Soft Recovery Diodes Lower Transformer Ringing by 10-20X

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Abstract

Power transformer secondary ringing was measured with 48 different semiconductor diodes; ringing amplitude was 10-20X lower with the best diodes than with the worst. They all rang, including Schottkys and HEXFREDs. A 1R + 2C snubber directly across the secondary completely eliminated ringing in every case.

Introduction

Solid state audio equipment very frequently contains a linear power supply with transformer, diodes, and filter capacitors. In these supplies the diode switch-off transient generates a current rate-of-change, dI/dt; excellent diodes generate small dI/dt and poor diodes generate large dI/dt. When dI/dt is large, it produces substantial voltage spikes across the leakage inductance of the transformer secondary (V = L*dI/dt). These voltage spikes stimulate the secondary RLC resonant circuit into oscillatory ringing.

Numerous audiophiles have reported improved sound quality when transformer secondary ringing is eliminated. Typical descriptions include: “Music just sounds cleaner, with a darker background”[1], “Quiet. Glorious quiet. This makes for a clarity and low level detail recovery that is quite amazing. Imaging has really taken leaps forward”[2], “There does seem to be an enhanced dynamic-I’m guessing from a lowering of the noise floor. I think there may be a better handling of signal peaks. Sibilance is handled more naturally”[3]. Several mechanisms for these subjective improvements have been proposed. High frequency transformer ringing can radiate RF noise into other circuits. Ringing can also capacitively couple into nearby conductors. Oscillatory currents in one secondary winding, induce oscillatory currents in the other windings. Viewed purely from an engineering perspective, transformer secondary ringing is an unwanted artifact; an unsightly wart. It *might* be harmless, or it might not; either way, surgically removing it eliminates all doubt.

Although it has been known for some time that different diode types produce different amounts of dI/dt at switch-off [4-5], there is little available data quantifying the amount of transformer ringing produced across a wide variety of diodes.

This is especially true for the more recently introduced types, such as soft recovery diodes with datasheet guaranteed softness ratio (tb/ta[1]), Super Barrier rectifiers, and silicon carbide diodes.

This paper presents measured data on power transformer secondary ringing, produced by 48 different semiconductor diodes. The exact same power supply test fixture is used in every measurement; only the diode changes. Therefore differences between the measured amounts of ringing are due to differences among the diodes. Many different diode types were measured, including standard silicon PN diodes, bridge rectifiers, high-Vf Schottkys, low-Vf Schottkys, hyperfast, ultrafast, HEXFRED, silicon carbide, Super Barrier, and soft recovery diodes. At today's distributor prices (qty=1000), the tested diodes span a 50-to-1 cost range.

Resonant Circuit

Figure 1(a) shows an AC-to-DC power supply using a single diode as a half-wave rectifier. This is the topology used for all measurements in this paper. With only one diode to remove and replace, experimental setup time is reduced, and parts cost is minimized since only 48 diodes need to be purchased. Other supply topologies would require purchasing (48*2) or (48*4) diodes.

The power transformer elements are enclosed by a dotted line. US 115VAC mains are connected to the primary, which consists of the leakage inductance LLp and the magnetizing inductance. The secondary's magnetizing inductance is perfectly coupled to the primary, at a turns ratio of n:1, and the secondary's

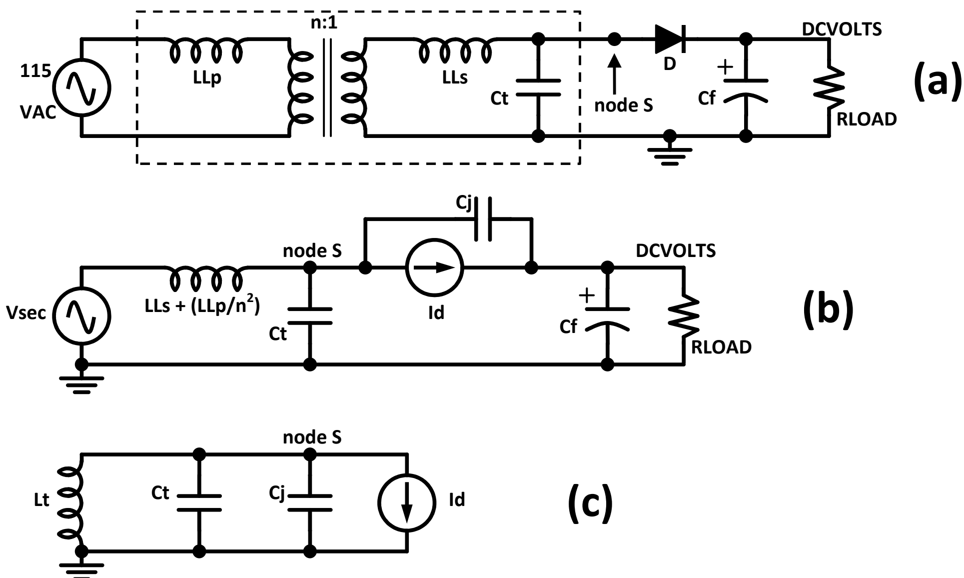


Figure 1 Resonant circuit in the transformer secondary.

[1] When a diode stops conducting, the current doesn't immediately return to zero but actually reverses to some value, THEN returns to zero. The time from start of current reverse to max reverse is called ta, the time from max reverse back to zero is called tb. The larger the ratio tb/ta, the softer the diode recovers.

leakage inductance LLs appears in series. The secondary winding capacitance is Ct. Rectifier diode D connects the secondary node S to the output node DCVOLTS, which is filtered by capacitor Cf. The supply delivers power to a resistor RLOAD.

The small signal incremental model is shown in Figure 1(b). Elements in the primary circuit (LLp and Vin) reflect into the secondary, by the square of the turns ratio n. The diode is modeled as a voltage controlled current source Id, in parallel with a junction capacitance Cj. It drives the output at DCVOLTS.

For small signal analysis, the input voltage source Vsec becomes a short circuit, and the output node DCVOLTS can be considered an AC ground; Cf acts as a short circuit at the frequencies of interest. These simplifications result in the small signal model shown in Figure 1(c). It is just a parallel LC resonant circuit, consisting of the transformer inductance Lt (= LLs + (LLP/n^2)), the transformer winding capacitance, and the diode junction capacitance at switch-off (where Vdiode ~ 0v). If the diode switches off abruptly, dI/dt is large, creating a large voltage spike across the inductor and stimulating the LC resonant circuit into oscillatory ringing.

Test Fixture

The AC-to-DC power supply used in these experiments is shown in **Figure 2** below. It is designed to maximize the amount of transformer secondary ringing, so that even small differences among excellent diodes (those producing very little ringing) will be detectable.

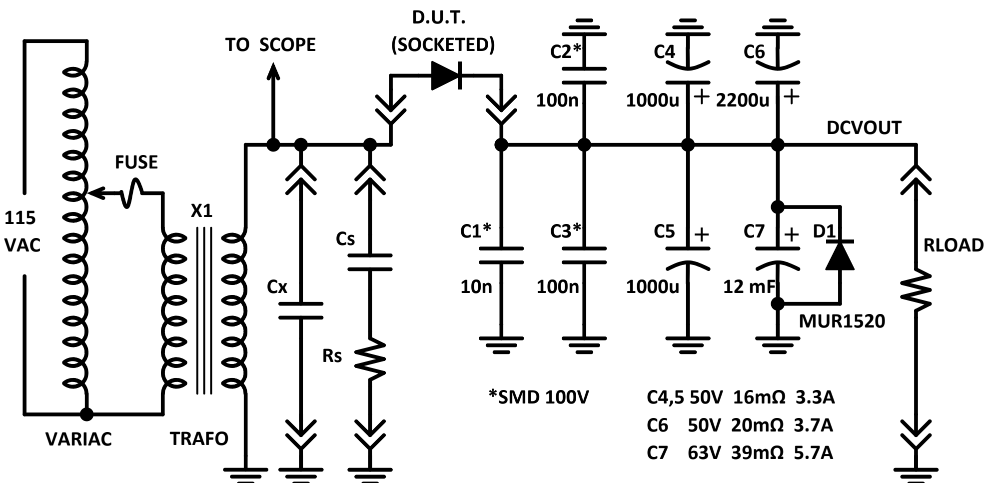


Figure 2 Test fixture.

A 600 watt autotransformer ("Variac") connects the test fixture to the AC mains, allowing fine-tuned adjustment of the output voltage. The Variac drives the primary of the power transformer X1 (115V primary, 24V secondary) through a 1 amp, fast-blow fuse. X1's secondary drives the diode (DUT) and, if used, the optional CRC snubber comprised of Cx, Cs, and Rs. These components are socketed and are usually removed from the circuit completely.

The diode charges seven parallel filter capacitors C1-C7, chosen for low ESR and high ripple current. Their total capacitance is high (16.2 mF), so the diode conduction angle is small. Therefore the diode current pulses are narrow and very tall, i.e., large peak current and large dI/dt. This provides a stronger stimulus to the secondary resonant circuit, increasing ringing amplitude. Low ESR capacitors ensure the current peaks are not compressed; high ripple current rating safely accepts the extremely tall peaks.

D1 protects the large electrolytic capacitors against reverse bias, if/when the DUT is accidentally installed backwards. The fuse blows immediately and the capacitors do not explode. This protection mechanism activated three times during the course of these experiments, with no detrimental effect.

The D.U.T. connects to the test PCB through a Phoenix 1935336 wire-to-board connector, rated for 17.5 amps. Screw-down terminals give mechanically solid connections and quick diode swapping. The 4-pin connector allows a variety of different size diode packages and lead spacings.

Ringing at Diode Switch-Off

In the power supply of Figure 2, the diode turns on when the secondary voltage exceeds the output voltage (DCVOLTS) by Vfwd or more. The diode remains on, charging the output capacitors, until the secondary voltage falls below (DCVOLTS + Vfwd); then the diode cuts off. As discussed above, diode cut-off produces a very large dI/dt which generates a large voltage spike across the transformer secondary inductance Lt (V = Lt*dI/dt). This is seen in the top trace of **Figure 3**; the voltage spike is about two vertical divisions tall: 20 volts! (Subsequent figures will show zoom-in magnifications of this region.) The bottom trace shows the secondary waveform when the diode is removed from its socket, disconnecting the secondary from the rest of the power supply. The mains outlet delivers a less than ideal sinewave in this laboratory.

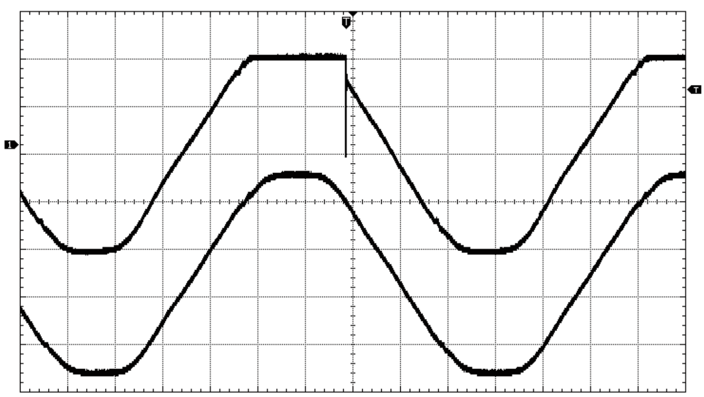


Figure 3 Secondary voltage in Fig.2 supply. Lower: unloaded. Upper: 1N5262GP + 100mA load. 10V/div, 2ms/div.

Measuring the Ringing Amplitude

Figure 4 shows a typical zoomed-in waveform of a typical "good" performing diode, the SBR12A45.

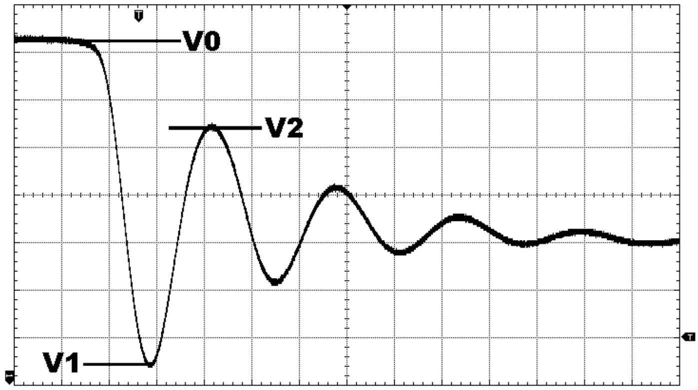


Figure 4 SBR12A45 at 100mA dc load current. Initial step down amplitude "V01" is 6.60V; first ringing amplitude "V12" is 4.88V. (Cursors + legend omitted). 1V/div, 5us/div.

When the diode switches off, it produces a voltage spike which stimulates the secondary resonant circuit into oscillatory ringing. The first step-down leg of the ringing is from V0 to V1; oscilloscope cursors are used to measure its amplitude, 6.60V. The first leg of post-stimulus ringing is from V1 to V2; scope cursors are again used to measure its amplitude, 4.88V. To maintain legibility at very small printed size, the cursors and legend were not displayed in Figure 4; instead, their values are included in the figure caption.

All 48 diodes were measured at a dc output current of 100mA. In each case the Variac was adjusted until the output voltage across the 150 ohm, 20 watt load resistor measured 15.0 volts. This ensured that all diodes operate at the exact same dc output current, regardless of their Vfwd. It also nulls out fluctuations of the mains voltage (on a timescale longer than the average measurement time, which was 2 to 5 minutes per diode). A rather low output voltage (15V) was deliberately chosen, so that very low Vfwd Schottky diodes, with very low max reverse voltage ratings (40V), could be included in the tests. This may not be representative of medium- and high-voltage power supplies, and different results might have been obtained with 80V output instead of 15V output. An opportunity for further research!

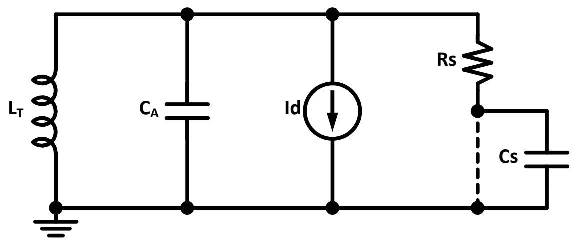
The amplitude V01 of the first step-down leg of ringing was measured @ 100mA, and so was the amplitude V12 of the first post-stimulus leg of ringing. These data are presented in **Table I**.

A second full set of tests were performed, operating the diodes at an average current of 2.0 amperes. This required a different power transformer with a higher power rating (80VA rather than 20VA). The second set of tests used an 8.0 ohm, 200 watt load resistor and the new transformer. In each test the Variac was adjusted to give 16.0 volts across the 8.0 ohm resistor, thus 2.0 amperes. Four of the 48 diodes were rated for only 1 ampere; so they were omitted, leaving 44 diodes to be measured at 2.0A. These data are also presented in Table I.

Adding a CRC Snubber Across the Secondary

Although the best diodes reduced ringing amplitude by a factor of 10-20X compared to the worst diodes, they *all* produced some oscillatory ringing in this sensitive test fixture. However the desired result is *zero* ringing. Fortunately, since the secondary is a parallel LC resonant circuit, it should be possible in theory to add a parallel resistance, and to tune the resistance value until the RLC resonant circuit is overdamped (damping ratio $\zeta > 1.0$). This should, in theory, eliminate ringing completely, even with the worst diodes.

Figure 5 shows a small signal circuit model of the transformer secondary. Lt (from Fig 1) is the transformer leakage inductance, and Ca is the total capacitance (= Ct + Cj), also from Fig.1. A parallel resistance Rs has been added. For the initial analysis assume Rs connects directly to ground, as shown with a dotted line. (This is equivalent to assuming that Cs = infinity).



$$\zeta = \frac{1}{2R_S}\sqrt{\frac{L_T}{C_A}} \qquad \frac{C_S}{C_A} \geq 15\,\zeta^2$$

Figure 5 Circuit model of transformer secondary including snubbing resistor.

The damping factor ζ can be calculated using Laplace Transforms (see Appendix):

$$\zeta = \frac{1}{2R_S}\sqrt{\frac{L_T}{C_A}}$$

To eliminate oscillation, the system is intentionally overdamped ($\zeta > 1.0$); plugging this in gives

$$R_S < \frac{1}{2}\sqrt{\frac{L_T}{C_A}}$$

And now in theory, the problem is solved: choose a suitably small Rs which gives ($\zeta > 1.0$). Unfortunately if this resistor Rs is connected directly across the transformer secondary, it sees the entire RMS secondary voltage, and so Rs dissipates an unacceptably large amount of power. Therefore a capacitor Cs is introduced in series with Rs, to reduce power dissipation. Cs presents a high impedance at the 60Hz mains frequency, limiting the current through Rs and reducing power dissipation. Cs presents a very low impedance (much lower than Rs) at high frequencies where the RLC circuit might oscillate. Theoretical calculations guide the selection of Cs (see Appendix).

In order to successfully overdamp the secondary,

$$C_S \geq C_A \cdot 15\zeta^2$$

To learn whether snubbers do eliminate ringing in practice, another set of measurements were taken in the 100 mA test setup. The 48 diodes were re-tested, with a 3-element snubber across the transformer secondary as shown in Figure 2. The values of the snubber were: Cx = 10nF / Rs = 150 Ω / Cs = 680nF. These gave a damping factor ζ of approximately 1.5 with this transformer; the secondary RLC circuit was overdamped.

Figures 6-10 below, show measured transformer waveforms from several diode types, with and without the 10nF / 150R / 680nF CRC snubber.

Figure 6 Super Barrier Rectifier SBR12A45 at 100mA. Lower (no snubber): V01=6.60V, V12=4.88V. Upper (with CRC snubber): no ringing. (Cursors + legend omitted). 2V/div, 5us/div.

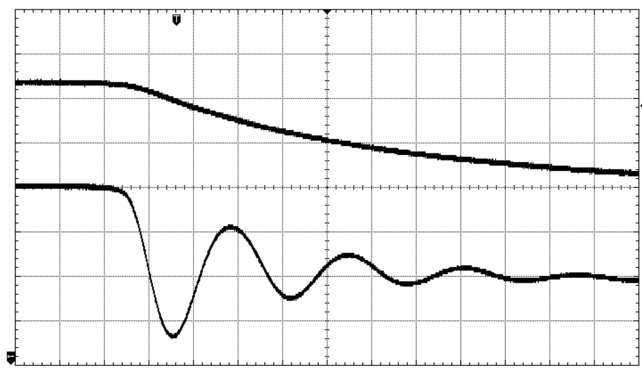
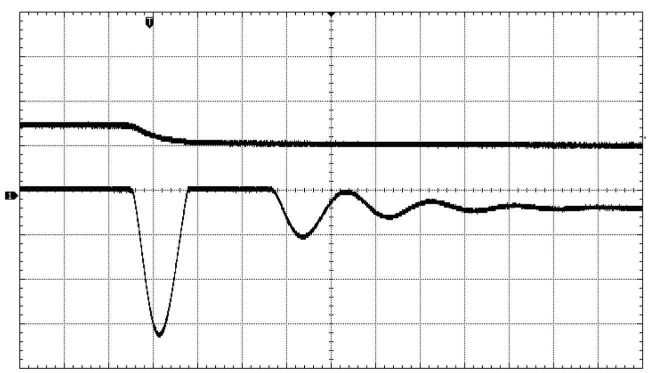


Figure 7 Bridge rectifier GBPC3510 at 100mA. Lower (no snubber): V01=32.8V, V12=32.8V. Upper (with CRC snubber): no ringing. 10V/div, 5us/div.



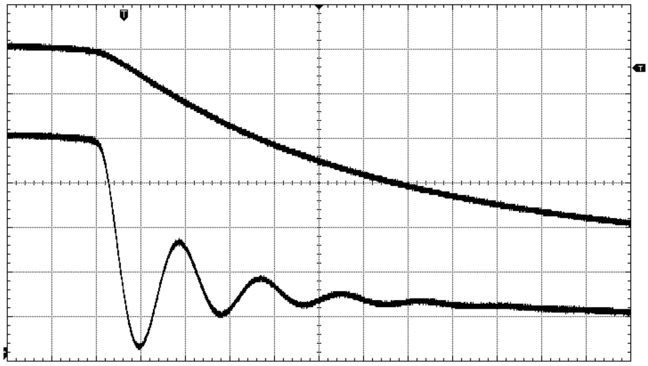


Figure 8 HEXFRED HFA08TB60 at 100mA. Lower (no snubber): V01=4.70V, V12=2.41V. Upper (with CRC snubber): no ringing. 1V/div, 5us/div.

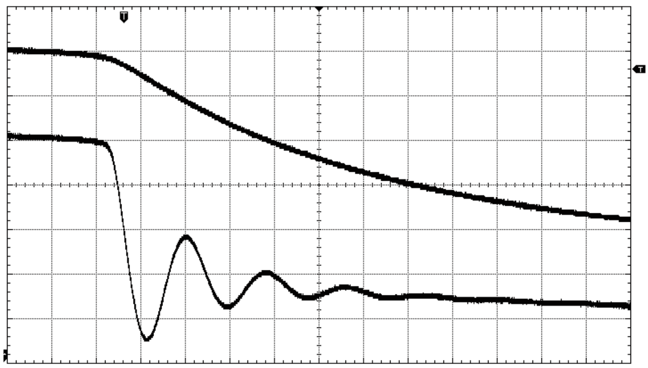


Figure 9 Soft Recovery diode ISL9R460 at 100mA. Lower (no snubber): V01=4.44V, V12=2.28V. Upper (with CRC snubber): no ringing. 1V/div, 5us/div.

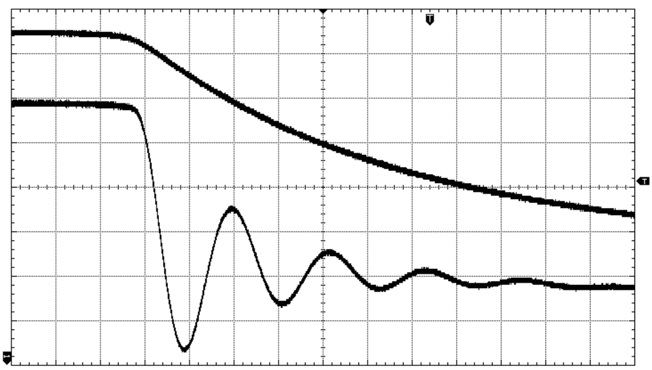


Figure 10 Schottky diode SB540 at 100mA. Lower (no snubber): V01=5.28V, V12=3.14V. Upper (with CRC snubber): no ringing. 1V/div, 5us/div.

Ringing with the 35 ampere, 1000V bridge rectifier diode (Figure 7) was especially brutal. The initial leg down V01 was so large (32.8V, in a 24VAC secondary!) that the subsequent ringing waveform rose high enough to actually turn on the diode again, and the ringing waveshape peak was clipped off, in the interval (18us < t < 28us). The diode in Figure 7 ran at a dc current of only 100mA; this diode behaved even worse at 2.0 amperes of dc current, see Table I. Several other diodes had similar “huge” ringing and peak clipping, they are the Table I entries whose V12=V01.

HEXFRED diodes (Figure 8) and Schottky diodes (Figure 10), both of which are passionately advocated by many DIY equipment builders, produced oscillatory ringing in this sensitive test fixture. So did soft recovery diodes (Figure 9). Fortunately the CRC snubber removed all traces of oscillation, for all 48 diodes tested.

Measured results

In Table 1, each row (each diode) shows four measurements: V01 and V12 at 100mA of dc output current, and also V01 and V12 at 2.0 amperes of dc output current. These ringing amplitudes are presented in the columns headed A, B, C, D respectively. The data was sorted and ranked by criteria B and D, giving the rankB and rankD columns. Rank=1 is best (smallest ringing amplitude), Rank=48 means the largest ringing amplitude. Four of the diodes were rated for only 1 ampere of dc current, so they are omitted from the 2 amp measurements and rankings.

Interpreting the data

The ringing amplitude data in Table I falls into two distinct zones. The first zone contains the *excellent diodes*; they have (V12@100mA < 4 volts) and (V12@2.0A < 8 volts). The second zone contains the *terrible diodes*; they have (V12@100mA > 6.5 volts) and (V12@2.0A > 15 volts). Although the sensitive test fixture has detected small differences among the excellent diodes, these differences are quite small compared to the huge differences between the excellent diodes and the terrible diodes. Randomly selecting any of the excellent diodes (RankB <= 40); (RankD <= 36), is vastly preferable to selecting the best of the terrible diodes, from the perspective of ringing amplitude. There is little performance penalty if an excellent diode is chosen based upon cost, or voltage rating, or Vfwd, or trendy fashion.

To select an excellent diode, avoid terrible diodes. In this test 100% of the terrible diodes were standard silicon PN diodes, either with a very slow reverse recovery time, or no tRR datasheet spec at all. A simple rule-of-thumb, which avoids all of the terrible diodes in Table I, is: either choose a Schottky, or choose a non-Schottky diode with datasheet tRR < 300 nsec. Remember that this is not a 200kHz switchmode power supply; these diodes operate from 60Hz AC mains, and engineering rules-of-thumb from SMPS designs are inapplicable here.

Part Number	MaxRating	type	Vf @ If	tb/ta ratio	A: V01 @ 100mA	B: V12 @ 100mA	C: V01 @ 2.0A	D: V12 @ 2.0A	rankB	rankD	top15?
SB160-E3	1A 60V	Schottky	0.65V 1A		5.00	2.69	---	---	25	44	
VS-MBR160	1A 60V	Schottky	0.75V 1A		5.12	2.63	---	---	22	44	
MBR1100	1A 100V	Schottky	0.79V 1A		4.92	2.58	---	---	14	44	✪
UF4004	1A 400V	Ultrafast	1.0V 1A		4.72	2.38	---	---	7	44	✪
1N4005	1A 600V	Standard	1.1V 1A		15.20	15.20	---	---	42	44	
SBYV27-200	2A 200V	Ultrafast	1.07V 2A		4.60	2.28	9.00	2.98	3	1	✪✓
GI851	3A 100V	Fast	1.25V 3A		4.64	2.38	9.92	4.60	6	28	✪
VSB3200	3A 200V	Schottky	0.86V 3A		4.96	2.60	9.24	3.44	19	12	✓
1N5404	3A 400V	Standard	1.0V 3A		5.28	3.18	12.32	7.96	32	36	
1N5626GP	3A 600V	Standard	1.0V 3A		20.60	20.60	45.00	45.00	45	40	
SBYV28-100-E3	3.5A 100V	Ultrafast	1.1V 3.5A		4.60	2.27	9.12	3.08	1	3	✪✓
BYV28-150	3.5A 150V	Ultrafast	1.1V 5A		5.08	2.74	9.68	3.68	26	19	
MUR420	4A 200V	Ultrafast	0.89V 4A		5.08	2.66	9.28	3.34	23	7	✓
MUR460	4A 600V	Ultrafast	1.28V 4A		4.86	2.54	9.72	4.02	12	24	✪
RURD460	4A 600V	Ultrafast	1.5V 4A	0.47	4.76	2.51	9.96	4.88	10	29	✪
C3D04060F	4A 600V	Si Carbide	1.5V 4A		5.12	2.74	8.60	3.06	27	2	✓
GBU4J	4A 600V	Standard	1.0V 2A		18.80	18.80	42.80	42.80	44	39	
ISL9R460	4A 600V	STEALTH-II	2.0V 4A	4.2	4.44	2.28	8.96	3.48	2	13	✪✓
SB540	5A 40V	Schottky	0.48V 5A		5.28	3.14	9.80	4.00	31	23	
SB5100	5A 100V	Schottky	0.85V 5A		5.10	2.79	9.60	3.52	28	16	
6A4	6A 400V	Standard	1.1V 6A		7.66	6.64	18.90	16.30	41	37	
RURD660S9A	6A 600V	Ultrafast	1.5V 6A	0.57	4.88	2.58	10.88	5.98	15	34	✪
FES8GT	8A 400V	Ultrafast	1.3V 8A		4.92	2.61	9.52	3.70	20	20	
GBU8G	8A 400V	Standard	1.0V 4A		27.30	27.30	57.80	57.80	46	41	

Table 1(a) Diode measurements at 100mA (columns A and B) and at 2.0A (columns C and D).

Part Number	MaxRating	type	Vf @ If	tb/ta ratio	A: V01 @ 100mA	B: V12 @ 100mA	C: V01 @ 2.0A	D: V12 @ 2.0A	rankB	rankD	top15?
APT8DQ60KG	8A 600V	Ultrafast	2.0V 8A		4.60	2.38	9.00	3.34	5	6	✪✓
DSR8U600	8A 600V	Schottky	2.5V 8A	1	5.54	3.70	10.20	5.12	38	30	
FFPF08H60S	8A 600V	Hyperfast-II	2.1V 8A	1.07	4.78	2.53	9.24	3.44	11	10	✪✓
HFA08TB60	8A 600V	HEXFRED	1.4V 8A		4.70	2.41	9.16	3.22	9	5	✪✓
ISL9R860	8A 600V	STEALTH-II	2.0V 8A	3.7	4.74	2.60	9.08	3.48	18	14	✓
MSR860	8A 600V	Ultrasoft	1.7V 8A	2.5	4.86	2.55	9.60	3.70	13	22	✪
RHRP860	8A 600V	Hyperfast	2.1V 8A	0.56	4.64	2.40	9.16	3.36	8	8	✪✓
RURP860	8A 600V	Ultrafast	1.5V 8A	0.66	4.82	2.59	9.56	3.60	17	17	
VS-ETH0806	8A 600V	FRED Pt	2.0V 8A		4.58	2.36	9.12	3.36	4	9	✪✓
VS-ETL0806	8A 600V	FRED Pt	0.97V 8A		5.32	3.19	11.40	6.32	33	35	
FFPF10UP30	10A 300V	Ultrafast	1.4V 10A	1.2	4.94	2.67	9.44	3.44	24	11	✓
SBL1040	10A 40V	Schottky	0.6V 10A		5.50	3.34	10.12	4.16	35	26	
UH10FT-E3	10A 300V	Ultrafast	0.96V 5A	0.36	4.96	2.62	9.56	3.50	21	15	✓
SBR12A45	12A 45V	SuperBarrier	0.43V 12A		6.60	4.88	11.36	5.96	40	33	
VSB1545	15A 45V	Schottky	0.33V 5A		5.86	3.84	11.56	5.92	39	32	
MUR1520	15A 200V	Ultrafast	1.05V 15A		5.26	2.93	9.76	3.66	29	18	
GBJ1506	15A 600V	Standard	1.05V 8A		17.30	17.30	42.80	42.80	43	38	
DSS16-01A	16A 100V	Schottky	0.79V 15A		5.52	3.42	10.20	4.40	36	27	
LQA16T300	16A 300V	Qspeed	1.6V 16A	0.7	4.96	2.59	9.00	3.16	16	4	✓
GBU2510	25A 1000V	Standard	1.0V 12A		31.30	31.30	65.60	65.60	47	42	
VF30100S	30A 100V	Schottky	0.39V 5A		5.80	3.56	11.32	5.48	37	31	
FFPF30UP20	30A 200V	Ultrafast	1.15V 30A	0.64	5.30	3.01	9.92	3.70	30	21	
GBPC3510	35A 1000V	Standard	1.1V 17A		32.80	32.80	67.80	67.80	48	43	
MBR40250	40A 250V	Schottky	0.86V 20A		5.44	3.26	10.08	4.14	34	25	

Table 1(b) Diode measurements at 100mA (columns A and B) and at 2.0A (columns C and D).

The amplitude of post-stimulus ringing (V12@100mA) varies from 2.27 volts to 32.8 volts, a ratio of 14.4 to 1. At 2.0 amperes, ringing amplitude (V12@2.0A) varies from 2.98 volts to 67.8 volts (!), a ratio of 22.7 to 1. These (worst diode/best diode) ratios are summarized as "10-20X" in the title of this paper. The two dc current levels were selected to broadly represent preamps and power amplifiers.

To accommodate readers who desire a very-best-of-the-best listing, the 15 most excellent diodes with the best rankings at 100mA, are indicated by a "✪" character in the final column. The 15 most excellent diodes with the best rankings at 2.0 amperes, are indicated by a "✓" character. Eight diodes were best-of-the-best at both 100mA and 2.0 amperes. Most of them have quite a large Vfwd voltage and would require a heatsink. The Soft Recovery diode with the highest datasheet "softness ratio" (tb/ta), was among the eight double gold medalists.

Summary

Excellent diodes perform excellently, reducing oscillatory ringing amplitude by as much as 22X compared to the poorest diodes. Diodes which performed well at 100mA also performed well at 2.0 amperes. Among the 48 diodes tested, 40 were deemed excellent at 100mA, and 8 "very best of the best" are identified. It was also found that CRC snubbers eliminate ringing completely, even with the poorest diodes.

References

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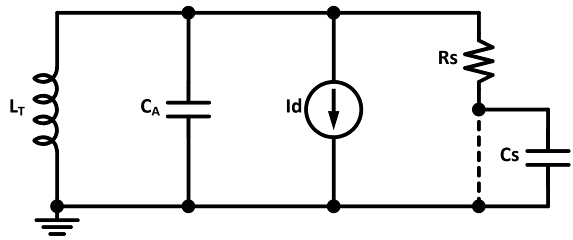
[5] Miller, Rick (1994), "Measured RFI Differences Between Rectifier Diodes in Simple Capacitor-Input Power Supplies," The Audio Amateur (magazine), Vol 1/1994, pp.26-27.

Further reading

"Rectifier snubbing – background and Best Practices" , Morgan Jones, Linear Audio Volume 5, April 2013, pp. 7 – 26.

Apppendix



In the RLC resonant circuit of Figure 5 (with Cs present), repeated here, consider the diode current to be the input and consider the voltage across the transformer secondary to be the output. The transfer function H(s) = (Vout/Iin) is simply the combined impedance of the three parallel circuit branches:

$$\frac{V_{OUT}}{I_D} = H(s) = \frac{1}{\frac{1}{sL_T} + \frac{1}{\left(\frac{1}{sC_A}\right)} + \frac{1}{\left(R + \left(\frac{1}{sC_S}\right)\right)}}$$

Simplifying the third term in the denominator,

$$H(s) = \frac{1}{\frac{1}{sL_T} + \frac{1}{\left(\frac{1}{sC_A}\right)} + \frac{sC_S}{(1 + sRC_S)}}$$

Multiplying numerator and denominator by (1 + sRCs),

$$H(s) = \frac{1 + sRC_S}{\frac{(1 + sRC_S)}{sL_T} + sC_A(1 + sRC_S) + sC_S}$$

Assume that (sRCs >> 1) so that (1 + sRCs) can be replaced by (sRCs) everywhere. Then

$$H(s) = \frac{sRC_S}{\frac{sRC_S}{sL_T} + sC_A(sRC_S) + sC_S}$$

Multiply numerator and denominator by (1/CaRCs) so the coefficient of s^2 is 1:

$$H(s) = \frac{\frac{s}{C_A}}{\frac{1}{L_T C_A} + s^2 + \frac{s}{RC_A}}$$

Term by term, match the denominator of H(s) with the denominator of a canonical second order system, $s^2 + (2\omega_n\zeta)s + \omega_n^2$

$$\omega_n = \sqrt{\frac{1}{L_T C_A}} \quad ; \quad \zeta = \frac{1}{2R_S}\sqrt{\frac{L_T}{C_A}}$$

We assumed that (sRCs >> 1), i.e., that (jωRCs >> 1). In the worst case, the radian frequency ω might be as low as the resonant frequency of Lt in parallel with *both* Ca and Cs (i.e. Rs is very small). Then

$$\omega = \sqrt{\frac{1}{L_T(C_A + C_S)}}$$

Since ωRCs >> 1 , RCs >> (1/ω) . Squaring both sides,

$$R^2 {C_S}^2 \gg \frac{1}{\omega^2} \quad \Rightarrow \quad R^2 {C_S}^2 \gg L_T(C_A + C_S)$$

Substituting , $R = \frac{1}{2\zeta}\sqrt{\frac{L_T}{C_A}}$,

$$\frac{{C_S}^2}{4\zeta^2}\frac{L_T}{C_A} \gg L_T(C_A + C_S) \quad \Rightarrow \quad \frac{{C_S}^2}{4\zeta^2} \gg C_A(C_A + C_S)$$

If Cs >> Ca then (Ca + Cs) reduces to Cs, and

$$\frac{{C_S}^2}{C_A C_S} \gg 4\zeta^2 \quad \Rightarrow \quad \frac{C_S}{C_A} \gg 4\zeta^2$$

Whew! Mathematical analysis says that (Cs/Ca) needs to be much much greater than 4 times zeta squared. But exactly how much greater? LTSPICE simulations, carried out at zeta values from 1 to 10, show that good damping behavior occurs whenever (Cs / Ca) is *fifteen* times zeta squared, or greater:

$$\frac{C_S}{C_A} \geq 15\zeta^2$$