

Overshoot as a Function of Phase Margin

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4/19/03

When an amplifier with a gain $A(s)$ is put in a feedback loop as shown in Figure 1, the closed loop gain,

$$V_o/V_{in} = A_{CL}$$

$$A_{CL}(s) = \frac{A(s)}{1 + \beta A(s)} \quad (1)$$

The system is unstable when the loop gain, $\beta A(s)$, equals -1. That is, $\beta A(s)$ has a magnitude of one and a phase of -180 degrees. An unstable system oscillates. A system close to being unstable has a large ringing overshoot in response to a step input.

The phase margin is a measure of how close the phase of the loop gain is to -180 degrees, when the magnitude of the loop gain is one. The phase margin is the additional phase required to bring the phase of the loop gain to -180 degrees. **Phase Margin = Phase of loop gain - (-180).**

The loop gain has a dominant pole at ω_{p1} . Higher order poles can be represented by an equivalent pole at ω_{eq} . The amplifier is approximated by a function with two poles as shown in Equation 2.

$$A(s) = \frac{A_o}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{eq}})} \quad (2)$$

Since for frequencies of interest where the loop gain magnitude is close to unity,

$$\omega > \omega_{p1} \quad (3)$$

And,

$$A(s) = \frac{A_o \omega_{p1}}{s(1 + \frac{s}{\omega_{eq}})} \quad (4)$$

Also, [it can be shown](http://www.ele.uri.edu/~daly/535/margin.html),

$$(5)$$

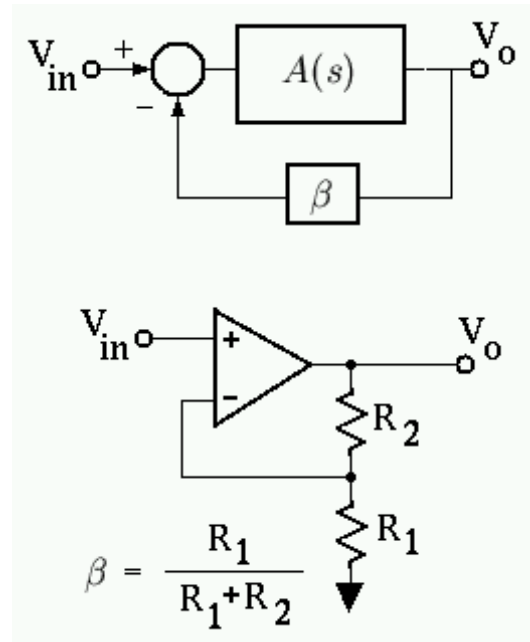


Figure 1

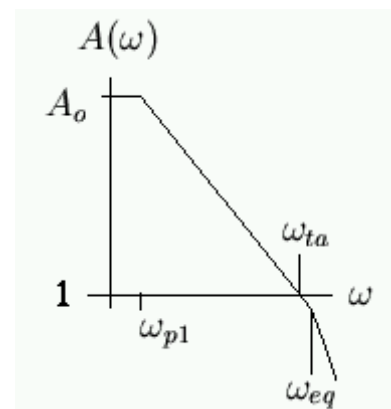


Figure 2 Amplifier frequency response.

PM	ω_t/ω_{eq}	Q	%OS
55°	0.700	0.925	13.3%
60°	0.580	0.817	8.7%
65°	0.470	0.717	4.7%
70°	0.360	0.622	1.4%
75°	0.270	0.527	0.008%

Table I

- **PM** is the phase margin.

$$\frac{1}{\omega_{eq}} = \frac{1}{\omega_{p2}} + \frac{1}{\omega_{p3}} + \dots$$

Defining ω_{ta} ,

$$\omega_{p1}A_o = \omega_{ta} \quad (6)$$

- ω_t is the unity gain frequency (rad/sec).
- ω_{eq} is the frequency of the equivalent higher order pole (rad/sec).
- Q is the system Quality factor.
- OS is the Over Shoot.

For frequencies of interest (frequencies close to the unity gain frequency), the amplifier gain can be written,

$$A(s) = \frac{\omega_{ta}}{s(1 + \frac{s}{\omega_{eq}})} \quad (7)$$

Plugging Equation 7 into Equation 1 results in the following expression for the closed loop gain.

$$A_{CL}(s) = \frac{1}{\beta} \frac{1}{1 + \frac{s}{\omega_{ta}\beta} + \frac{s^2}{\omega_{ta}\beta\omega_{eq}}} \quad (8)$$

Equation 8 is the transfer function for a second order system. The general form for the response of a second order system, where system properties are described by its Q and resonant frequency ω_o , is shown in Equation 10.

$$A_{CL}(s) = \frac{K}{1 + \frac{s}{Q\omega_o} + \frac{s^2}{\omega_o^2}} \quad (9)$$

By comparing Equation 8 to Equation 9 we can get an expression for the resonant frequency and Q of the amplifier closed loop gain. (Equate coefficients of like powers of s in the dominators.)

$$\omega_o = \sqrt{\beta\omega_{ta}\omega_{eq}} \quad (10)$$

$$Q = \sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}} \quad (11)$$

The loop gain is the feedback factor β multiplied by the amplifier gain $A(s)$.

$$\beta A(s) = \frac{\beta\omega_{ta}}{s(1 + \frac{s}{\omega_{eq}})} \quad (12)$$

The phase margin is a function of the phase of the loop gain at the frequency where the magnitude of the loop gain is unity.

$$\beta A(\omega_t) = 1 \quad (13)$$

where ω_t is the loop gain unity gain frequency. It follows from Equations 12 and 13 that,

$$\beta^2 \omega_{ta}^2 = \omega_t^2 \left[1 + \frac{\omega_t^2}{\omega_{eq}^2} \right] \quad (14)$$

Also, solving for ω_{ta} and dividing by ω_{eq} ,

$$\frac{\omega_{ta}}{\omega_{eq}} = \frac{1}{\beta} \frac{\omega_t}{\omega_{eq}} \sqrt{1 + \frac{\omega_t^2}{\omega_{eq}^2}} \quad (15)$$

It follows from Equations 11 and 15 that,

$$Q = \sqrt{\frac{\omega_t}{\omega_{eq}} \left(1 + \frac{\omega_t^2}{\omega_{eq}^2} \right)^{\frac{1}{2}}} \quad (16)$$

The phase of the loop gain (Equation 13) is.

$$\text{Phase of loop gain} = -90^\circ - \tan^{-1} \left(\frac{\omega_t}{\omega_{eq}} \right) \quad (17)$$

The phase margin is the additional phase required to bring the phase of the loop gain to -180 degrees.

Phase Margin = Phase of loop gain - (-180).

$$\text{Phase Margin} = 90^\circ - \tan^{-1} \left(\frac{\omega_t}{\omega_{eq}} \right) \quad (18)$$

A well known property of second order systems is that the percent overshoot is a function of the Q and is given by,

$$\%OS = 100e^{-\frac{\pi}{\sqrt{4Q^2-1}}} \quad (19)$$

Both phase margin (Equation 18) and Q (Equation 16) are a function of ω_t / ω_{eq} . This allows us to use Equation 19 to create tables and plots of percent overshoot as a function of phase margin. As shown in Figures 3 and 4, and in Table I.

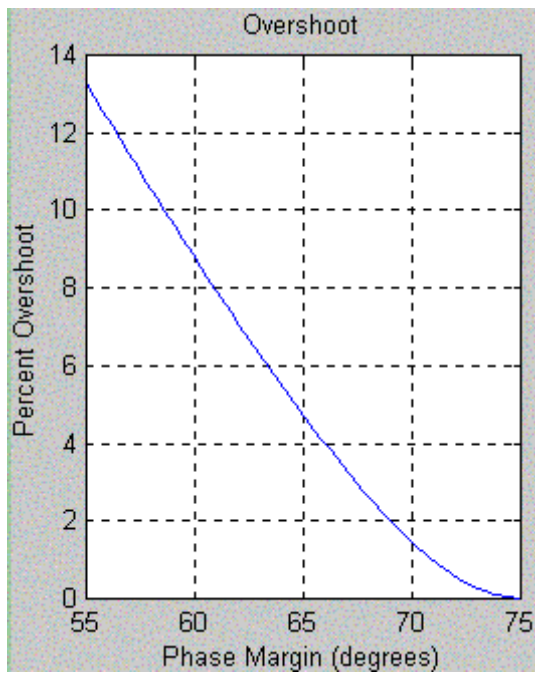


Figure 3 Overshoot as a function of phase margin.
Plot generated using [MATLAB code](#).

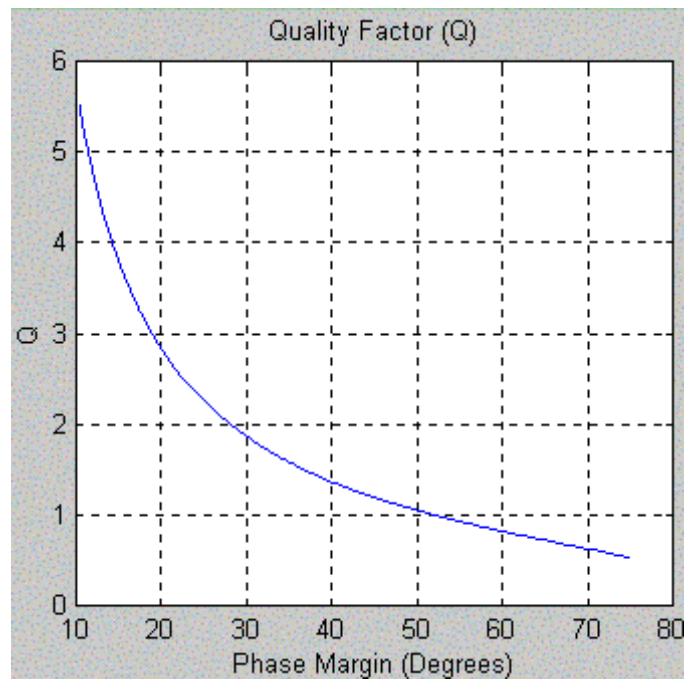


Figure 4 **Q** as a function of phase margin.
Plot generated using [MATLAB code](#).