An Analysis of Bending Wood via Kerf Cuts

Introduction

A 1996 article in *The Family Handyman* [1] describes a method of forming circular arches out of strips of plywood by making a series of kerf cuts. The author, David Radtke, wrote, "Kerf bending is not the only method of curving wood, but it's the simplest and the most direct, " and gave the following algorithm.

1. Find the middle of your wood and make a slice in the wood leaving approximately 1/32" in the wood. (This slice is known as a kerf, and hence the name of the method.)
2. Bend the wood at the cut and measure along the raised surface the desired inner radius (r) of the arch. From this point measure the vertical distance (W) to the flat surface below. (See Figure 1.) This is the distance between cuts.
3. Make repeated cuts distance W from each other on either side of the initial cut until sufficient cuts are made. Alternate sides as you make these cuts to ensure symmetry.

Mr. Radke "does the math" to determine how many cuts are necessary: C = 3.14*(r +3/4") gives the circumference of the arch, where 3/4" is the thickness of the plywood and 3.14 is the approximation of the "magic repeating decimal B". C divided by W gives an approximation to N (the number of cuts necessary).

Fortunately for a pair of inquisitive undergraduate students, one mathematics teacher, and one professor, Mr. Radke didn't justify his algorithm. Why does the kerf cut method above yield a circular arch?

Preliminary Investigations

In an experimental phase of the investigation, we created several samples of kerf cut arches and it became apparent that we weren't getting semicircles, but rather polygonal arches (See figure 2.) which looked like semicircles from a distance. Our conviction that we were, in fact, only going to get an approximation when applying this method was confirmed by checking a handbook on wood-based materials [2]:

The deflection of beams is increased if reductions in cross-section dimensions occur such as are caused by holes or notches.

That is, the curvature of the bent wood at the locations of the kerf cuts is different from the curvature at other locations. Without a constant rate of curvature, we can not possibly attain a perfect semicircle.

Our experiments also showed that the angle formed in Step 1 of the algorithm was a function of the type of wood used, with plywood generating an angle of approximately 11 degrees, and solid oak generating an angle of only 9 degrees. This means that W (distance between cuts) is a function of r (inner radius of the arch) and the type of wood used. It further indicates that we will consistently use 17 to 19 cuts to form a semicircle - bending the wood 180 degrees, and that the number of cuts is independent of C (the circumference of the arch).

So is the method really any good? Or, equivalently, how well does the polygon that we obtain approximate a semicircle?

The First Calculation

Since the lengths of both sides of the unbent wood are the same, we have the outer circumference of the semicircle equal to the inner circumference minus the wood removed in the cuts. That is, BR = Br - BN, where R represents the outer radius of our arch and B represents the width of the saw blade. Solving for N, we get

N= (BR - Br)/B = BT/B

where T = R-r is the thickness of the wood.

For standard values for T and B (T = 3/4" and B = 1/8") we find that N = 18.84, consistent with our experimental data.

Analysis of the Polygonal Approximation

To determine the amount of error involved in the kerf cutting algorithm, we decided to calculate the ratio of the area of the resultant polygon to the area of the semi-circle that we hoped to obtain, as illustrated in Figure 2.

The area of the polygon is found by summing the areas of its isosceles triangles. There are N-1 of these triangles (where N is the number of kerf cuts made.) plus one-half of another at each end of the polygon. Since the central angles of the polygon are " = B/N radians each, the other two angles of each triangle are $ = B *(1/2 - 1/(2N)) radians. The area of each triangle is then given by

r^2sin($)sin("/2)

and the area of the polygon is therefore

$$A = N*r^2\sin(\$)\sin("/2)$$

$$= N*r^2\sin(B *(1/2 - 1/(2N)))\sin(B/2N)$$

$$= N*r^2\cos(B /(2N))\sin(B/2N)$$

$$= (N*r^2/2)\sin(B/N)$$

The ratio of the area of the polygon to the semicircle can then be found by dividing A by $B*r^2/2$, yielding $(N/B)\sin(B/N)$. The error in our approximation is given by

$$E = 1- (N/B)\sin(B/N) .$$

From this we see that the error is not dependent on the radius of the arch we are making. That is, we get the same ratio of error whether we are making a small arch or a very large arch, and the observer will have the same illusion of viewing a semi-circle.

In particular, when using T =3/4" and B=1/8", E = .0175. The areas of the polygon and the semicircle differ by only 1.75% ! Since a finished product will generally involve some sanding and the addition of a flexible veneer surface, we see that Mr. Radke's algorithm does provide a remarkably good approximation to a true semicircle.

Back to the Algorithm

The question still remains as to how the algorithm, in which the carpenter uses the distance from the bent wood to the level surface to determine the distance between her cuts, is used to construct the polygon.

Basically, since the carpenter doesn't know the angle ", she uses W to approximate the length of a side of the polygon, S. Bending the wood creates an exterior angle which supplements the angle 2$ of the polygon with $ = B *(1/2 - 1/(2N)). See Figure 3. This supplementary angle is the same as the central angle ". The measurement W is the length of the perpendicular dropped from one of the two equal angles in the isosceles triangle, so that she has W/S = sin($). Given integer values of N in the interval [17, 21] as suggested by our experimentation, we see that sin ($) is in the interval [.996, .997]. Hence the carpenter's approximation to the polygon is off by less than four-tenths of one percent. A pretty good approximation!

Conclusion

This investigation into an old carpenter's trick required that we do some experimentation, use trigonometry, as well as begin to become materials engineers. Finally, it is interesting to note that though Mr. Radke was working with wood, we found the same kerf cut method demonstrated in *LEGO*s. When inverted, rubber *LEGO* tires demonstrate the same kerf cut pattern that we've described here.

References

[1] Radke, David. "Making Curves in Wood". *The Family Handyman.* September, 1996, p104, 105, 127.

[2] Forest Products Laboratory, Forest Service, US Department of Agriculture. *Handbook of Wood and Wood-based Materials for Engineers, Architects, and Builders.* Hemisphere Publishing Co., New York, p11-7 to 11-12 (1989).

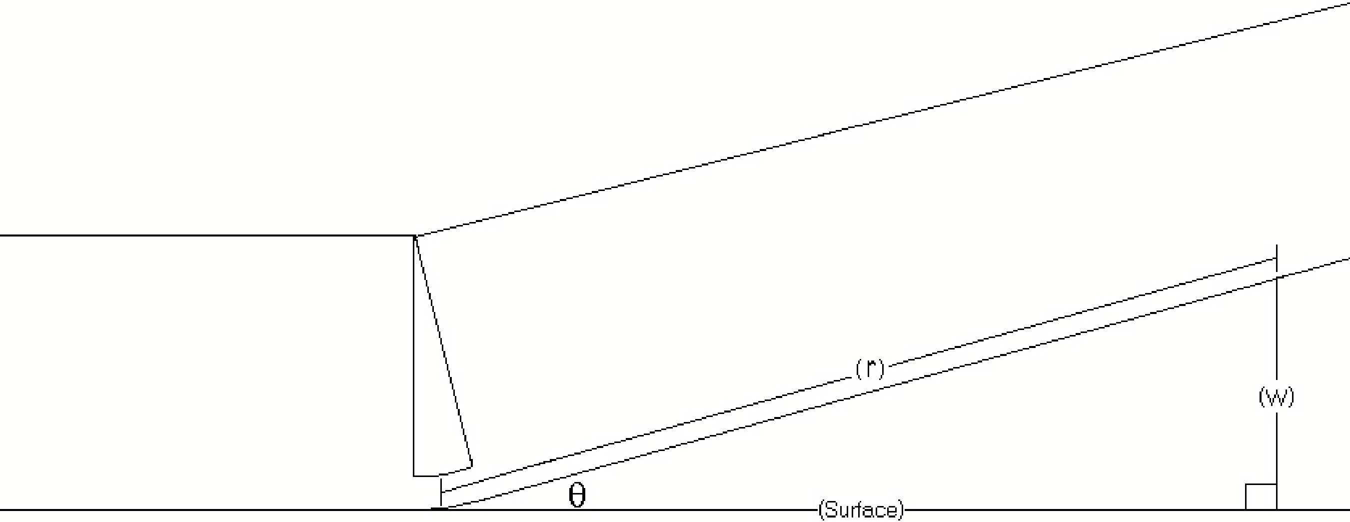

Figure 1

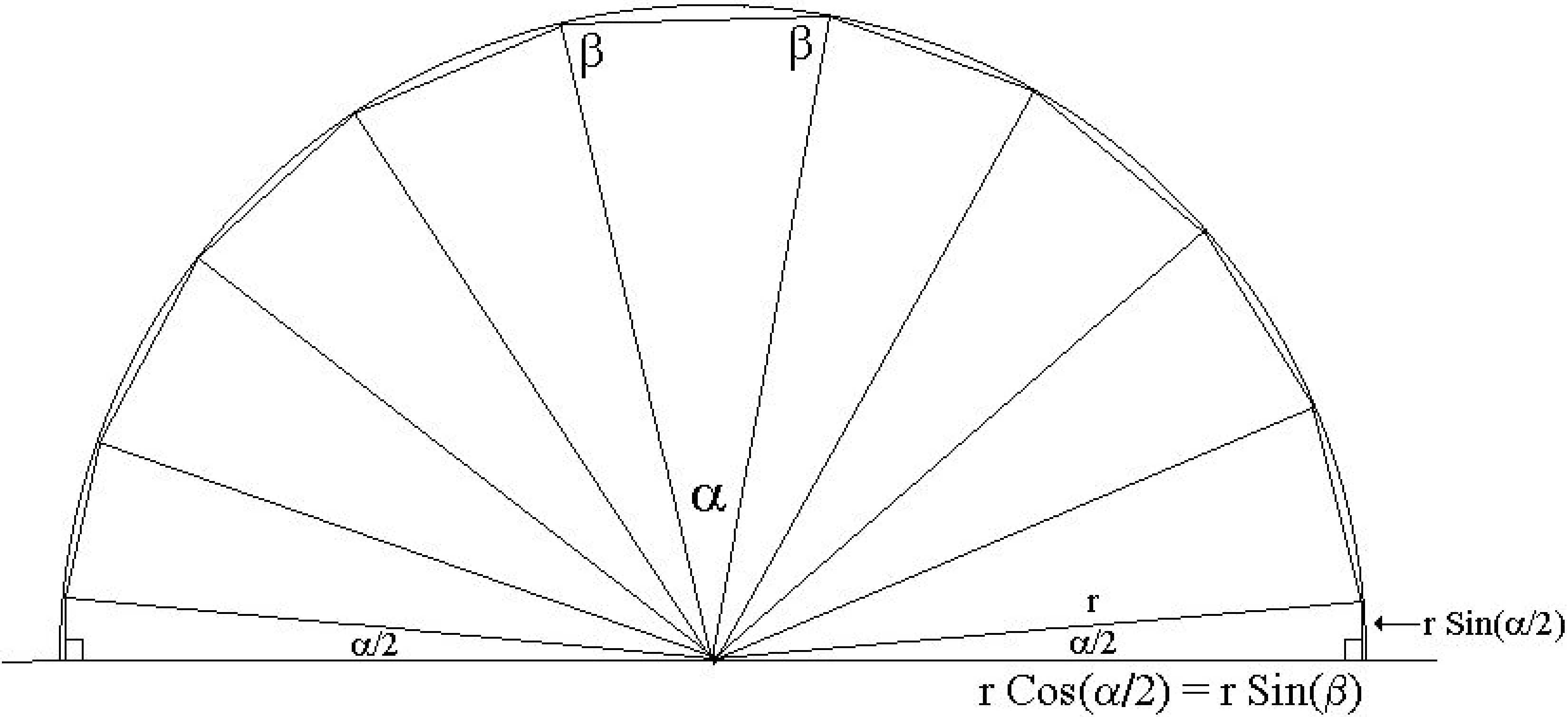

Figure 2

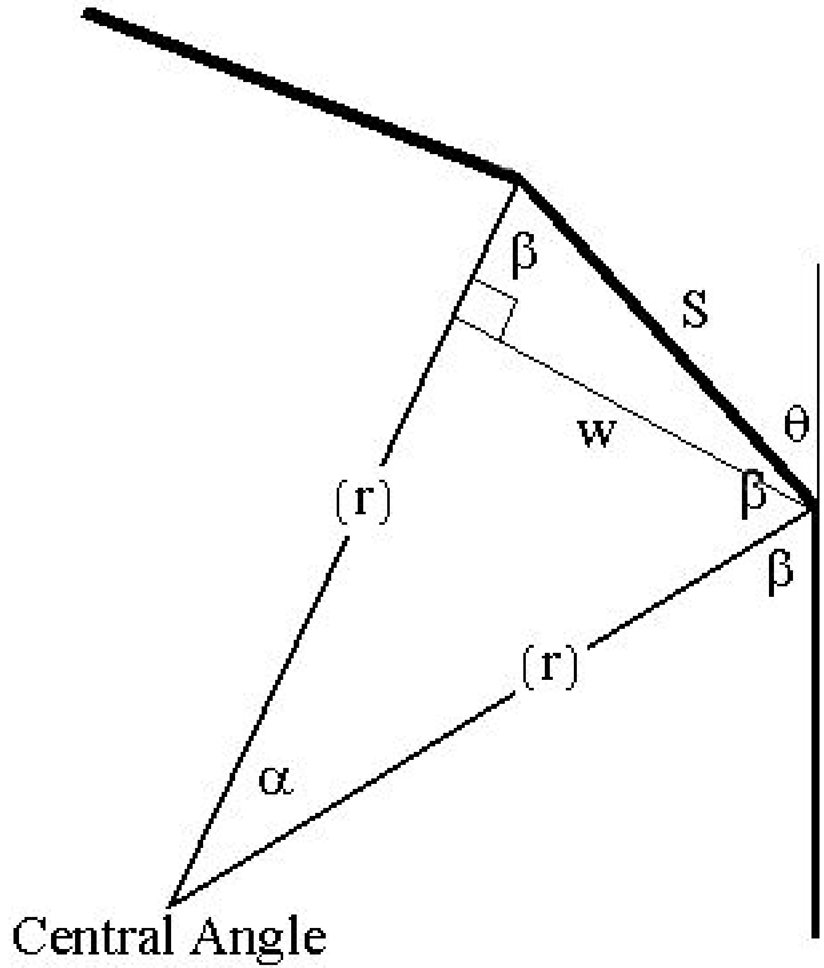

Figure 3