the input signals). As the level rises, the distortion is thus weighted toward the higher-order terms, and mostly toward higher frequencies.[6] Further experiments (not shown) with the FET amplifier demonstrates this point.

If we change the fundamental signal amplitudes, the amplitudes of the distortion products change so that the spectrum has essentially the same shape in that it is made up of tiers of distortion products, but the amplitude difference between the tiers changes. As the fundamental signal amplitudes decrease the spacing increases, so small signals are *relatively* less distorted than large ones.

BJTs are different

Consider the two Bipolar Junction Transistor (BJT) circuits shown in figure 6. BJTs are quite different from FETs in that, while FETs have a square-law relationship between the controlled current and the control voltage, the relationship for a BJT is exponential. For a correctly biased BJT a good approximation of the collector current i_C in terms of the base-emitter voltage v_{BE} is

$$i_C = I_0 \left(e^{\frac{q}{kT} v_{BE}} - 1 \right), \tag{32}$$

where I_0 is a very small number, such as 10^{-12}, that depends on the geometry of the transistor; and $\frac{q}{kT}$ is about 38.68 for a temperature of 300 K.

Circuit **d** is a simple BJT common-emitter amplifier with no feedback, while circuit **e** is the same but with emitter feedback created by the inclusion of non-zero R_E. We consider two values of R_E, 100 and 1000 ohms, and refer to the two resulting amplifiers, having respectively about 14 and 32 dB of

[6] This strong dependence of the amplitude of a distortion product on its order is not accidental, but it is inherent in the mathematics of the situation: each distortion product arises from terms of the series expansion that have powers of the input equal to the sum of the absolute values of the integer weights of the fundamental components. So frequency $17 = 4 \cdot 3 + 1 \cdot 5 = -1 \cdot 3 + 4 \cdot 5$ has integer weights 4 and 1 which sum to 5. Thus the exponent associated with it is 5 and so a 12 dB change in the input produces a $5 \times 12 = 60$ dB change in the output at this frequency. Of course, $17 = 7 \cdot 5 - 6 \cdot 3$ as well, with an exponent of 13. So this term grows even faster than the fifth-order ones already described, but it starts out so much smaller that under reasonable assumptions it never exceeds the fifth-order terms. The lowest-order combinations of input frequencies that give any particular component are usually dominant. The conditions under which this series converges, and under which the low-order terms are dominant are mathematically very interesting, but they are beyond the scope of this paper.

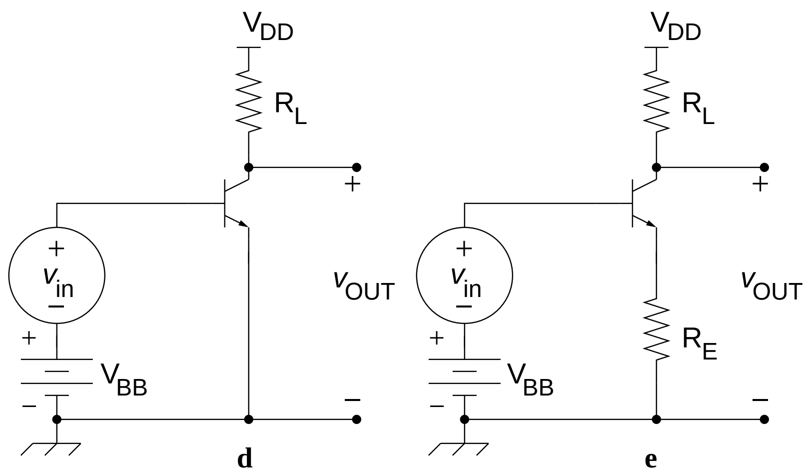

Figure 6: Amplifier **d** has no feedback; amplifier **e** has emitter feedback.

feedback, as **e** and **f**. We bias all three amplifiers to have a gain of -10 with a collector current of about 1 mA. The analysis of these BJT amplifiers is similar to the analysis of the FET amplifiers. The operating conditions are summarized in the table below. The numbers are not round because we wanted to bias circuit **d** to 1 mA.

	d	**e**	**f**	
V_{BB}	0.5453	0.6453	1.5453	V
R_L	263.2	1264	10270	Ω
R_E	0	100	1000	Ω
I_C	1.0	1.0	1.0	mA

The nonlinearity of a BJT is extremely sharp, so we drive the input with a peak voltage of only 0.0004 Volts in each frequency. If we used 0.05 Volts, as with the FETs, the distortion of the no-feedback BJT amplifier would be very bad and we would not be able to understand the effect of the feedback. To avoid this distortion in real-world circuits that use BJTs, degenerative feedback is employed to ensure that the signal appearing across the base-emitter junction of the transistor is very small.

The spectrum of **d**, the BJT amplifier without feedback (figure 7), is complicated, and remarkably similar to the spectrum of the FET amplifier *with* feedback, showing several tiers of distortion products. We can understand

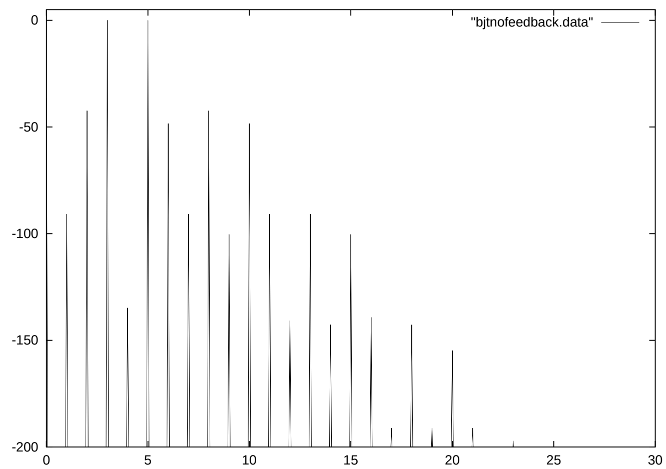

Figure 7: The spectrum of amplifier **d**: a single-ended BJT without feedback; the two-tone input has 0.0004 peak volts in each component.

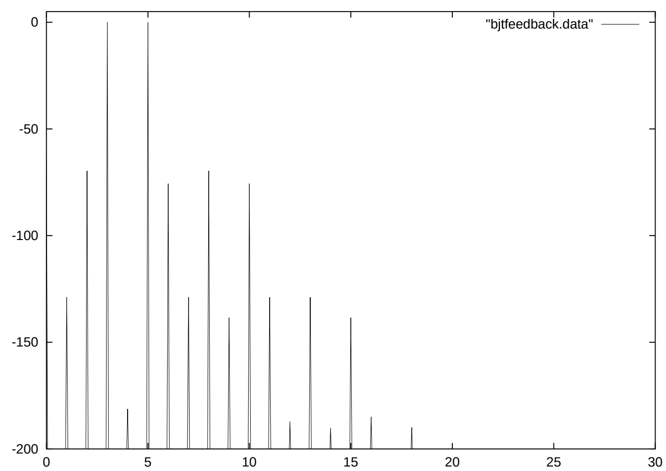

Figure 8: The spectrum of amplifier **e**: a single-ended BJT with emitter feedback; the two-tone input has 0.0004 peak volts in each component.

this by realizing that the power-series expansion of the BJT's exponential characteristic contains terms of all order in the input voltage. In the FET amplifier the high-order terms are constructed by the feedback process; in the BJT, they are inherent.

The spectrum of **e**, the BJT amplifier with feedback (figure 8) is like the spectrum without feedback except that all of the distortion products are substantially attenuated. When we increase the feedback, as in circuit **f**, we see (figure 9) that the performance of the amplifier again improves dramatically, so that the principal distortion lines are suppressed below -100 dB and the next tier is suppressed below -180 dB. Thus, in the case of the BJT, feedback improves the nonlinear distortion of the amplifier.

Although the no-feedback BJT amplifier started out much worse than the no-feedback FET amplifier it is vastly improved by feedback; and because the transconductance of the BJT is much higher than the transconductance of the FET, we can use much more feedback and still obtain the same overall gain.

The extra gain also allows enough feedback so that a BJT amplifier can accomodate a greater range of input voltages without exceeding a given distortion level, even though the intrinsic nonlinearity of the BJT is much sharper than the nonlinearity of the FET, seriously restricting the input range of the bare BJT relative to the bare FET. For example, if we try to drive BJT circuit **f** (with 32 dB of feedback) with the same size input that we used with the FET amplifiers (peak voltage 0.05 in each component), the distortion in the BJT amplifier (figure 10) is 13 dB lower than the distortion in the FET amplifier **c** with only 9.5 dB of feedback.

Comparison of figure 9 and figure 10 illustrates another interesting point: For the BJT stage with or without feedback, as for the FET stage wth feedback, higher-order distortion products are emphasized as the amplitude of the input signals is increased.[7] This is generally true of any nonlinear system that shows tiers of intermodulation products.

Vacuum triodes

To analyze a triode we start with the behavior of a vacuum diode. This can be approximately characterized by the Child-Langmuir law, which gives the

[7]See the footnote 6

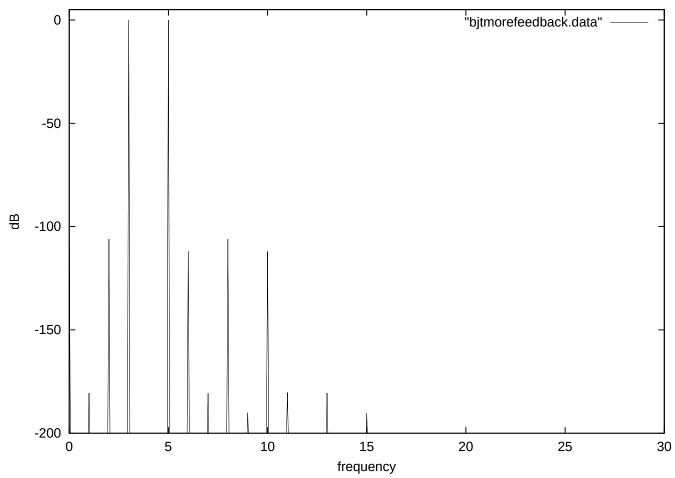

Figure 9: The spectrum of amplifier **f**: a single-ended BJT with more emitter feedback; the two-tone input has 0.0004 peak volts in each component.

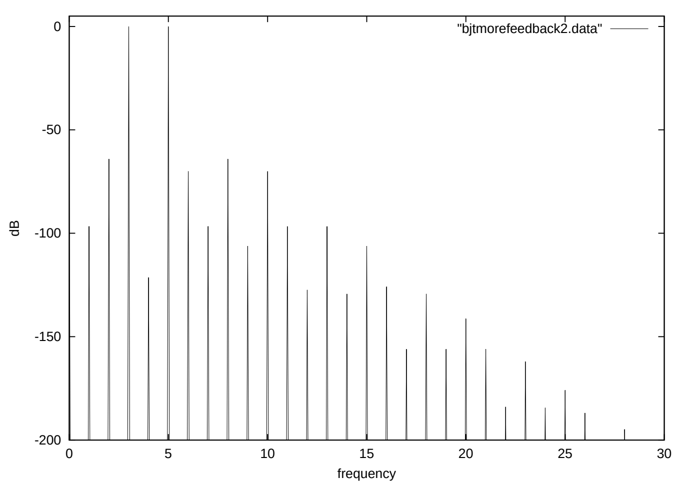

Figure 10: The spectrum of amplifier **f**: a single-ended BJT with more emitter feedback; the two-tone input has 0.05 peak volts in each component.

anode current i_P as a function of the voltage from anode to cathode v_{PK}

$$i_P = \frac{\sqrt{2}}{9\pi d^2}\sqrt{\frac{e}{m}}v_{PK}^{3/2}, \tag{33}$$

where d is the distance from the anode to the cathode, and e/m is the charge-to-mass ratio of the electron. This forms a theoretical basis for a model for the behavior of a vacuum triode by modifying equation (33) to include the influence of the control grid [6]. The influence of the control grid is parameterized by the amplification factor μ, which is defined to be

$$\mu = \frac{\partial v_{PK}}{\partial v_{GK}}, \tag{34}$$

where v_{GK} is the grid-to-cathode voltage. With this, the model is

$$i_P = K(v_{PK} + \mu\, v_{GK})^{3/2}, \tag{35}$$

where the perveance K and the amplification factor are determined by the geometry of the device. In published specifications, the amplification factor and the transconductance

$$g_m = \frac{\partial i_P}{\partial v_{GK}} \tag{36}$$

are often provided for a given operating point, and we have to deduce the perveance.[8]

For the FET amplifiers we had single equations (14, 30) that gave v_{OUT} in terms of v_{in}. For the BJT, the situation was similar, though we did not give the equations. In the FET case, equation (30) was an explicit solution, but in the BJT case a numerical solution was necessary. But for the tube circuit **h** (amplifier **g** is the same, but with $R_K = 0$; see figure 11), we have instead a fairly complicated set of equations:

$$\begin{aligned} v_{PK} &= V_{BB} - (R_L + R_K)i_P & (37)\\ v_{GK} &= v_{in} - V_{CC} - R_K i_P & (38)\\ i_P &= K(v_{PK} + \mu\, v_{GK})^{3/2} & (39)\\ v_{OUT} &= V_{BB} - R_L i_P. & (40) \end{aligned}$$

[8] For example, for a section of a 6DJ8 [1] double triode, the data sheet gives $g_m \approx 0.0125$ amps/volt and $\mu \approx 33$ when operating at $V_{PK} = 90$ volts and $V_{GK} = -1.3$ volts, so we have $K \approx 3.7 \times 10^{-5}$.

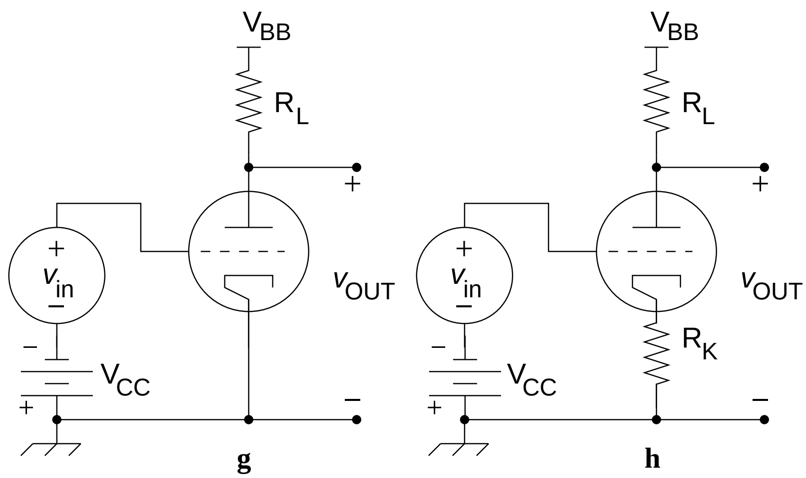

Figure 11: Amplifier **g** has no feedback; amplifier **h** has cathode feedback.

It is not illuminating to attack these equations analytically. However, the 3/2 power law in equation (39) can be expressed as a power series that has terms of all order, and this will be part of the answer even when $R_K = 0$. Thus, we can expect the triode results to be something like the BJT results obtained above, in that distortion products of all orders should appear in the spectrum of the simple amplifier **g**. The question is: Does the addition of feedback make things better or worse?

The numerical results are not surprising. We simulated the amplifier with a 6DJ8 triode[9] under the following operating conditions:

	g	**h**	
V_{BB}	100	140	V
V_{CC}	-1.2	0	V
R_L	1200	3500	Ω
R_K	0	150	Ω
I_P	11.7	11.1	mA

Both of these amplifiers have a gain of approximately −10. The spectrum of the no-feedback tube amplifier **g** has all orders of distortion products, as predicted. But the distortion products are generally smaller than the

[9]See footnote 8

corresponding products in either the no-feedback FET or BJT amplifiers (**a** and **d**), because the triode is fundamentally more linear than either the BJT or the FET.

The spectrum of amplifier **h**, with cathode feedback (figure 13), is cleaner than that of **g** (figure 12). All of the products are pushed down considerably, making it comparable to the spectrum of the BJT with emitter feedback (figure 8). However, here the amplitude is 0.05 volts for each component; the comparable BJT spectrum is for a drive of .0004 peak volts. If we compare with the spectrum of a BJT with "more" emitter feedback driven with 0.05 peak volts (figure 10) we find that the tube amplifier with small cathode feedback is still somewhat cleaner.

Additional experiments (not shown) show that smaller signals produce smaller distortion components, for triode amplifiers with and without cathode feedback.

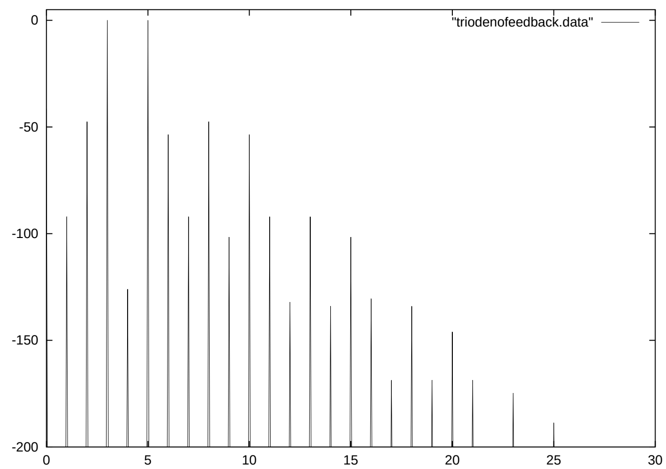

Figure 12: The spectrum of amplifier **g**: a single-ended vacuum triode without feedback; the two-tone input has 0.05 peak volts in each component.

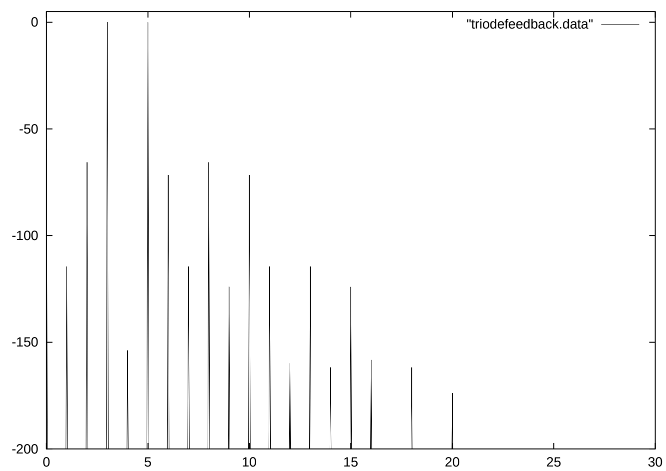

Figure 13: The spectrum of amplifier **h**: a single-ended vacuum triode with cathode feedback; the two-tone input has 0.05 peak volts in each component.

Details of single-ended amplifiers

In the following table we summarize the relative amplitudes of the principal spectral lines that occur in the output of each of the elementary amplifiers that we have considered:

	FET			BJT			Triode	
ckt	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**
drive	0.05	0.05	0.05	0.0004	0.0004	0.0004	0.05	0.05
fb dB	0	1.8	9.5	0	14	32	0	4.2
1		-78.9	-92.7	-90.7	-128.9	-182.1	-92.1	-114.5
2	-32.0	-34.5	-51.1	-42.4	-69.6	-106.0	-47.5	-65.6
3	0	0	0	0	0	0	0	0
4		-114.3	-125.4	-134.8	-181.3	-193.7	-126.0	-153.8
5	0	0	0	0	0	0	0	0
6	-38.1	-34.5	-57.1	-48.4	-75.7	-112.0	-53.5	-71.6
7		-78.9	-92.7	-90.8	-128.9	-177.9	-92.1	-114.5
8	-32.0	-34.5	-51.1	-42.4	-69.6	-106.0	-47.5	-65.6
9		-88.4	-102.2	-100.3	-138.4	-187.0	-101.6	-124.0
10	-38.1	-40.5	-57.1	-48.4	-75.7	-112.0	-53.5	-71.6
11		-78.9	-92.7	-90.8	-128.9	-180.7	-92.1	-114.5
12		-120.3	-131.4	-140.8	-187.2	-197.8	-132.1	-159.8
13		-78.9	-92.7	-90.8	-128.9	-182.6	-92.1	-114.5
14		-122.3	-133.4	-142.7	-190.2	-200.5	-134.0	-161.8
15		-88.4	-102.2	-100.3	-138.4	-184.8	-101.6	-124.0
16		-118.7	-129.8	-139.2	-185.0	-188.8	-130.5	-158.2
17		-159.2	-167.6	-191.1		-201.4	-168.7	-202.4
18		-122.3	-133.4	-142.7	-189.1	-209.7	-134.0	-161.8
19		-159.2	-167.6	-191.1			-168.7	-202.4
20		-134.3	-145.4	-154.8	-199.0		-146.0	-173.8

Summary of single-ended amplifiers

For all cases, input signals were at frequencies 3 and 5 (unscaled). Input voltage levels were always equal for the two components and were as given below. Unless otherwise stated, (a) Higher input level raises the relative distortion and (b) emphasizes higher-order distortion products. (c) Adding more feedback lowers all distortion products.

1. FET: 0.05 peak volts in each input component

   Without feedback, distortion consists of only 2nd harmonics and first-order sums and differences of the two input components; that is, only the four frequencies 2, 6, 8, 10. Feedback creates complex new distortion products extending over the full bandwidth of the amplifier and thus constituting a kind of "noise floor."

2. BJT: 0.0004 peak volts in each input component

   Distortion without feedback is complex, resembling that of the FET amplifier *with* feedback.

3. Triode: 0.05 peak volts in each input components

   Distortion without feedback is complex, resembling that of the FET amplifier with feedback, but distortion products are distinctly lower in level that in the no-feedback FET amplifier at the same input level, or even the no-feedback BJT amp at its much lower level.

Output stages using two devices

The push-pull output stage is ubiquitous in audio amplifiers. It provides symmetrical low-impedance drive, and it introduces no even-harmonic distortion.

BJT complementary-pair output stage

Consider the idealized BJT stage in figure 14. We assume that the PNP and NPN transistors have identical behavior, except for signs, and that they are symmetrically biased and operated in class A (that is, neither transistor is cut off for any input signal). One way to understand this stage is as a push-pull emitter follower; it has no voltage gain, but plenty of power gain.

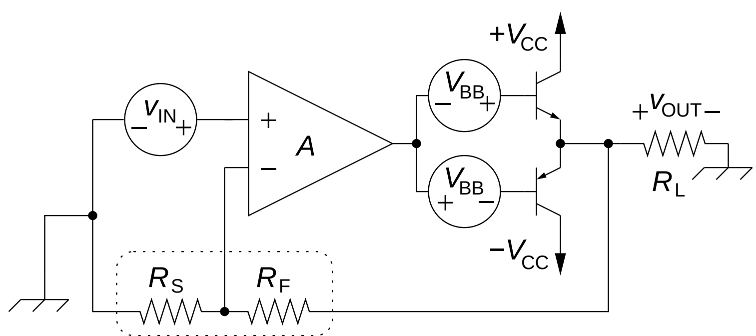

Figure 14: An idealized complementary-pair stage constructed from BJTs, driven by an ideal linear differential amplifier of voltage gain A.

The analysis is then quite simple. For the two transistors we have

$$i_{CN} = +I_0 \left(e^{+\frac{q}{kT} v_{BEN}} - 1 \right) \tag{41}$$

$$i_{CP} = -I_0 \left(e^{-\frac{q}{kT} v_{BEP}} - 1 \right), \tag{42}$$

where

$$v_{BEN} = A\Delta v - v_{OUT} + V_{BB} \tag{43}$$

$$v_{BEP} = A\Delta v - v_{OUT} - V_{BB} \tag{44}$$

and

$$i_{OUT} = i_{CN} + i_{CP} \tag{45}$$

$$\Delta v = v_{IN} - \frac{R_S}{R_F + R_S} v_{OUT}. \tag{46}$$

From these we can deduce that

$$v_{OUT} = 2R_L I_0 e^{\frac{q}{kT} V_{BB}} \sinh\left(\frac{q}{kT}\left(Av_{IN} - \left(\frac{AR_S}{R_F + R_S} + 1\right) v_{OUT}\right)\right). \tag{47}$$

For each v_{IN} we must numerically solve for v_{OUT}.

By adjusting V_{BB} this complementary-pair stage could be biased for any amount of crossover distortion. The stage has approximately unity gain, but by adjusting the prescalar gain A and the resistive divider (R_S and R_F) we can see how the stage would behave in an amplifier with any gain and feedback we please. We tested the BJT pair stage only in class A, however. As in the single-ended cases, we set things up so that the overall gain is always 10. In our experiments we set $v_{BB} = 0.6$ volts. So, with $I_0 = 10^{-12}$ amperes we have a resting bias current of about 8 mA through the transistors.

In the no-feedback case, we prescale the input by 10 (to give the desired overall gain), so the actual signal driving the bases of the complementary pair had a peak voltage of 0.05 volts for each component. (The variation in the base-emitter voltage remains small here because this is a follower circuit.) With this we got the spectrum of the output that appears in figure 15.

We introduce feedback by boosting the prescalar gain to 100, and feeding back 0.089 of the output signal. With this we get the spectrum of the output that appears in figure 16. As we would expect, the feedback improves the result by suppressing all of the distortion products. Whether or not we apply feedback in this class-A amplifier the relative distortion decreases with the signal amplitude.

FET complementary-pair output stage

We can replace the complementary-pair of BJTs in figure 14 with FETs. The most common way to do this is to use the P-channel FET as the pullup and the N-channel FET as the pulldown. This yields a symmetrized inverting amplifier stage with voltage gain, as in a CMOS inverter stage. An unusual alternative, which is more analogous to the complementary BJT stage analyzed above, is to make a symmetrical source follower, as in figure 17.

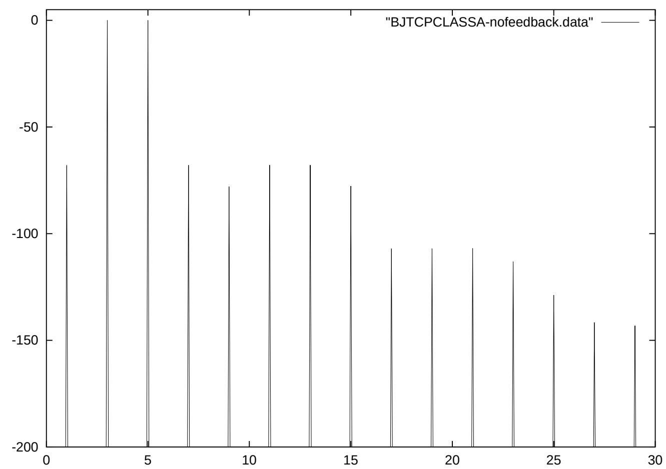

Figure 15: The spectrum of the complementary-pair BJT amplifier without feedback; the two-tone input has 0.005 peak volts in each component.

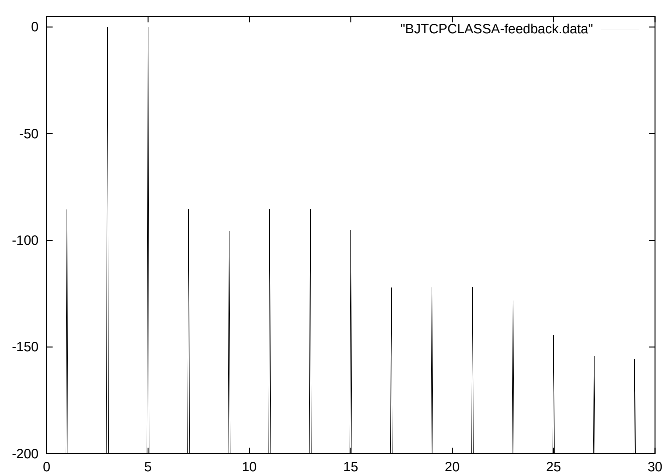

Figure 16: The spectrum of the complementary-pair BJT amplifier with 19 dB of feedback; the two-tone input has 0.005 peak volts in each component.

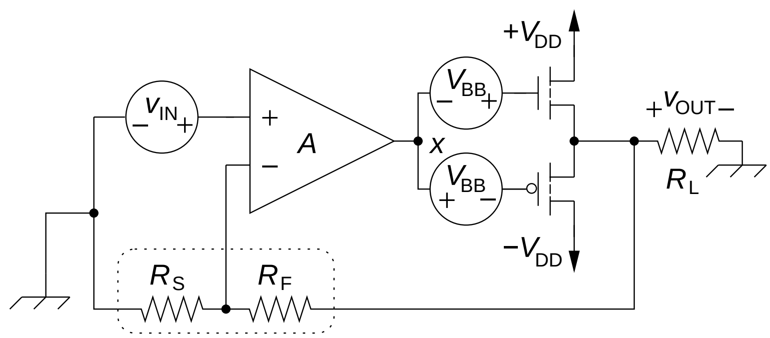

Figure 17: An idealized complementary-pair stage constructed from FETs, driven by an ideal linear differential amplifier of voltage gain A. The node potential at the output of the differential amplifier is labeled x.

If we let the excess bias $V_B = V_{BB} - V_T$ and we let $v = x - v_{out}$, where x is the node potential at the output of the differential amplifier, then

$$i_{DN} = \begin{cases} 0 & \text{if } v + V_B < 0 \\ +\frac{k}{2}(v + V_B)^2 & \text{otherwise} \end{cases} \tag{48}$$

$$i_{DP} = \begin{cases} 0 & \text{if } v - V_B > 0 \\ -\frac{k}{2}(v - V_B)^2 & \text{otherwise} \end{cases}. \tag{49}$$

By adjusting V_{BB} we can choose to make this stage operate class A or class B (for any input, one transistor is always cut off and the other is active) or any combination.

A remarkable feature of this complementary pair is that if we set $V_{BB} >> V_T$, so the stage is operated class A, then the distortions exactly cancel and the result is linear! In this case

$$v_{OUT} = \frac{2R_L V_B k}{1 + 2R_L V_B k} x; \tag{50}$$

feedback has no effect except to control the gain.

If we set the bias $V_{BB} = V_T$ then $V_B = 0$, the amplifier operates in class B, and there is distortion. Where the BJT emitter follower had voltage gain close to unity, the FET source follower has substantially less gain. So to attain an overall gain of 10 with a load of 1000 Ω and no feedback, the prescalar must have a gain of about 20.