As there becomes more and more interest in polar response (I remember when I seemed to be the only one who cared), it's important to understand some facts about how sound gets radiated. Understanding these facts can really help in understanding the problem, but even more so can be used to create a far better scheme for actually measuring the polar response.

Let's look at the simplest possible example, a flat piston in an infinite wall. The polar response for this is well known and this classic "beaming" response is the result of the sound radiation from a single "mode". This mode is the zeroith mode, or the mode where the piston is rigid and moves as a single unit.

But what if the piston is not rigid and moves in complex ways? It's well-known that for a circular disk, or membrane (a membrane is the easier situation), any response, no matter how complex can be broken down into a series of modes. It is also know that for any given upper frequency limit that there are a finite number of modes in the membrane that will describe the entire situation. Lest say that we are interested in some maximum frequency X, there will be some number of modes N which uniquely describe the motion of the membrane up to X. What is not so well known is that there is a one-to-one correspondence between the modes of vibration in the membrane and the modes that get radiated. For each membrane mode there is exactly one radiation mode for that vibration mode.

This means that for all frequencies below X, if I know the N vibration modes in the membrane then I will know the sound radiation at every field point with infinite precision up to the frequency X. It is also well known that if I have N modes in the membrane then there is some set of N points which when measured will yield the N modes – N equations in N unknowns. Now it's also true that if I measure the right N points of sound in the far field (or any distance actually) then I can calculate the N radiation modes and from this determine the N vibration modes. In other words I can uniquely define the vibration shape of the membrane from N points of sound radiation.

How do I know if I have the right N points? Simple, just take more points than N, say, on the extreme, I take 2N points. Then If I reconstruct the membrane vibration and I add in one mode at a time, I will find that at some point adding in a mode does not change the vibration pattern. At this point I have a sufficient number of modes to characterize both the membrane and the radiation field (at any point near or far mind you).

How does this work for a real loudspeaker? Well you first need to understand that the technique that I described above only works when the radiation device is a surface that is one of the orthogonal coordinate systems. Otherwise the modes are not orthogonal and "spanning" the space is not assured. But lets say that I enclose the real source in a hypothetical sphere. Then there is guaranteed to be some velocity distribution on that sphere that will generate the sound field that I measure. This spear will not exactly correspond to the exact device under test, but that does not actually matter. The only caveat is that when I reconstruct the motion from the measured sound field, I can only do so on some sphere. This requirement then limits the accuracy of the location of the radiated sound ON the actual source, but in no way limits the resolution or accuracy of the radiated sound when some distance from this sphere.

What this buys us is this: by measuring the sound field I can calculate the radiation modes, basically what I get is the frequency response of the N modes. How do I know if I have enough modes? It's exactly the same technique that I described before: add in one mode at a time until there is no longer any change in the response at the highest frequency of interest. Now that I know that I have a sufficient number of modes to represent both the sources velocity and the sound field, I know the sound level everywhere below the same frequency X to infinite resolution (note that resolution is not the same thing as precision.)

In my testing thus far I have found that about 8 modes are required to get to 10 kHz, hence, in theory it would only take 8 points to get a complete polar response (I am assuming here that the device is axisymmetric. This is not a serious limitation, it just makes the calculations easier. Things get very tedious with a full 3-D calculation, but certainly no more complicated, just more tedious.) I have found that since the largest rate of change of SPL with angle occurs very hear the axis, one needs to put most of the points near the axis. I test at 0, 5, 10, 15, 20, 30, 40, 50, 60, 80, 100, 120, 150 and finally 180 degrees (note that if we are assuming a sphere, then I need to do a full sphere, there is no way out of this.)

If any of this seems odd then consider this: At low frequencies, if I measure the SPL at a single point, then I actually know it everywhere – that's because at LFs only the zeroith order mode will radiate and hence only this mode matters. If I know this mode then I know everything – at least up until the dipole or first mode becomes significant.

Sure there are lots of details to all of this, not the least of which is that the problem is unstable at LFs because all of the modes are ineffective and any of them can account for the errors, or noise in the data if the levels are large enough. Basically you just have to cut off the higher order modes at the lower frequencies. There is not a precise way of doing this, but it turns out to not be so critical. But, a simple computer calculation without some interference will simply blow up at these lower frequencies because of these singularities. (Oh the trials and tribulations of numerical analysis!).