

Midcom's Tips for Transformer Modeling
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Transformer characterization is important if you plan to model your circuit in PSPICE or other simulation programs. Unfortunately, not all transformer manufacturers provide circuit models for their products, so you may find you need to determine the equivalent circuit model based on lab measurements. This technical note will help you characterize almost any transformer so you can develop its equivalent circuit model to suit your analysis needs. It is intended to help circuit designers, component engineers understand transformer capabilities and limitations. For further information on transformer behavior and modeling, refer to Midcom Technical Note #69 which may be found on the Midcom web site, www.midcom-inc.com.

Characterizing a transformer of unknown origin can be tricky, but if you know the application in which it is intended to be used, you can make some educated guesses regarding the working impedances, voltages and bandwidth. Here is the equivalent circuit on which we will base our measurements for the purposes of assigning values to the as-yet unknown elements:

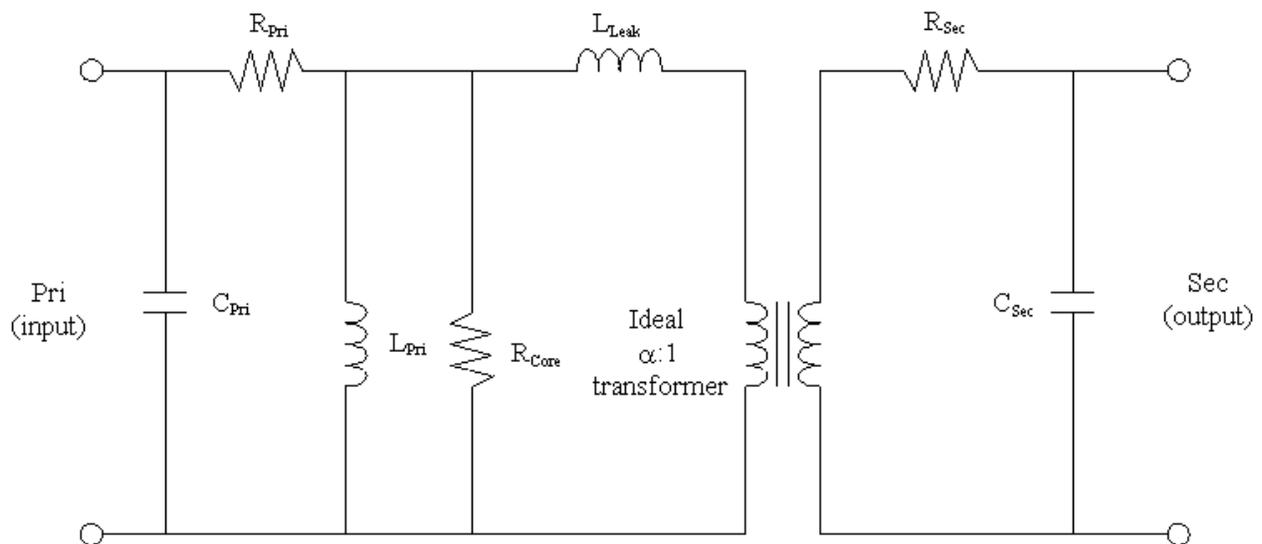


Figure 0-1

The "complete" transformer equivalent circuit

FIRST-ORDER APPROXIMATION

This technique will give you useful results for most power or audio transformers and is relatively quick to perform. You will need a standard ohmmeter and an LCR bridge capable of displaying series and parallel complex impedances ($R \pm jX$). If you're willing to convert your readings, you may use any means available that will provide complex impedance measurements. If you are patient, you can even do this with a simple VOM using the method described in Midcom Technical Note #35, [VOM Measures Complex Impedances](#).

1. DC Resistances (R_{DC})

Measure DC resistances, R_{Pri} and R_{Sec} using a standard ohmmeter. If the ambient temperature is within five degrees of 20°C, your measurements will be within 2% of the 20°C value. (Temperature compensation is covered in a later section)

R_{Pri} : _____ R_{Sec} : _____

2. Magnetizing Inductance and Core Loss Resistance

Set up the LCR bridge to measure inductance and resistance in the parallel mode: L_p , R_p . Measure the inductance and resistance at the primary winding at or near the midband of the transformer's frequency range of operation. You can use 55 Hz for 50/60 Hz linear power transformers while 1kHz is a good choice for analog telecommunications magnetics. The values you measure will correspond approximately to the elements L_{Pri} and R_{Core} shown in the equivalent circuit.

L_p : _____ R_p : _____ Use L_p for L_{Pri} , R_p for R_{Core} .

3. Leakage Inductance

Measure leakage inductance, L_{Leak} , by shorting the secondary and measuring the inductance at the primary. Try 1kHz first, then 10kHz. If you are measuring a transformer with very small turn counts less than about 25 turns on each winding, take a reading at 100kHz. Use whichever value is lowest, *but not negative* (which would indicate capacitance). The reason you need the lowest reading is explained in section on "Improved Accuracy Modeling". Note: If you insist on using coupling coefficient (k) instead of leakage inductance, please refer to Midcom Technical Note #69 for a means of converting between leakage inductance and coupling coefficient.

L_{Leak} : _____ at _____ Hz.

4. Turns Ratio

If you don't know the nominal turns ratio from the transformer's spec sheet, you'll need to measure it. Turns ratio, α , is best measured by using precision decade resistor substitution boxes in a bridge configuration that will be described later. You can make a rough estimate of turns ratio by applying an ac voltage at the primary, then measuring the voltage at the secondary. Armed with this, you then calculate turns ratio by $\alpha = V_{Pri}/V_{Sec}$. This method is prone to several kinds of errors, so a better approach is to employ your LCR bridge once again.

Set the LCR bridge to measure L_p and Q, then connect the bridge to the primary terminals. Vary the frequency of measurement until the Q is at its highest reading. Laminated sheet steel transformers will typically have Q values of perhaps less than 1 to about 5; maybe 10 if you're lucky. Ferrite-based transformers will have substantially higher Q values than those with steel cores, perhaps as high as 100. The higher your Q value, the more accurate will be your turns ratio estimation. If your Q value is less than 5, you can't count on this method to provide better than a few percent of accuracy.

Measure the secondary inductance at the same frequency as you did the primary, noting the Q value.

L_{Pri} at _____ Hz, Q = _____
 L_{Sec} at _____ Hz, Q = _____

Assuming Q is high enough, turns ratio may be calculated by:

$$1) \quad \alpha = \frac{N_{Pri}}{N_{Sec}} = \sqrt{\frac{L_{Pri}}{L_{Sec}}}$$

If this calculation returns a value less than 0.5 or greater than 2, consider adjusting the voltage applied to either winding to account for changes in flux density.

The goal here is to keep maintain the following equivalence:
$$\frac{V_{pri}}{N_{Pri}} = \frac{V_{Sec}}{N_{Sec}}$$

Laminated sheet steel transformers may exhibit significant change in inductance depending on the excitation voltage applied to their windings. Ferrite-core transformers also exhibit this behavior, but to a lesser extent. The reason inductance depends on excitation level is explained in Midcom Technical Note #69.

To correct for effects of flux density, reduce the voltage applied to the winding with the lower inductance value (either L_{Pri} or L_{Sec}) by the ratio you first calculated in equation 1. For example, if you calculate α to be 4.0, reduce the measurement voltage of L_{Sec} from 1.0V to 0.25V. Recalculate α if this yields a significantly different (usually lower) value of L_{Sec} . Call this L'_{Sec} to differentiate it from the uncorrected flux density.

Turns ratio, corrected for flux density, B_m :
$$\alpha = \frac{N_{Pri}}{N_{Sec}} = \sqrt{\frac{L_{Pri}}{L'_{Sec}}}$$

If your uncorrected value for α was less than 0.5, raise V_{Sec} by a factor of measured α , then remeasure and recalculate. Watch out for saturation symptoms such as falling or unstable inductance readings. A single iteration of level adjustment/remasurement is usually sufficient.

5. Distributed Winding Capacitances

The primary and secondary distributed winding capacitances manifest their effects at the higher frequencies. Most well-designed communications transformers will have negligibly small values of distributed capacitance such that measuring them may be more work than the results are worth. This is less true with switchmode power transformers. To determine if the values of C_{Pri} and C_{Sec} are significant, connect your LCR bridge to the transformer's primary and increase the frequency until the impedance seen at the primary becomes capacitive. The frequency at which the impedance changes from inductive (+jX) to capacitive (-jX) is the transformer's self-resonant frequency. If this frequency is well above the passband of interest, it may not be worthwhile to continue. If self-resonance is not too far (within a decade or two) above the midband frequency of the transformer's passband, you can make a quick estimate of distributed capacitance as follows: Raise the frequency of the LCR bridge beyond self-resonance (where the reactance changes from inductive to capacitive) until you find the frequency at which the Q of the capacitance is at its maximum. Use the series (C_s) mode. If the Q of the capacitance is at least 5, preferably 10 or more, you have a useful value of the total distributed capacitance. You can assume that all of this capacitance exists on the primary side and call it C_{pri} , then assume C_{Sec} is zero. Conversely, you could assume that the entire capacitance exists at the secondary, but to move it over to that side, you must multiply the measured primary capacitance by the square turns ratio, α . You must multiply by α^2 instead of dividing (as you would with an impedance) because you are working with capacitance which is proportional to the reciprocal of impedance. This effect allows a transformer with a 0.5:1 turns ratio to effectively multiply by four a capacitance placed on its secondary.

That's it. You now have a first-order approximation of the transformer's equivalent circuit model. For an improved accuracy model, keep reading.

IMPROVED ACCURACY MODELING

Let's start again with the easy ones: R_{pri} and R_{sec} . If you want to get fancy you can use a temperature-compensated ohmmeter with a thermal-sensor matched to the metal comprising the transformer's windings. Copper is the most common metal used in transformer windings and its temperature coefficient of resistance (T_{cc}) is about +0.4% per degree Celsius. You could also measure resistance with an uncompensated ohmmeter, then consult a nearby thermometer to convert to Standard Temperature of 20°C (**not** 25°C like the rest of the electronics world). As it turns out, we actually need the *uncompensated* reading for later use, so record it anyway. Unless you work outside in the Arctic or Death Valley, take your readings with reasonable precision (3 places for xxx, 4 places for 1xxx) on a standard ohmmeter. Use of a four-terminal (kelvin) ohmmeter is recommended for resistances less than about 0.1 ohm. If you don't have a four-terminal ohmmeter, but you do have a stable and accurate current source and a high-accuracy voltmeter, you can use the current source to develop a voltage across the transformer winding. Measure the resulting voltage drop with your high-accuracy voltmeter. This is exactly the same as a four-terminal test, but without the convenience of doing it all with the same meter.

WARNING: Applying current to an inductor can result in dangerous and possibly lethal voltages when the source of the current is

suddenly removed. (For you calculus buffs, remember that $V_{inductor} = L \frac{di}{dt}$)

For safety's sake, use just enough current to provide an adequate reading on the voltmeter. Keep the power dissipated in the winding low to prevent the winding's resistance from climbing due to a current-induced increase in temperature. Stay below a few tens of milliwatts for transformers larger than your thumb and one or five milliwatts for those that are smaller. A level of 100mW may be appropriate for fist-sized and larger transformers with large wire diameters. Connect the inductor or transformer to the current source with the source turned off. Slowly (over the course of several seconds) raise the current until the voltage read on the voltmeter is stable and comfortably within its accuracy range. If you notice the voltage reading start to climb, you may be heating the winding. Either that or your current source hasn't stabilized.

If the winding resistance is less than an ohm, you may have to settle for a voltage less than 100mV to stay below the 100mW power level. Resistances less than 0.1 ohm require even lower voltages, so make sure you have chosen an appropriate voltmeter and current source for the job.

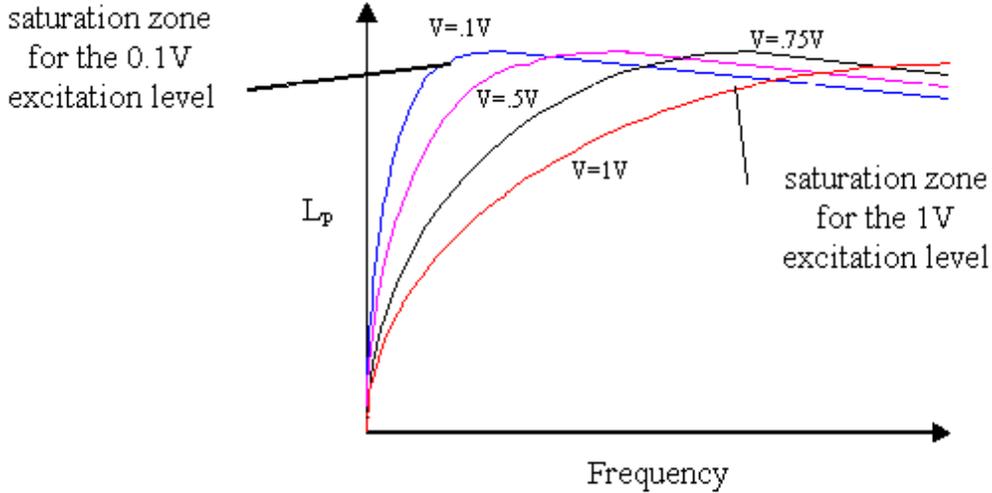
After the voltage and current have stabilized, note the voltage and current values, then slowly reduce the current to zero. Physically small inductors and transformers (smaller than your fist) are probably safe enough that several seconds of ramp up and down will keep your hair in place. An inductor the size of a breadbox (does anyone *have* a breadbox anymore?) is as dangerous when carrying high currents as it would be if someone dropped it your head.

INDUCTANCE

For improved accuracy, we need to describe how the transformer's winding inductance changes with varying frequency. We also need to remove effects of R_{pri} in our attempt to measure L_{pri} .

To begin this process, it is important to know the upper and lower band limits of the transformer's operational frequency range. We also need to know something about the transformer's operational voltage range. If we apply too much voltage at a frequency below the transformer's lower frequency limit, we run the risk of causing core saturation. While saturation does no permanent damage to the core, it won't allow us to accurately model the transformer.

To check for saturation, first measure the transformer's inductance. We'll arbitrarily choose the series equivalent at its lowest frequency and highest drive level. Call this operating point f_{LOW} , V_{HIGH} . The inductance obtained under this set of conditions will be the highest inductance value we measure for any subsequent combination of higher frequency and lower voltage. If we find the inductance goes up significantly as we raise the frequency above f_{LOW} , the transformer is operating in an unstable region near saturation. Similarly, if we reduce the voltage below V_{HIGH} and find inductance increases, we have again shown that the transformer is operating near its saturation point. The following family of curves shows the effect of saturation on inductance:



Inductance versus Frequency at various voltages

Figure 0-2

The equivalent circuit model of a single winding of a transformer is the same as that of an inductor. That equivalent circuit looks like this:

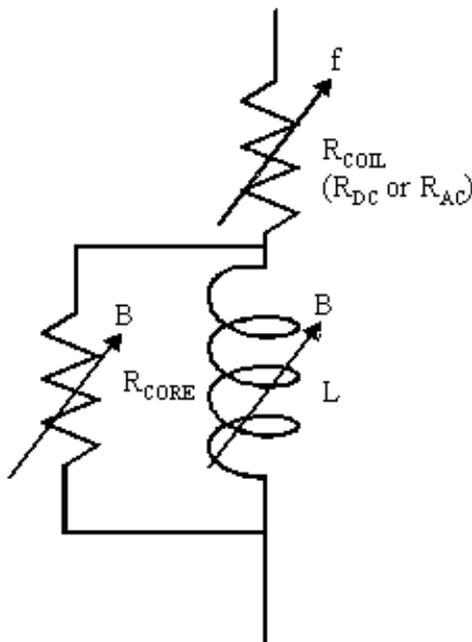


Figure 0-3

The arrows show the dependency of each element on flux density and frequency. The skin effect is responsible for the frequency dependence of R_{COIL} , but this is beyond the scope of this technical note. Skin effect is covered in greater detail in Midcom Technical Note #69.

We found R_{COIL} as our first step (where we called it R_{DC} for simplicity). We now need to account for it in our model. We need the actual resistance, not compensated for temperature, to determine the true core loss resistance and magnetizing inductance values.

1. With your LCR bridge in the series equivalent mode, measure L_s and R_s . If your bridge does not have this capability, or your measurements were recorded using the parallel equivalent, you must convert your measurements to L_s and R_s . The conversions between bridge parameters are readily available from various sources. Refer to the manual for your LCR bridge if you can't find them anywhere else. Five common conversions are shown here for reference:

$$L_p = L_s \left(1 + \frac{1}{Q^2} \right) \quad L_s = \frac{L_p Q^2}{1 + Q^2} \quad R_p = R_s (1 + Q^2) \quad R_s = \frac{R_p}{1 + Q^2} \quad Q = \frac{2\pi f L_s}{R_s} = \frac{R_p}{2\pi f L_p}$$

2. Subtract the DCR from the measured value of R_s . Call this R_s' to differentiate it from the measured reading.
3. Calculate the intrinsic inductance and core loss resistance by converting L_s and R_s' into L_p and R_p using the following conversions, noting that core loss, R_c , is the parallel element R_p in the L_p/R_p combination.

$$Q' = \frac{2\pi f L_s}{R_s'} \quad L_p = L_s \left[1 + \frac{1}{(Q')^2} \right] \quad R_c = R_p = R_s' \left(1 + (Q')^2 \right)$$

Example:

Find L_p and R_c given measured L_s and R_s at 1000 Hz are 1.138 henries and 6809 ohms at a frequency of 1 kHz. The DC resistance of the coil is 143.24 ohms:

$$R_s' = R_s - R_{DC} = 6809.15 - 143.24 = 6665.9 \text{ ohms}$$

$$Q' = \frac{2\pi f L_s}{R_s'} = \frac{2\pi 1000(1.138)}{6665.9} = 1.073$$

$$L_p = L_s \left[1 + \frac{1}{(Q')^2} \right] = 1.138 \left[1 + \frac{1}{(1.073)^2} \right] = 2.127 \text{ henries}$$

$$R_c = R_p = R_s' \left(1 + (Q')^2 \right) = 6665.9 \left(1 + (1.073)^2 \right) = 14,334 \text{ ohms}$$

MAGNETIZING INDUCTANCE AND CORE LOSS RESISTANCE FACTORS

Magnetizing inductance and core loss resistance change over frequency. The factors by which they do this were fit to curves by Midcom engineers in the early 1980s. The factors **do not** include effects due to saturation and are only approximations designed primarily for voiceband applications. The formulae are reproduced here for reference:

Inductance change versus frequency

$$L' = L_{fR} \left[\frac{f}{f_R} \right]^{\left(\frac{\ln(\alpha_L)}{\ln\left(\frac{f_R}{f_L}\right)} \right)}$$

where:

f is the frequency at which we need to determine the inductance

L' is the inductance at a given frequency, f

L_{fR} is the inductance at a reference frequency, f_R

f_R is a reference frequency, usually chosen to be near the transformers mid-band frequency

f_L is a frequency lower than f_R, usually about one-fifth the value of f_R

α_L is ratio of inductances at a low frequency, f_L to the reference frequency, f_R, or $\alpha_L = \frac{L_{fL}}{L_{fR}}$

Core loss change versus frequency

$$R_{cF} = R_{cFR} \left[\frac{f}{f_R} \right]^{\alpha_{Rc}}$$

where:

f is the frequency at which we need to determine the core loss resistance

R_{cF} is the inductance at a given frequency, f

R_{cFR} is the core loss resistance at a reference frequency, f_R

f_R is a reference frequency, usually chosen to be near the transformers mid-band frequency

α_{Rc} is the core loss resistance factor, which usually ranges from 0.35 to 0.45

We need to be able to calculate the two factors, α_L and α_{Rc} from measured values. The two equations, solved for their respective factors are shown here:

$$\alpha_L = \exp \left[\frac{\left[-\ln\left(\frac{L_f}{L_{fR}}\right) \right]}{\ln\left(\frac{f}{f_R}\right)} \ln\left(\frac{f_R}{f_L}\right) \right] \qquad \alpha_{Rc} = \frac{\ln\left(\frac{R_{cF}}{R_{cFR}}\right)}{\ln\left(\frac{f}{f_R}\right)}$$

Example:

A transformer has measured inductances of 1.8 H at 400 Hz and 1.2 H at 1000 Hz. Find its inductance ratio, α_L:

$$\alpha_L = \exp \left[\frac{\left[-\ln\left(\frac{L_f}{L_{fR}}\right) \right]}{\ln\left(\frac{f}{f_R}\right)} \ln\left(\frac{f_R}{f_L}\right) \right] = \exp \left[\frac{\left[-\ln\left(\frac{1.8}{1.2}\right) \right]}{\ln\left(\frac{400}{1000}\right)} \ln\left(\frac{1000}{200}\right) \right] = 2.04$$

Example:

A transformer has measured core loss resistances of 20k ohms at 500 Hz and 26k ohms at 1000 Hz. Find its core loss resistance ratio, α_{Rc} :

$$\alpha_{Rc} = \frac{\ln\left(\frac{R_{cF}}{R_{cR}}\right)}{\ln\left(\frac{f}{f_R}\right)} = \frac{\ln\left(\frac{20k}{26k}\right)}{\ln\left(\frac{500}{1000}\right)} = 0.379$$

LEAKAGE INDUCTANCE

Direct measurement of leakage inductance can be difficult if the magnetizing inductance is low. The quick-and-dirty approach described earlier will give you useable results in most cases, but you should understand *why* you raise the frequency to find the most valid reading. An abbreviated equivalent circuit for the leakage inductance case looks like this at frequencies below self-resonance:

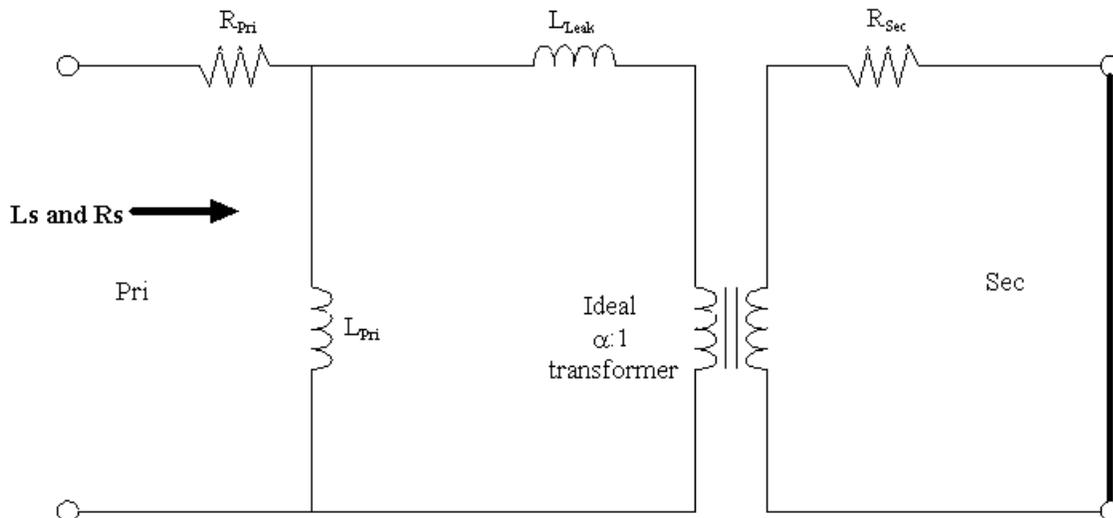


Figure 0-4

Leakage Inductance equivalent circuit

When L_{Pri} is two or more orders of magnitude larger than L_{Leak} , we can simply raise the measurement frequency until the effects of L_{Pri} do not significantly affect the reading of L_{Leak} . Most communications transformers have ratios of L_{Pri}/L_{Leak} on the order of 200 to 1000. Flyback and certain digital telecommunications transformers with gapped cores may have ratios between 20 and 100. Ratios greater than 200 or so order allow direct reading of L_{Leak} with minimal loss of accuracy due to effects of L_{Pri} . The trick is to find the frequency at which the impedance of L_{Pri} is high enough not to interfere with the reading of L_{Leak} . One way to do this is to set your LCR bridge to read L_s and sweep the frequency until the inductance reaches its lowest value. A spot-frequency version of this was outlined in the earlier section. Another way is to "back out" the value of L_{Pri} from the L_s and R_s readings, but this ends up being fairly complicated. I worked up a MathCad sheet that calculates 'true' leakage inductance given L_{Pri} , R_{Pri} , R_{Sec} , turns ratio and measured input inductance and resistance. The MathCad sheet works fine, but when the frequency is on the low side of the plateau you must make extremely precise measurements of the input parameters to achieve believable results.

The example shown in the MathCad sheet worked fine for 1kHz and 10kHz, but at 100Hz an error of -0.5% in the reading of R_{Sin} gave a result of 165uH (when the correct number was about 14uH). An error of +0.1% yielded an impossible -26uH result. The moral of the story: stay reasonably close to the plateau range shown in figure 5.

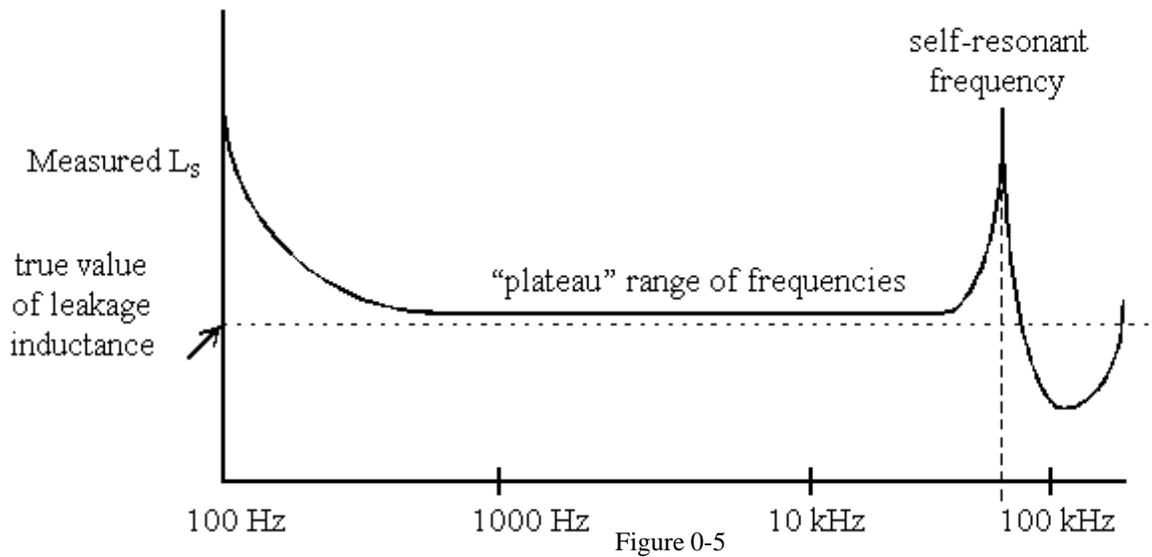


Figure 0-5
Apparent Leakage inductance versus frequency

URNS RATIO

Turns ratio errors generally result from poor coupling between the windings being measured. You can ameliorate this by putting the two windings in series then measuring the voltage ratio of one winding to the two in series. If the windings happen to be 1:1 you'll get a nice, round number of 2 for turns ratio when you use this method. Common-mode chokes are commonly tested for turns ratio this way since they typically have very poor coupling between the windings. At the "coil" stage of the manufacturing process, transformers may be tested with ungapped cores to reduce the effects of fringing. (The root word is fringe, making the first "g" in fringing a soft sound)

Fringing can make turns located near a gap seem to disappear in a magnetic sense. Thus we have turned an electrical conundrum into a philosophical question: Is the turns ratio of a transformer equal to the quotient of each winding's turn count or is it the measured ratio after the appropriate gapped core is placed on the coil? The answer depends on what you are trying to accomplish. As transformer manufacturers, we need to know the exact turns count our winding machines are applying to each coil. That is why we (usually) test our coils with ungapped cores to reduce the effects of fringing. The fully-assembled product, however, may react differently. Remembering that magnetic flux is actually a gradient, it is possible for any portion of a turn, or more than one turn, to be magnetically absent from the flux path. In a power supply, for example, the fringing effect may lead to lower output voltages than that predicted by the actual turns count applied to the coil. Knowing this, a transformer designer could add physical turns to make up for the apparent loss of turns. In this instance, it is more important that the *effective* turns ratio be measured since the output voltage is a function of the turns included in the magnetic flux path. This is true regardless of the actual number that are wound on the coil.

So the question remains: what is the turns ratio of a coil in the presence of fringing effects? The answer: whatever its effective voltage ratio happens to be. How do we measure this ratio? Any of several ways, but one that works very well in the absence of fringing and reasonably well in the presence of fringing is the *resistive bridge* method.

To use the resistive bridge method, connect the windings as shown at N_x and N_{ref} . Choose an oscillator level and voltage consistent with the midband frequency and drive level appropriate for the transformer under test. At Midcom, we use 1kHz or 10kHz and occasionally 100kHz, and we set V_1 to 1.0V. The exact values for frequency and voltage aren't critical. The 671-0018 isolates the generator from the circuit, but it won't work very well below 100Hz or above 100kHz. Set R_{REF} to some convenient decade value (10, 100, 1k, etc) ohms. Adjust R_x until V_2 is nulled as low as it will go. You should be able to get nulls less than $0.001V_1$. If you can't, you may have poor coupling between N_x and N_{REF} or the polarity of one winding is reversed. (This method will also verify polarity - an added benefit of the test) The resistive bridge method is Midcom Test Specification 999-2357. Refer to that specification for more details.

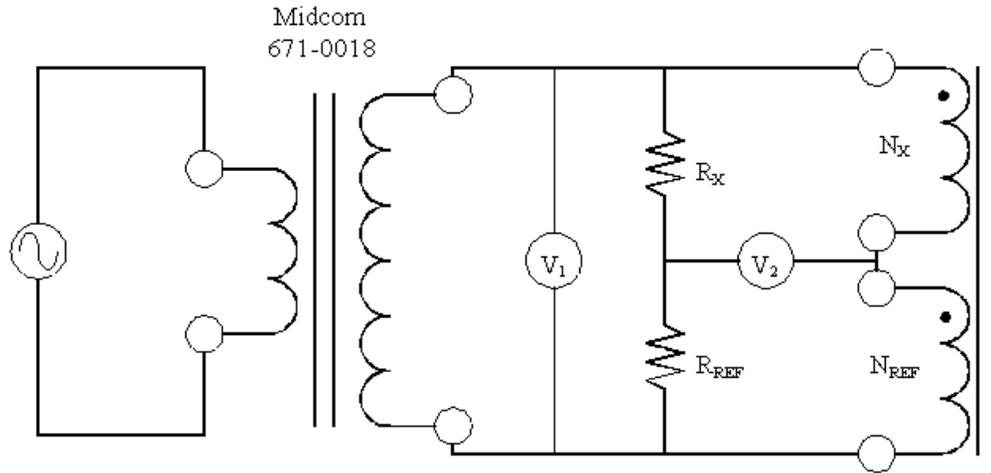


Figure 0-6

Resistive Bridge Turns Ratio Test Method

If all else fails, you can use the method described in the first section to determine turns ratio using inductance readings. What that method lacks in accuracy is made up in its convenience. If winding resistance is reasonably low, the inductance ratio method will give pretty decent results.

DISTRIBUTED CAPACITANCES

Only in the case of very loosely-coupled windings is the result of attempting to determine the individual primary and secondary capacitances worth the effort. Double-tuned RF transformers and some common-mode chokes are the only cases I can think of where it might be beneficial to know the individual values versus the lumped-constant capacitance value. Remember that the lumped-constant equivalent circuit is really only a convenience for us to visualize all of the elements. A distributed-parameter model is more appropriate, but much harder to analyze. In my lab experience, shorting one winding will cause the capacitance one would normally attribute to the shorted winding to "float" (reflect) over to the other side and appear at the measured winding's terminals.

As a method of double-checking the capacitance value you found in the first section, you can measure the transformer's self-resonant frequency and then calculate the distributed capacitance by solving the resonance formula for capacitance:

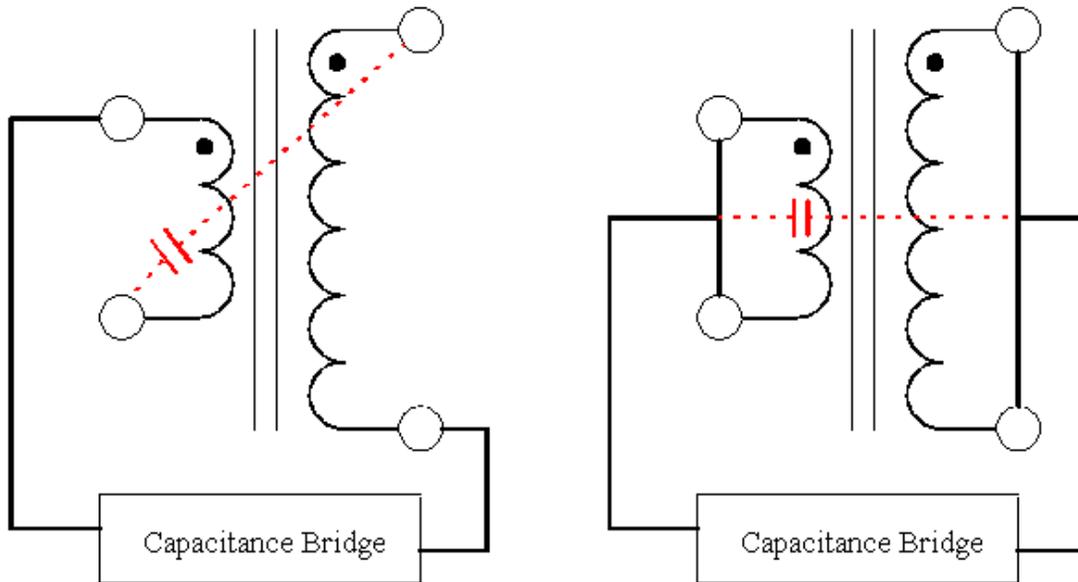
$$C = \frac{1}{4\pi^2 f^2 L}$$

Note that you must make some assumptions about L at the resonant frequency. Remembering that inductance is not always constant as frequency is varied, you may need to use the inductance factor to estimate the inductance at the resonant frequency since it cannot be measured directly. By making a series of measurements of inductance below self-resonance, you can see the trend of inductance shift versus frequency, then plug in an appropriate value to the formula shown above. If your impedance bridge is only capable of discrete frequency measurements, none of which is particularly close to the resonant frequency, you can estimate the resonant frequency by noting the two frequencies on either side of resonance that are conjugates of each other and taking the average of them. (You could take the geometric mean, $\sqrt{f_L f_H}$ instead if you feel the additional accuracy is warranted)

INTERWINDING CAPACITANCE

This one is so easy I should have put it in the first section. In practice, interwinding capacitance is rarely a concern in telecommunications transformer since it affects only common-mode signals (assuming the transformer is properly designed for good balance). In switchmode power and certain balanced inductors, interwinding and winding-to-core capacitance may result in low self-resonant frequency, reduced rise time and extended ringing.

To measure interwinding capacitance, short the primary windings, short the secondary windings, then measure capacitance between the two. If you don't short the windings, you run the risk of error due to a voltage differential being developed across the transformer's windings. If this happens, you will see some inductance inside your capacitance. This effect is most pronounced when you measure from a 'dotted' terminal on the primary side and an 'undotted' terminal on the secondary side. If this happens you will notice a self-resonance at fairly low frequencies. This is true particularly if the interwinding capacitance is fairly high or the winding inductance is high.



Improper measurement method: will give erroneous results due to inductance.

Proper method: eliminates possibility of exciting the magnetizing inductances.

Figure 0-7

FINAL WORDS

If you notice any unusual behavior when you are measuring transformer parameters, send your results to me. As time permits, I will update this technical note with your input, which is welcomed.

Best regards,

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