Analysis of an Uncontrolled Single-Phase Power Supply Rectifier Circuit

with Source Inductance and Resistance, a Reservoir Capacitor with ESR, and a Constant-Current Load

Revision dV8

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Introduction:

Several authors have attempted to analyze AC-to-DC power supply rectifier circuits. But apparently none has included all of the components of a practical circuit of the type that might be used with an audio amplifier or a voltage regulator as the load, while also not using coarse approximations that make the results inaccurate when the ripple voltage is not small compared to the average DC output level. The effects of source inductance, which can cause the output voltage peaks to be higher than the input voltage, have usually also not been included.

This paper's analysis includes the inductance and resistance of a power transformer or other source, which enables the effects of transformer size, e.g. Volt-Amp Rating and Output Voltage Rating, to be included, and also enables the output voltage minima and maxima to be more-accurately predicted.

A reservoir/filter capacitance is also included, with the capacitance's ESR (Equivalent Series Resistance), which improves the accuracy of the results and enables a designer to better-specify a real capacitor and also to better-evaluate the trade-offs involved when considering using multiple smaller parallel capacitances.

Also, rather than a simple resistive load, a power amplifier or voltage regulator in series with a resistive load is assumed. This not only makes the analysis more useful. It also adds a degree of simplification, because in order to enable the determination of the minimum reservoir capacitance value that will guarantee that the power rail voltage cannot violate the amplifier or regulator's "dropout" region, a constant DC or square wave signal (in the case of an amplifier load), *with an "on" voltage equal to the peak sine voltage for a given output power rating*, is a convenient quasi-worst-case signal for analysis. And considering only a portion of the positive part of one output cycle with a square wave (or constant DC) signal is equivalent to considering a constant current load.

In the case of an amplifier load with a resistive load in series, it is assumed that the amplifier behaves as a time-varying resistance, which maintains the output current through its driven load resistance at a

constant level during the "on" portions of a square wave (or constant DC) signal, within its ability per its PSRR (Power Supply Rejection Ratio), which is easily verified to be the case, with a Spice simulation, where it can also be seen that the minima of the power supply's output voltage ripple must not fall below the sum of the voltages across the load resistance and the amplifier's power output stage, which for the purposes of this analysis might, for example, consist of a bipolar transistor's collector-emitter voltage in series with a sub-one-Ohm resistor.

A voltage regulator load has a very similar requirement for the power supply output voltage minima to stay above the region where its dropout voltage clearance would be violated.

Attempting a closed-form mathematical analysis of simple rectifier circuits is surprisingly complex and tedious and most authors resort to using one or more approximations, in order to simplify or make possible the analysis. One common approximation technique that is used is to assume that the filter capacitance discharges linearly, rather than exponentially. In this paper's analysis, the use of a constant-current load makes that approximation unnecessary, while still providing the benefit of that type of simplification. Another common assumption, that the ripple voltage amplitude is very small compared to the average output voltage, will be avoided, here.

The circuit equations herein will necessarily include the voltage, current, and resistance of one or more rectifier diodes. Since this analysis is intended to be implemented in computer software, if necessary it will be assumed that if given the diode voltage or current, then the diode's voltage, current, and resistance can all be known, either using a table-lookup function or an equation.

Typically, when attempting to derive a closed-form solution for these types of circuits, a significant problem is encountered when, inevitably, the point in time (or phase angle) when the decaying exponential capacitor voltage intersects the rising sinusoidal transformer secondary output voltage must be found, to determine when the rectifier diodes begin conducting again. The resulting equation is a transcendental equation, which has no closed-form mathematical solution.

Some authors have used clever methods to circumvent that problem, such as using the first few terms of a Taylor Series approximation for the sinusoid or the exponential or both. And some authors have used the LambertW function, we^w, to advantage.

While a closed-form solution will remain as a desired goal, enabling engineering work to proceed is also a worthy goal. Therefore, this current paper will focus on defining the relevant differential equations and implementing a numerical solution. The transcendental equation will be avoided and, in addition to the differential equations that describe the circuit behaviors, at most, only equations that define the rectifier mode-transition conditions will need to be derived. And those equations will even be allowed to contain differential terms, since all of the derivatives will be available, at all times during the numerical solver software's iterations.

Variables and Symbols Used:

|	
t	time (seconds)
f	AC Mains frequency (Hertz)
ω	AC Mains angular frequency (= $2\pi f$ Radians per Second)
T	AC Mains period (= $1/f$ Seconds)
t_{on}	time when diode conduction starts (Seconds)
t_{off}	time when diode conduction stops (Seconds)
t_c	tcharge: time interval during which diode conducts and capacitor charges
t_d	tdischarge: time interval during which capacitor discharges
$V_{out}(t)$	Output voltage at time t (Volts)
$V_{cap}(t)$	Capacitor voltage at time t (Volts)
$Vs_{in}(t)$	Transformer secondary's input voltage at time t
$R_d(t)$	Resistance of diode at time t
$i_d(t)$	Current from transformer secondary winding at time t, when diode is conducting (Amps)
i_L	Constant current through load and amplifier output stage
C	Capacitance value of power supply's reservoir capacitance (Farads)
$i_c(t)$	Current through capacitor at time t
R_E	ESR (Equivalent Series Resistance) of filter capacitance (Ohms)
L_s	Leakage inductance of transformer secondary, or source inductance (Henrys)
$V_{Ls}(t)$	Voltage across inductance L_s at time t
R_s	Resistance of transformer secondary, or source (Ohms)
$V_{Rs}(t)$	Voltage across resistance R_s at time t
$V_E(t)$	Voltage across capacitor's equivalent series resistance, R_E, at time t

Analysis:

One possible version of the type of circuit being considered is shown in Figure 1. (Note that the current source labeled "iload" is not strictly an ideal current source, as might be assumed from the figure, since it contains an active load and that active circuit's driven load resistance. Thus the depicted current source can have resistance and can have a non-zero voltage across it.)

This analysis will mainly consider the power supply behavior during the positive "on" portion of a constant DC or square wave output signal. Only the positive voltage rail of a dual-polarity power supply supplying a single active load with a series load resistance will be considered. The equivalent of a push-pull type of amplifier load, driving a single series load resistance, drawing a constant current during the portions of the constant DC or square wave output being considered, will be assumed.

Circuit operation:

When the absolute value of the transformer secondary's output voltage amplitude is large-enough, relative to the power supply's output voltage, Vout, a pair of the rectifier diode(s) will be forward biased and will allow current to flow from the transformer secondary to the filter capacitor and the load, as needed. When the absolute value of the transformer secondary's output voltage amplitude is not large-enough, none of the rectifier diode(s) will be forward biased and no current will flow from or to the transformer secondary through the rectifier diode(s) and any needed load current will then be supplied by only the reservoir capacitor.

At a minimum, we desire to be able to accurately predict the power supply output voltage's and current's minimum, maximum, and average steady-state values, given the circuit parameters.
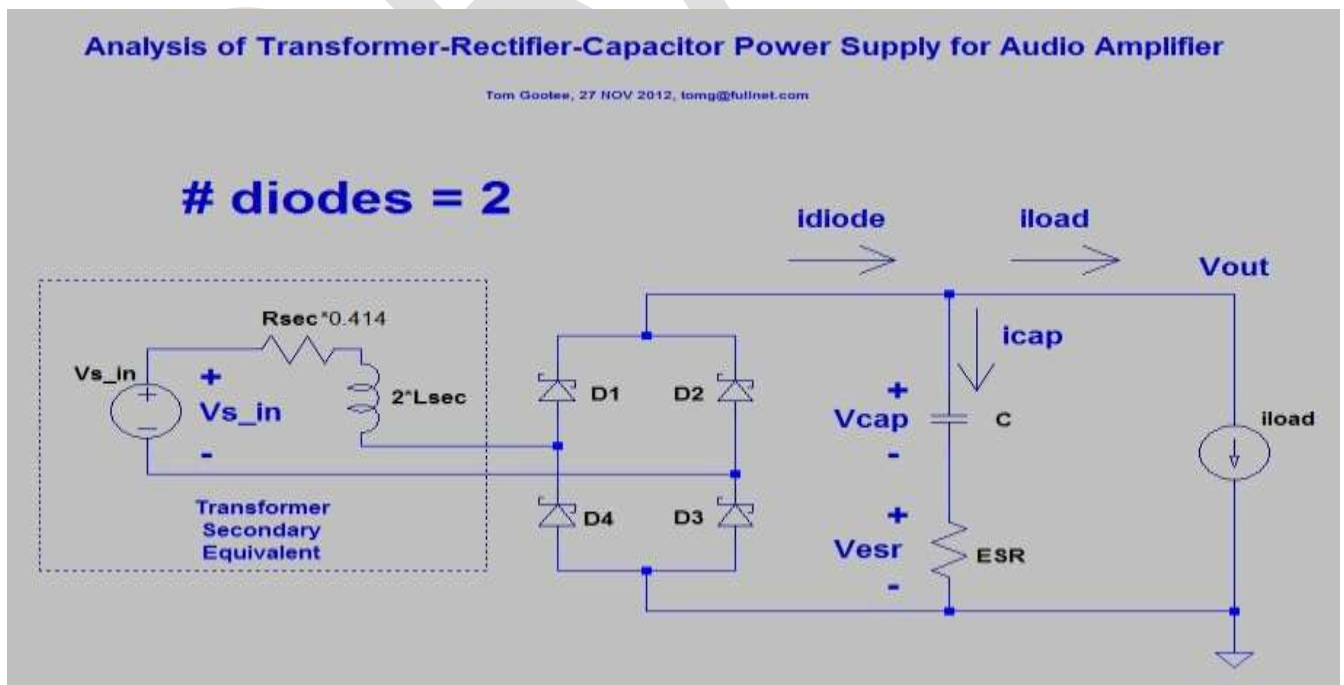
Figure 1

By inspecting the circuit of Figure 1 and using Kirchoff's Current Law, we can write

$$i_d(t) = i_C(t) + i_L \quad (1)$$

Also, we can see that

$$V_{out}(t) = V_{cap}(t) + V_E(t)$$

$$V_{out}(t) = V_{cap}(t) + i_C(t) \cdot R_E \quad (1a)$$

Using the equation that relates the current and voltage of an ideal capacitor,

$$C \cdot \frac{dV_{cap}(t)}{dt} = i_c(t)$$

and substituting into (1a), we could also use, equivalently:

$$V_{out}(t) = V_{cap}(t) + R_E \cdot C \cdot \frac{dV_{cap}(t)}{dt} \quad (2)$$

Diodes OFF:

During the portion (if any) of each half of the AC cycle when none of the rectifier diodes are conducting, the capacitance, its ESR, and the constant-current load can be considered, alone:

When the diodes are not conducting,

$$i_d(t) = 0$$

$$i_C(t) = -i_L$$

$$C \cdot \frac{dV_{cap}(t)}{dt} = -i_L \quad (3)$$

$$\frac{dV_{cap}(t)}{dt} = -\frac{1}{C} i_L \quad (4)$$

Equation (4) is a first-order differential equation with a known solution for $V_{cap}(t)$ as a function of time, for any initial capacitor voltage, $V_{cap}(0)$.

We can also see that V_{out} can be expressed in terms of only V_{cap} and constants:

$$V_{out}(t) = V_{cap}(t) - i_L \cdot R_E$$

Diodes ON:

When the diodes are conducting,

$i_d(t) = i_C(t) + i_L$, which is the same as

$$i_d(t) = C\frac{dV_{cap}(t)}{dt} + i_L \quad (5)$$

And therefore

$$\frac{di_d(t)}{dt} = C\frac{d^2V_{cap}(t)}{dt^2} \quad (6)$$

Noting that the equation relating voltage and current in an ideal inductor is

$$V = L\frac{di(t)}{dt}$$

we can use Kirchoff's Voltage Law and equations (5) and (6) and sum the voltages around the loop to zero, assuming the input voltage for the transformer's secondary winding is $Vs_{in} \cdot \cos(\omega t + \varphi)$:

$$Vs_{in} \cdot cos(\omega t + \varphi) - \left(C\frac{dV_{cap}(t)}{dt} + i_L\right)R_s - L_sC\left(\frac{d^2V_{cap}(t)}{dt^2}\right) - \left(C\frac{dV_{cap}(t)}{dt} + i_L\right)R_d =$$

$$V_{cap}(t) + R_EC\frac{dV_{cap}(t)}{dt} \quad (6a)$$

Regrouping and rearranging, we get

$$\frac{d^2V_{cap}(t)}{dt^2} + \left(\frac{R_s+R_d+R_E}{L_s}\right)\frac{dV_{cap}(t)}{dt} + \frac{V_{cap}(t)}{L_sC} = \left(\frac{Vs_{in}}{L_sC}\right)cos(\omega t + \varphi) - \left(\frac{R_s+R_d}{L_sC}\right)i_L \quad (7)$$

which is a standard form for a non-homogeneous second-order ordinary differential equation that is typical of driven series RLC circuits.

The solution, i.e. the equation for $V_{cap}(t)$, for such linear second-order equations is well known. However, the diode resistance, R_d, is not constant, and is a non-linear function of the time-varying current through the diode or the voltage across the diode (we could choose either one). Rather than proceeding with possible solution methods for the non-linear differential equation (7), at this time, which could result in equations that would give the circuit behavior given the component values and enable the desired analysis and design work to be performed, this paper will, instead, focus on the quick creation of a usable, practical solution, for the benefit of those who merely want to find component values that solve their real-world power supply design problems. Therefore, a simple numerical solution engine will be developed, which can be implemented on almost any personal computer.

Numerical Solution:

For implementation of the numerical solutions of equations (4) and (7), a particular diode was chosen and a p-Spice model of the diode, supplied by the diode's manufacturer, was used, in LT-Spice, to produce data points for the diode's current versus the voltage across the diode. The data points were exported from LT-Spice and MS Excel was then used to calculate the resistance for each pair of current and voltage points, using Ohm's Law. Then, curve-fitting software was used to determine an equation that approximated the diode's resistance, versus the current through the diode, which was valid over a sufficient current range of interest.

For an OnSemi MBR20100CT diode, one of the equations found to be sufficiently accurate is:

Y = (A+B*X**C) / (D+X**C) with

A=0.48953E+00   B=0.62604E-02   C=0.87411E+00   D=0.44516E-03

or

$$R_d(i_d) = \frac{0.48953 + 0.0062604 \cdot i_d^{0.87411}}{0.00044516 + i_d^{0.87411}} \quad (8)$$

The plots of R_d versus i_d from (8), and the plotted percent error, are given in Figure 2 and Figure 3. Note that the plot for the resistance versus the current was truncated so that resistances above ten Ohms are not shown, in order to enable viewing more detail for low resistances. In reality, equation (8) is valid for diode resistances up to at least 100 to 500 Ohms, which corresponds to a diode current of 2 mA to 0.2 mA, and the equation could probably be used for diode resistances up to 2500 Ohms, which corresponds to a diode current of about 0.5 µA.
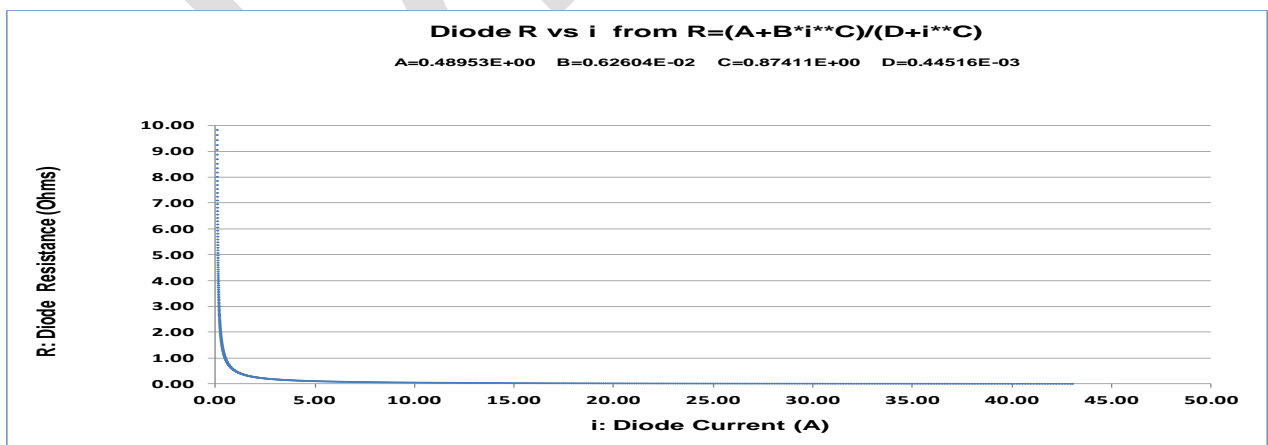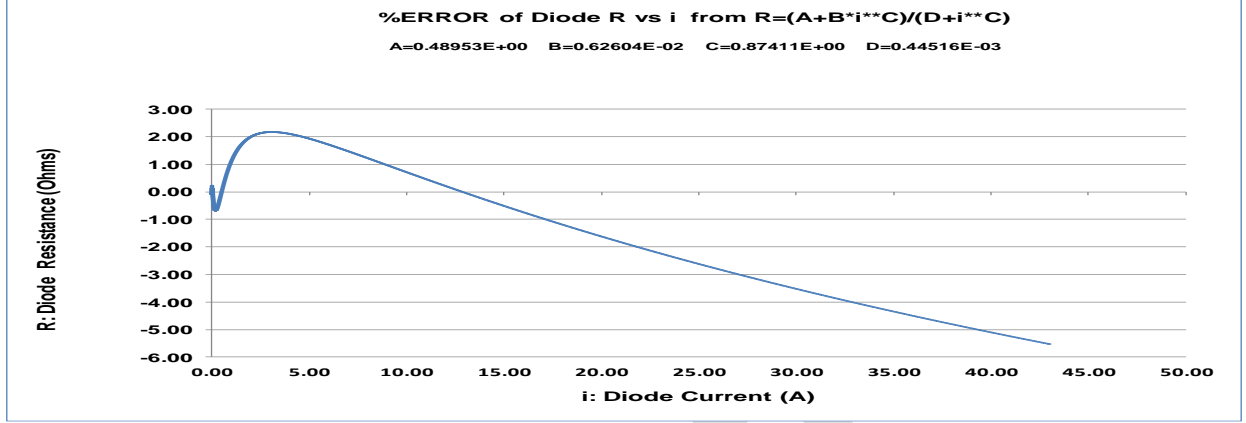

Figure 2: Diode Resistance vs Forward Current, from Curve-Fitted Equation (8)

Figure 3: Error Percentage for Diode Resistance from Curve-Fitted Equation (8)

In order to implement a numerical algorithm to generate $V_{cap}(t)$ and related values, a fourth-order Runge-Kutta method was selected, which is capable of solving two simultaneous first-order differential equations. In order to implement the algorithm, equation (7) was converted from a second-order equation into a system of two first-order equations, where

$$y = V_{cap}(t) \quad (9)$$

$$z = \frac{dV_{cap}(t)}{dt} \quad (10)$$

and

$$g = z \quad (11)$$

$$f = \frac{dz}{dt} \quad (12)$$

Substituting (9)-(12) into equation (7), while allowing for ndiodes in series instead of just one, gives:

$$f = \left| \left(\frac{Vs_{in}}{L_s C} \right) cos(\omega t + \varphi) \right| - \left(\frac{R_s + ndiodes \cdot R_d(i_d(t))}{L_s C} \right) i_L - \left(\frac{R_s + ndiodes \cdot R_d(i_d(t)) + R_E}{L_s} \right) z - \frac{y}{L_s C}$$

$$(13)$$

where $R_d(i_d(t))$ is the diode resistance, from equation (8), a non-linear function of the time-varying diode current, and the absolute value of the input voltage is used in order to simulate the behavior of having both pairs of diodes, more easily, and gives equivalent results.

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Some background information: (simplified to the point of being slightly incorrect)

Numerical "solution" of differential equations is conceptually simple. Often, differential equations arise because we know, from physical laws, for example, how to write mathematical expressions and equations involving the derivatives (rates of change) of variables of interest, but we then desire an equation for the behavior of the variable itself, as a function of time for example, in the particular situation for which the differential equation is applicable. The desired mathematical equation for the variable of interest (as a function of time, in our case) IS "the solution" of whatever differential equation we have written.

There are several formal methods for solving linear differential equations with time-invariant coefficients. But not much is known about solving most non-linear differential equations. Therefore, iterative numerical methods are often used, to simply "run" (i.e. simulate) the system described by the differential equation, or set of equations, in order to see how the system would behave.

The basic idea is that if small-enough time steps are chosen, and initial values of the variable of interest and some of its derivatives are chosen or calculated, or guessed-at, for the first several instants of time, *then the value of the variable at each "next" instant of time, from then on, can simply be calculated*, by calculating the derivatives and applying the coefficients in the differential equation(s), and the calculated values will be accurate-enough if the time steps used are small-enough.

After all, the variable's "first derivative" (e.g. dV/dt) is simply the slope of the plot of the values (e.g. of the waveform) of the variable itself, and we can use those values, from two adjacent time steps, and the time step length, to calculate that slope. The "second derivative" (e.g. d^2V/dt^2) is just the slope of the plot of the first derivative. So we can calculate its new value the same way, and so on for any higher-order derivatives. If we set the time steps to be small-enough, and calculate the values of the variable and each of its needed derivatives as we iterate forward to each new time step, and plug those values into the differential equation(s) at each step and find the new values of the variable and its derivatives for the next time step, the calculated values will follow the trajectory that the actual system would follow, and the resulting set of times and values will be a plot of the actual solution of the equation versus time, which in our case will be the waveform that would result from running the actual circuit!

It turns out that the obvious way of calculating the derivatives, using the simple slope formula, which is called Euler's Method, is not the best we can do. While it can almost always be made to work well-enough, Euler's Method has a built-in problem which can make it require excessively-small time steps if we want it to be accurate-enough as time gets farther from t=0. A slightly more-complicated but more-accurate and better-behaved method of calculating the new value (at the next time step) of a variable and its first derivative is the Runge-Kutta type of algorithm. It comes in more than one level of complexity but the fourth-order version is great for our purposes, here.

For that case, the "new" values of the variable and its first derivative (ynew and znew, in our case) would be calculated, for each time step, using the following standard Runge-Kutta equations, shown here as part of a Visual Basic subroutine named rk2sr. Note that "h" is the time-step duration, and that

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the f() and g() functions contain the differential equations, solved for the second and first derivatives of Vcap(t), respectively.

Fourth-Order Runge-Kutta subroutine for solving two first-order differential equations:

Sub rk2sr(h, t, y, z, ynew, znew) ' Fourth-Order Runge-Kutta equations

k1 = h * g(t, y, z)

l1 = h * f(t, y, z)

k2 = h * g(t + h / 2, y + k1 / 2, z + l1 / 2)

l2 = h * f(t + h / 2, y + k1 / 2, z + l1 / 2)

k3 = h * g(t + h / 2, y + k2 / 2, z + l2 / 2)

l3 = h * f(t + h / 2, y + k2 / 2, z + l2 / 2)

k4 = h * g(t + h, y + k3, z + l3)

l4 = h * f(t + h, y + k3, z + l3)

znew = z + (l1 + 2 * (l2 + l3) + l4) / 6

ynew = y + (k1 + 2 * (k2 + k3) + k4) / 6

End Sub

Back to the current problem:

We have equation (4) for the times when the diodes are not conducting and equation (7) for when the diodes are conducting (or equations (11) and (13)). Therefore, the software that iterates through the time steps will need to keep track of which of the two differential equations it is currently using and will have to calculate when to switch to the other equation.

In general, we could use terms from equation (6a) or (7) and check whether or not the diodes are forward-biased, by comparing the voltages just upstream and downstream from the diode(s):

$$|Vs_{in} \cdot cos(\omega t + \varphi)| - \left(C \frac{dV_{cap}(t)}{dt} + i_L \right) R_s - L_s C \left(\frac{d^2 V_{cap}(t)}{dt^2} \right) \leq V_{cap}(t) + R_E C \frac{dV_{cap}(t)}{dt} \quad (14)$$

would indicate a non-conducting diode or diode pair.

However, when considering only a fixed constant load current, some simpler conditions can be derived.

Determination of when a "Diode Off → On" transition occurs:

By inspecting the circuit in Figure 1 we can see that the voltage at the upstream end of the diode is the transformer secondary's input voltage minus the voltages across the secondary's resistance and inductance. However, if the diodes are not conducting and we need to determine when they should start conducting again, then we can use the fact that the current through each diode has been zero for some time, and thus the current through the secondary winding and its resistance and inductance has also been zero for some time. Therefore the first derivative (rate of change) of the same current must also be zero, as we approach the diode turn-on time. And therefore the voltages across both the resistance and the inductance of the secondary are zero, in that situation. Therefore the voltage at the upstream end of the diode must be equal to the input voltage of the secondary winding.

We also know that when the diodes are not conducting, the capacitor is discharging linearly into the load, due to the constant load current pulling charge out of the capacitor. The voltage at the downstream end of the diode in the forward path is designated as the output voltage, Vout. We can now say that if the diode is not conducting then it will start conducting when:

$$|Vs_{in} \cos(\omega t + \varphi)| \geq V_{out}(t) \quad (15)$$

Using equation (2), we get

$$|Vs_{in} \cos(\omega t + \varphi)| \geq V_{cap}(t) + R_E \cdot C \cdot \frac{dV_{cap}(t)}{dt} \quad (16)$$

In terms of the numerical solution, we then have

$$|Vs_{in} \cos(\omega t + \varphi)| \geq y + R_E \cdot C \cdot z \quad (17)$$

Equation (17) must become true at the diode turn-on time.

Determination of when a "Diode On → Off" transition occurs:

In the case when the diode(s) is(are) conducting and the numerical algorithm software needs to determine when it(they) should "turn off":

We assume that a diode's current is zero, or goes to zero, at the moment the diode stops conducting.

But

$$i_d(t) = i_C(t) + i_L$$

So, when $i_d(t)$=0:

$$0 = i_C(t) + i_L$$

$$i_C(t) \rightarrow -i_L$$

$$C \cdot \frac{dV_{cap}(t)}{dt} \leq -i_L$$

$$\boldsymbol{C \cdot \frac{dV_{cap}(t)}{dt} \geq i_L}$$

In terms of the numerical solution, we then have:

$$\boldsymbol{C \cdot z \geq i_L} \quad \textbf{(18)}$$

Equation (18) must become true at the diode turn-off time.

Equations (17) and (18) were used for the first version of the numerical algorithm and worked well. But when the algorithm was changed to enable it to handle step inputs, it became more convenient to use equation (17) and an equivalent of equation (14).

Initial Conditions:

The differential equations specify the rate of change of the variable of interest, and those of its derivatives. But in order to simulate a specific solution, a starting point must be provided, for the variable's value and for each derivative except the highest-order one. This is true for both closed-form solutions and iterative numerical solutions.

For the second-order differential equation that was derived for the circuit in Figure 1, the initial capacitor voltage and its first derivative must be supplied, in order to generate a particular solution.

The initial capacitor voltage can be chosen to be any voltage. But if we are only interested in seeing the steady-state behavior, i.e. after the circuit has been running for some time and the response has settled to a steady condition, then we would probably want to select an initial voltage that is near-enough to the final average output voltage, to minimize the number of time steps needed to reach steady state. If we want to see the startup behavior of the power supply, then we could choose zero as the initial voltage.

The software implementation described farther below provides an initial guess for the capacitor voltage, which is intended to be near the final steady-state average value. But a user-entry field is provided, so that the user can override the initial capacitor voltage value.

For the software implementation described below, a simple method was used for specifying the initial value of dV_{cap}/dt, by assuming that the diodes were not conducting (which was originally going to be a requirement imposed by the very simple original algorithm), so that dV/dt is, from (4):

$$\frac{dV_{cap}(0)}{dt} = -\frac{1}{C}i_L \quad (19)$$

While (19) will not always be strictly correct (e.g. if the diodes are initially conducting), in practice for the circuits being considered the numerical algorithm quickly forces the solution to follow the correct trajectory, anyway.

The formal method for selecting the initial conditions would be to use the applicable closed-form solution of equation (7), differentiating the solution and using the resulting equation to help determine dV/dt at t=0, given V(t) at t=0. The non-linear time-varying diode resistance might complicate that process. And since we have chosen to use a numerical solution method instead of attempting to derive a complete closed-form solution, and since experimentation with the resulting numerical solver software has shown that the results are almost identical when using a large negative value for dV/dt and when using zero, the initial conditions will probably be handled sufficiently-well by setting up the software to allow user override of any automatically-calculated initial conditions.

Software Implementation:

The MS Excel platform was used to implement the numerical algorithm and a user interface. An image of the main user interface portion is shown in Figure 4.
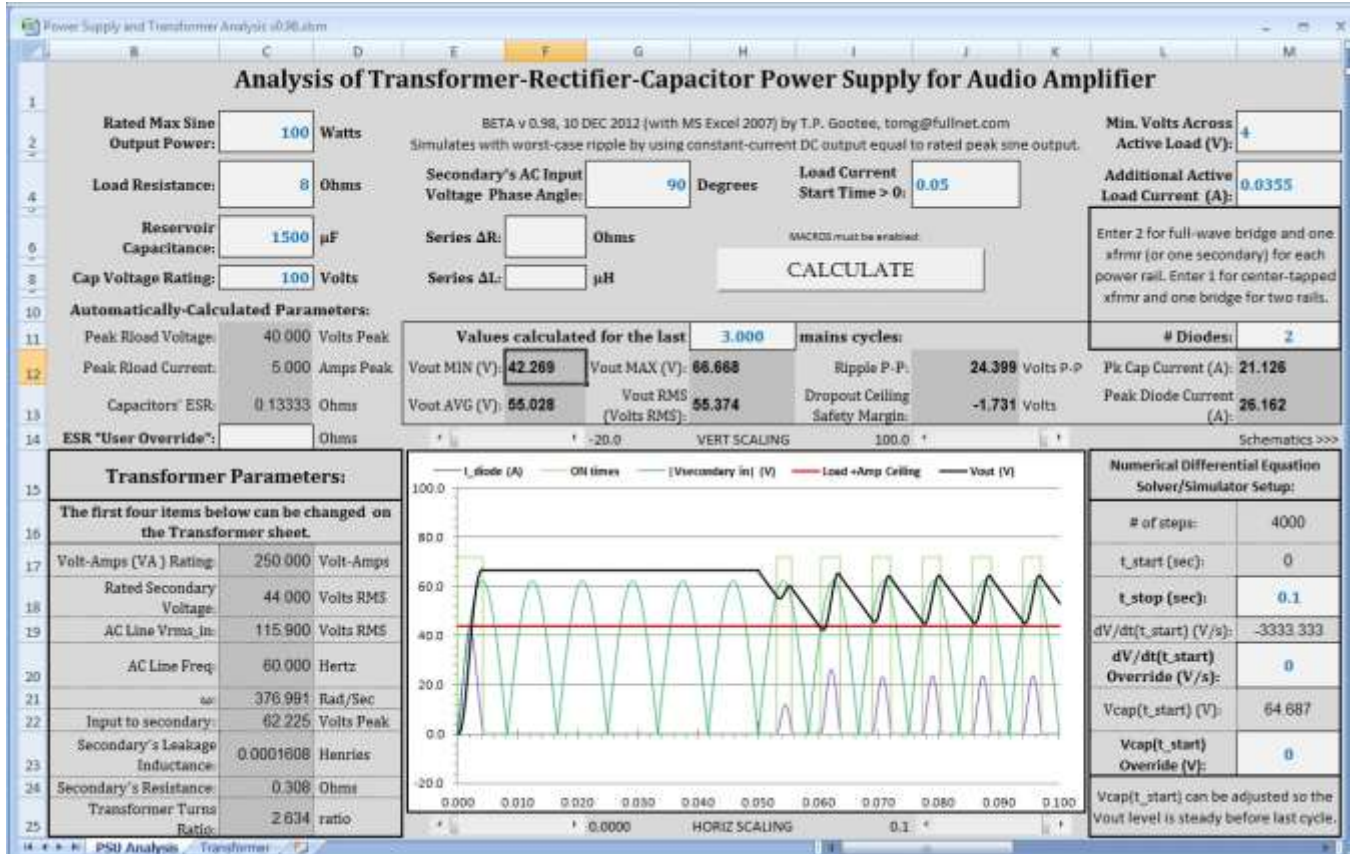

Figure 4 – Main user interface screen for numerical solver in MS Excel

A VBA (Visual Basic for Applications) subroutine macro was written, which runs each time the "CALCULATE" button is selected. The VBA subroutine reads and writes values from the cells of the spreadsheet depicted in Figure 4 and implements the numerical solution algorithm. The values for the plots shown in Figure 4 are also calculated in the VBA subroutine, and are written in spreadsheet rows below the rows shown in Figure 4, with one set of calculations (for one time-step) per row. An example of a small portion of the output rows is shown in Figure 5.

Software Limitations:

In a case where the user has entered an initial capacitor voltage that is not close to the final average output voltage level of the power supply, such as when zero is entered in order to see the start-up behavior, it could be the case that the capacitor voltage will not be in a steady state when the stop time occurs, especially when a large capacitor value has been entered. In that case, many of the performance measures that are calculated and displayed, which are calculated using the data for the last half-cycle of

the AC line voltage, will not be representative of the power supply's actual performance. The software does not currently check whether or not steady state has been reached and also does not calculate the possible error due to the time-step duration.

To mitigate the problem, the user could increase the value of the stop time, t_stop. But unless the value of the number of time steps was also increased, then the time duration of each step would increase, which would result in diminished accuracy, to some unknown extent. The number of steps could also be increased, to avoid diminishing the accuracy of the results. But then the user would need to manually edit the plot settings, to include the additional rows of data that would be generated.

t	vcap	dvcap/dt	Vout	id	non-zero = yes	Vs_in* cos(wt+p)	ic	Ceiling Voltage	Vd sum
time (sec)	Capacitor Voltage	Cap Volts per Second	Output Voltage	Diode Current	Diode Conducting?	Secondary Voltage	Capacitor Current	Peak Load + Amp Voltage	Vdiodes
0.0000000	0.0000	-500.0000	-0.1000	0.0355	71.5592	0.0000	-5.0000	44.0000	0.638315
0.0000208	-0.0104	-501.3777	-0.1107	0.0217	71.5592	0.4885	-5.0138	44.0000	0.597281
0.0000417	-0.0209	-500.1646	-0.1209	0.0339	71.5592	0.9770	-5.0016	44.0000	0.634275
0.0000625	-0.0312	-496.6735	-0.1306	0.0688	71.5592	1.4654	-4.9667	44.0000	0.696591
0.0000833	-0.0415	-490.9548	-0.1397	0.1260	71.5592	1.9538	-4.9095	44.0000	0.753807
0.0001041	-0.0517	-482.9931	-0.1483	0.2056	71.5592	2.4420	-4.8299	44.0000	0.803413
0.0001250	-0.0616	-472.7751	-0.1562	0.3077	71.5592	2.9300	-4.7278	44.0000	0.846867

Figure 5 – Examples of the output data generated for each time step

The original hope, for the software implementation, was only to be able to display a few cycles of the steady-state response, with total run-times of up to, perhaps, 0.0375 second. The original default mode was to have the software provide an initial guess for Vcap(0) and also allow the user to override that value, in case it wasn't close enough to ensure that steady state would be reached before the last one-half mains cycle, for which the software reported the minimum and maximum Vout, as well as the average and RMS values, the peak-to-peak ripple voltage, and the peak diode and capacitor currents.

While the anticipated short-time steady-state waveforms showed errors of less than 2% when compared to LT-Spice simulations of identical circuits, formal error predictions versus time-step size were not implemented for this software, so the characteristics of the degradation of the accuracy, as the run time was increased without increasing the number of steps and thereby increasing the time-step size, was and is unknown.

However, somewhat-thorough informal testing was performed, with typical component values. The run time was first increased from 0.0375 second to 0.1 second and it was found that there was no significant difference in the accuracy of the waveforms that were produced. And although small spurious oscillations of the peak output voltage sometimes started to appear at durations of around 0.2 seconds or more, while still using the same number (4000) of time steps, the reported peak value was still within 99.8% of the correct value even with 2.0 seconds of run time.

It was also discovered that when the initial capacitor voltage is set to zero, then the transient response, including the initial inrush current pulse waveforms, is almost equally accurate, typically well within

two percent error, when compared to LT-Spice simulations. And the two percent error is likely due to the curve-fitted diode resistance equation's error.

The robustness of the software's performance is much better than expected and therefore it could be a useful tool for the design and analysis of transformer-rectifier-capacitor power supply circuits. It was made even more useful by the addition of several parameters that can be entered by users, such as an override of the calculated ESR value, and positive or negative "delta" values to simulate changes in the circuit's series inductance and resistance, the secondary input voltage's phase angle (-90 to +90 degrees), the number of diodes in series when conducting (which enables also simulating full wave rectifier circuits with center-tapped transformers, and the minimum voltage across the active load that would be needed in order to preclude dropout or clipping types of behaviors. Also, the plot's usefulness was significantly enhanced by adding "zooming" capability, by providing a "slider" control for each axis' minimum and maximum scale values. Also added was the capability of setting a non-zero start time for the constant load current, which provides "Step Input" capability. The number of final mains cycles for which to calculate was also provided as a user-enterable input.

Transformer Modeling:

The Excel-based software tool that was created also includes a Transformer sheet, which incorporates a convenient transformer measurement procedure, automatic spice model parameter extraction, per-unitization, and subsequent automatic scaling of the spice model parameters upon entry of different desired values for the transformer's Volt-Amp rating, Output Voltage rating, AC Line voltage, and AC Line frequency.

Since the effects of the transformer's leakage inductances and winding resistances are included in the differential equations that are used to model the power supply, the effects of using different transformer ratings, e.g. Volt-Amps and Output Voltage, can be explored, using this software implementation of the numerical solution of the equations.

For example, the effects of using a transformer with a Volt-Amps rating that is too low, even though the output voltage rating is high-enough, can become obvious from the simulation results, and can be explored quantitatively. That could be a very valuable capability, since many designers initially have only "rules of thumb" to use, to try to choose a Volt-Amps rating for a new power supply's transformer, which typically only consider the average output voltage, which might not be accurate-enough, especially if it is also desired to minimize the size of the reservoir capacitance while still guaranteeing that the ripple voltage's minima can not violate the "dropout ceiling" voltage of an amplifier, under worst-case load conditions.

However, those "worst-case load conditions" are only the worst-case steady-state conditions. The software's usefulness was further increased by allowing entry of a non-zero start time for the load current, which enables the step response to be analyzed, providing the ability to also consider the transient-response to worst-case disturbances during steady-state operation.