Refracting Sound Waves

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Structures are described which refract and focus sound waves. They are similar in principle to certain recently developed electromagnetic wave lenses in that they consist of arrays of obstacles which are small compared to the wave-length. These obstacles increase the effective density of the medium and thus effect a reduced propagation velocity of sound waves passing through the array. This reduced velocity is synonymous with refractive power so that lenses and prisms can be designed. When the obstacles approach a half wave-length in size, the refractive index varies with wave-length and prisms then cause a dispersion of the waves (sound spectrum analyzer). Path length delay type lenses for focusing sound waves are also described. A diverging lens is discussed which produces a more uniform angular distribution of high frequencies from a loud speaker.

INTRODUCTION

IN the course of the investigation of artificial dielectric microwave lenses comprising arrays of small conducting objects,[1] much recourse was had to the early analytical work of Lord Rayleigh on the scattering of energy from objects which are small compared to the wave-length. In dealing with this subject, Rayleigh indicated that many of his results were applicable both to aerial (sound) waves and to electric (electromagnetic) waves.[2] It seemed probable, therefore, that the same principles which were applied in the focusing of electromagnetic waves could be applied to sound waves. Preliminary experiments showed that certain existing microwave lenses using rigid elements in an open construction did focus sound waves over a similar range of wave-lengths. An investigation of this subject was therefore begun not only on convergent lenses but also on other optical counterparts such as divergent lenses and prisms. This paper describes the course and results of this investigation.

[1] W. E. Kock, Bell Sys. Tech. J. **27**, 58 (1948).
[2] See for example, Lord Rayleigh, Phil. Mag. **43**, 259 (1897); *ibid.* **44**, 28 (1897); [collected Papers IV, pp. 283 and 305].

OBSTACLE ARRAYS

The Concept of Wave Refraction in Obstacle Arrays

The customary concept of an optical lens is that it is a continuous medium such as glass. One generally associates the term refraction with the continuous
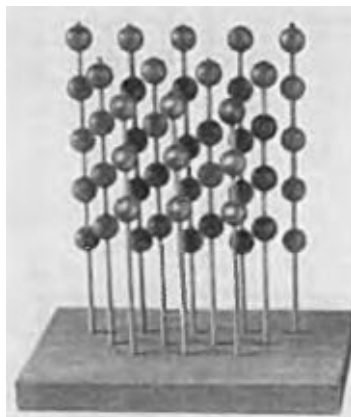

FIG. 1. An array of spherical obstacles.

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nature of the medium and associates the term diffraction with non-continuous optical devices such as gratings and Fresnel zone plates. However, refraction can and does occur in media which are assemblages of individual, discrete particles, if the particles and the distances between them are small compared to the wave-length of the wave propagation under consideration. Max Born has presented[3] an elegant proof that the various extensions of Maxwell's equations which describe the behavior of electromagnetic waves in a continuous dielectric medium can be arrived at by assuming the medium to be an assemblage of discrete re-radiating particles (dipoles).

In both the electromagnetic and acoustic cases, the mechanism of refraction can be explained by two approaches: (1) the re-radiation from the individual obstacles, (2) the alteration of the properties of the original medium brought about by the immersion of the obstacles. Figure 1 shows a simple version of an obstacle array of the type we are considering. Here the obstacles are in the form of spheres; for refracting electromagnetic waves, they are made electrically conducting, while for acoustic waves they are assumed to be rigid (immovable). In the re-radiating dipole picture the spheres, under the influence of the impressed electromagnetic (or sound) waves, become small electric (or acoustic) dipoles and the resultant of the original wave and the re-radiated waves manifests itself as a new wave having a lower velocity inside the array. The acoustic dipole action of the spheres can be looked upon in this way. If they were very light and free to move to and fro with the pulsations of the sound wave, they would not influence the progress of the wave. However, when rigidly mounted, air which normally would have passed back and forth through the space occupied by the spheres is prevented from so doing and the re-radiated wave thus produced is equivalent to that generated by spheres moving back and forth in the direction of propagation of the sound waves.

The second approach lends itself more easily to quantitative evaluation of the effective velocity reduction produced by a given array. In this picture, the extreme nature of the elements (perfectly conducting in the electric case and perfectly rigid in the acoustic case) is used to arrive at a mean refractive effect for the mixture of obstacles and the original medium. In the electromagnetic case, the relative dielectric constant is unity for free space and infinite for a perfect conductor. An array of conducting obstacles in free space thus appears to have a dielectric constant somewhere between unity and infinity. Consequently, its index of refraction is also greater than unity. Similarly, for our acoustic consideration, a perfectly rigid or immovable object has an effectively infinite mass, so that the combination of rigid (infinitely dense) spheres immersed in air (whose relative density is unity) results in a new medium having an index of refraction greater than unity.

The increased effective mass or density of a fluid caused by the immersion of an array of obstacles in it can be visualized in the following way. When one moves a paddle through a fluid, the paddle acquires an effective increased inertia depending upon the amount of fluid moved. Conversely when one holds a paddle rigidly in a moving fluid, the fluid acquires an effective increased inertia or mass. The increased mass of the fluid caused by the presence of an array of obstacles can be used to calculate the increased density of this artificially produced medium. Since the velocity of sound is dependent upon the density of the medium, the velocity of propagation through an obstacle array is less than that in free air.

Evaluation of the Index of Refraction

The increased inertia of a sphere moving through a fluid is known to be equal to $\frac{1}{2}$ the mass of the displaced fluid.[4,5] If, instead, the fluid is in motion and the sphere fixed, the fluid acquires this increased effective mass. A fluid moving past an array of N spheres per unit volume would thus appear to have its original density ρ_0 increased to the value

$$\rho = \rho_0 + \tfrac{1}{2}N\rho_0 V, \tag{1}$$

where V is the volume of one sphere. That is, the ratio of the effective density of the sphere array to that of the free medium is

$$\rho/\rho_0 = 1 + \tfrac{1}{2}N(4\pi/3)a^3, \tag{2}$$

where a is the radius of the spheres.

Since the velocity of propagation of sound in a medium is inversely proportional to the square root of
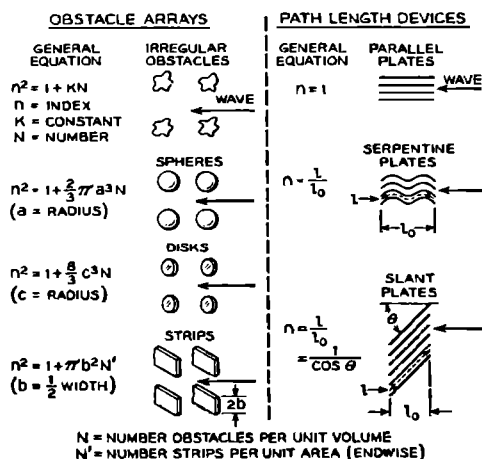

FIG. 2. Refractive index for delay mechanisms using arrays of rigid elements in an open construction.

[3] M. Born, Optik (Verlag Julius Springer, Berlin, 1933), Chapter 7, Article 74.

[4] H. Lamb, Hydrodynamics (Cambridge University Press, London, 1916), p. 116.
[5] Lord Rayleigh, Theory of Sound (MacMillan Company, London, 1940), Vol. II, p. 248.