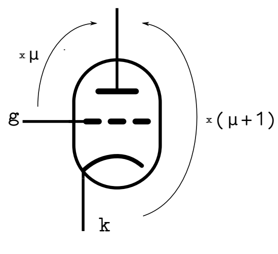
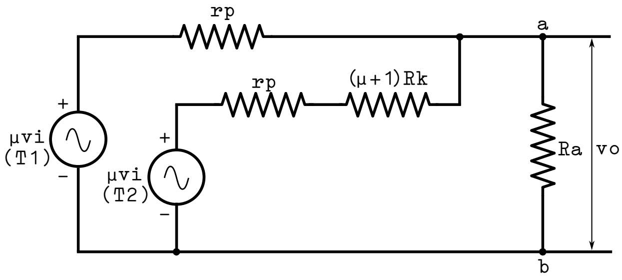


Fig.5

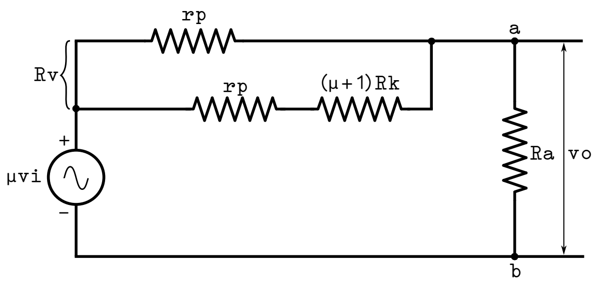
And then draw the ac-equivalent configuration for both the T1(=left) and T2(=right)plate-loop network as in Figure17:

Fig.17



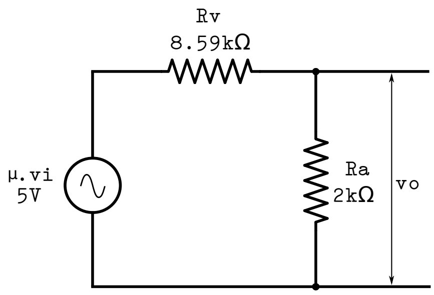
Looking at Figure17 we can calculate vo for instance by applying the superposition theorem, but after taking a closer look we can replace both voltage sources by one voltage source $\mu \cdot v_i$ with a source resistance Rv consisting of rp in parallel with rp+(μ+1)Rk,the way it is is drawn in Figure18 and then immediately calculate vo:

Fig.18



If rp=10kΩ, gm=5mA/V, μ=50, Ra=2kΩ, Rk=1kΩ, Rv=8.59kΩ and $vg=0{,}1\ V_{RMS}$ then the value of vo is given by

Fig.19

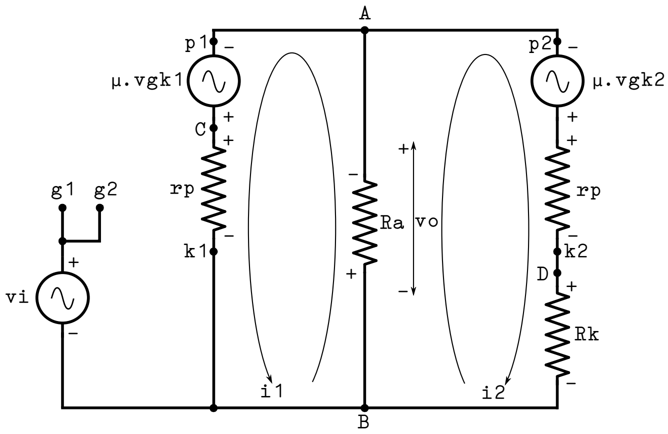


$$\boxed{v_o = \frac{R_a}{R_a + R_v} \cdot \mu \cdot v_i = \frac{2}{10{,}59} \cdot 5 = 0{,}944\,V}$$

1. Sietsma, A.J.(1959). Radiotechniek DEEL I p.257,pp.262-263,pp.267-268.
2. Amos,S.W and Birkenshaw,D.C.(1959). Television Engineering Principles and Practice. Part4 Appendix K.
3. Milton Boone,E. (1953).Circuit Theory of Electron Devices pp.22-24.

APPENDIX

We start from Figure16:



and according to the 2nd Law of Kirchoff we can write for loop current i1 in mesh network BACB:

$$-(i_1+i_2)R_a+\mu\cdot v_{gk1}-i_1\cdot r_p=0$$

$$\mu\cdot v_{gk1}=i_1(r_p+R_a)+i_2\cdot R_a \qquad (1)$$

and for the loop current i2 for mesh network BADB :

$$-(i_2+i_1)R_a+\mu\cdot v_{gk2}-i_2(r_p+R_k)=0$$

$$\mu\cdot v_{gk2}=i_1\cdot R_a+i_2(r_p+R_k+R_a) \qquad (2)$$

For the input loop network of T1:

$$v_i-v_{gk1}=0$$

$$v_{gk1}=v_i \qquad (3)$$

and for the input loop network of T2:

$$v_i-v_{gk2}-i_2\cdot R_k=0$$

$$v_{gk2}=v_i-i_2\cdot R_k \qquad (4)$$

Substituting (3) into (1) gives

$$\boxed{\mu\cdot vi=i_1(r_p+R_a)+i_2\cdot R_a} \quad *) \qquad (5)$$

Substituting (4) into (2) gives

$$\mu(v_i-i_2\cdot R_k)=i_1\cdot R_a+i_2(r_p+R_k+R_a)$$

$$\boxed{\mu\cdot v_i=i_1\cdot R_a+i_2\{r_p+(\mu+1)R_k+R_a\}} \quad *) \qquad (6)$$

Now we subtract (6) from (5):

$$\begin{array}{llr} \mu\cdot v_i=i_1(r_p+R_a) & +\, i_2\cdot R_a & (5) \\ \mu\cdot v_i=i_1\cdot R_a & +\, i_2\{r_p+(\mu+1)R_k+R_a\} & (6) \quad - \\ \hline 0=i_1\cdot r_p & -i_2\{r_p+(\mu+1)R_k\} & \end{array}$$

$$\boxed{i_1\cdot r_p=i_2\{r_p+(\mu+1)R_k\}} \qquad (7)$$

This results in the following relationship between ip1 and ip2:

$$i_1=\frac{r_p+(\mu+1)R_k}{rp}\cdot i_2 \qquad (7a)$$

It is at this stage easier to substitute all the known parameters into (5),(6) and (7) before proceeding further.

$r_p=10\,k\Omega$; $\mu=50$; $R_a=2k\Omega$; $R_k=1\,k\Omega$; $v_i=0{,}1\,V_{(RMS)}$

Giving:

$$5=12\cdot i_1+2\cdot i_2 \qquad (5)$$

$$5=2\cdot i_1+63\cdot i_2 \qquad (6)$$

$$i_2=0.164\cdot i_1 \qquad (7a)$$

Substituting (7a) into (5) gives for i2:

$$5=75.2\cdot i_2$$

$$i_2=\frac{5}{75.2}=0.066\,mA$$

This again substituting in (5) gives i1:

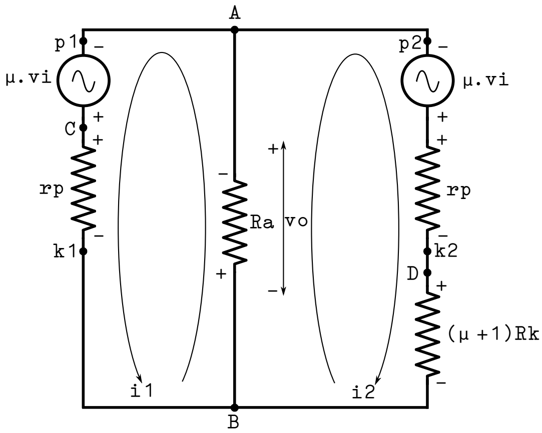
$$i_1 = \frac{5 - 0.132}{12} = 0.406\,mA$$

So we finally arrive at vo:

$$v_o = (i_1 + i_2)\,R_a \qquad \boxed{v_o = 0.944\,V}$$

*)
If we redraw Figure16 using the above described faster method and then calculate the loop currents i1 and i2:

Fig.17a



The i1 loop current for mesh network BACB:

$$-(i_1 + i_2)\,R_a + \mu \cdot v_i - i_1 \cdot r_p = 0$$

and <u>immediately arrive at equation (5)</u> in the Appendix:

$$\mu \cdot vi = i_1 (r_p + R_a) + i_2 \cdot R_a$$

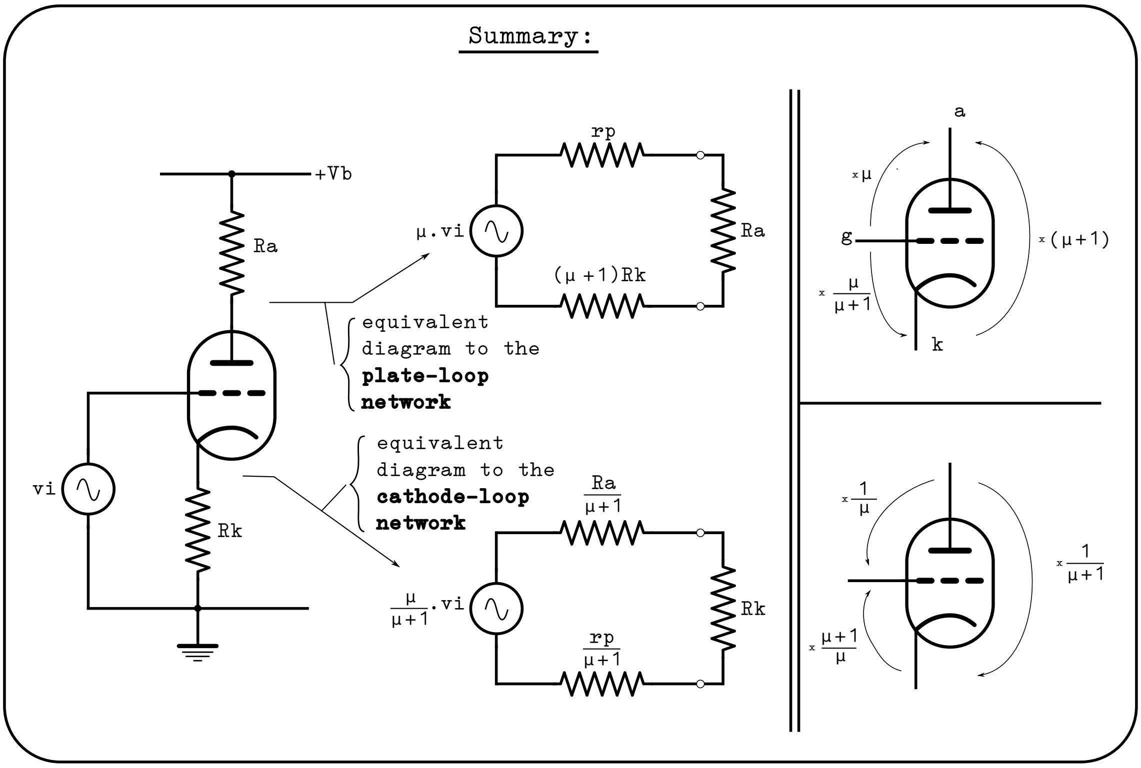
The i2 loop current or mesh network BADB:

$$-(i_2 + i_1)\,R_a + \mu \cdot v_i - i_2 (r_p + R_k) = 0$$

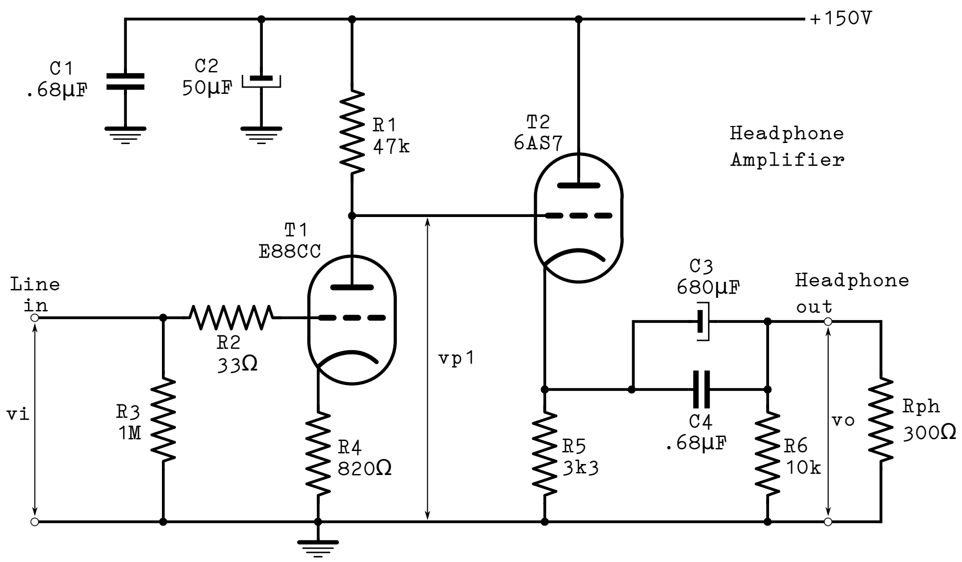
<u>arriving immediately at equation (6)</u> in the Appendix:

$$\mu \cdot v_i = i_1 \cdot R_a + i_2 \{ r_p + (\mu + 1)\,R_k + R_a \}$$

In the Figure below, I give a schematical summary of the above described procedure to use the fast calculation method.

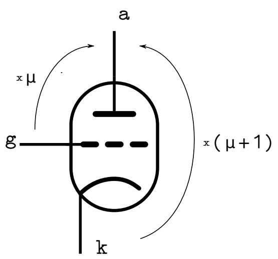


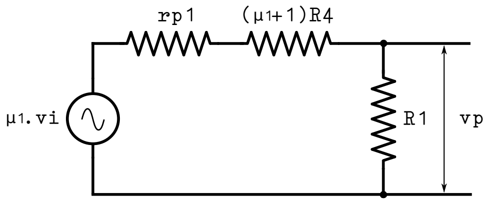
We will finally use the above described method for AC-voltage gain calculations on a headphone amplifier and compare the results with those when using LTspiceIV.



The first step is calculating the voltage gain for the first gain stage with T1.

The ac-equivalent circuit is derived with the help of Figure5:





The relevant tube data for T1=E88CC : μ=33 ; rp1=2.64kΩ ;
For the used passive components : R4=820Ω ; R1=47kΩ.
Calculating the voltage gain A1:

$$v_{p1}=\frac{R_1}{r_{p1}+(\mu_1+1)R_4+R_1}\cdot\mu_1\cdot v_i$$

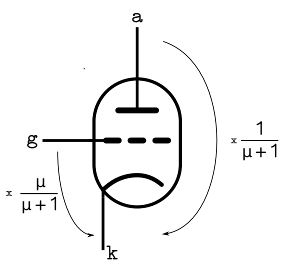
$$A_1=\frac{v_{p1}}{v_i}=\frac{\mu_1\cdot R_1}{R_1+(\mu_1+1)R_4+r_{p1}}$$

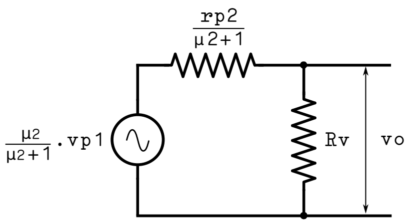
$$A_1=\frac{33\cdot 47}{47+(33+1)0.82+2.64}=\frac{1551}{77.52}=20$$

Expressing the voltage gain in dB gives:

$$A_1=20\log 20=26\,dB$$

Proceeding further deriving the ac-equivalent circuit for the second gain stage T2 with the help of Figure9:





The relevant tube data for T2=6AS7 : μ=1.54 ; rp2=280Ω ;
For the used passive components : R5=3k3 ; R6=10kΩ ;
Rph=300Ω.

The equivalent resistor Rv is $\frac{1}{R_v}=\frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{R_{ph}}$. Rv=268Ω.

Calculating the voltage gain A2 gives

$$v_o = \frac{R_v}{R_v + \frac{r_{p2}}{\mu_2 + 1}} \cdot \frac{\mu_2}{\mu_2 + 1} \cdot v_{p1}$$

$$A_2 = \frac{v_o}{v_{p1}} = \frac{R_v}{R_v + \frac{r_{p2}}{\mu_2 + 1}} \cdot \frac{\mu_2}{\mu_2 + 1}$$

$$A_2 = \frac{0.268}{0.268 + \frac{0.280}{1.54 + 1}} \cdot \frac{1.54}{1.54 + 1} = \frac{0.164}{0.378} = 0.43$$

Expressing the voltage gain in dB gives:

$$A_1 = 20 \log 0.43 = -7.3\, dB$$

The table below gives the results obtained by LTspice and hand calculations.

Headphone-amplifier	VOLTAGE GAIN	
	Hand calculated	LTspiceIV
E88CC gain stage	26 dB	25 dB
6AS7 gain stage	-7.3 dB	-7.4 dB
Total voltage gain	18.7 dB	17.6 dB

1. ECC88:Electronic Tube Handbook (1971) Muiderkring-Publishers, Netherlands.
2. 6AS7 :Svetlana 6AS7 specification, Svetlana Electron Devices USA.

J.Alders
Grave,the Netherlands, 31/XII/2016.