8.12 Curved Horns

Only a few musical wind instruments have straight horns; for very many of them it is necessary to bend the horn in order to bring its length within a reasonable compass for playing and transport. This bending is significant in instruments such as the bassoon and saxophone, and extreme in the case of brass instruments. It is therefore important to know what effect, if any, it has on the acoustic properties of the horn.

To an obvious first approximation we should expect to measure the effective length of the horn along a curved axis passing through the centroid of its cross-section at every point. As expected, this is a good approximation for cases in which the bend radius R is large compared with the tube radius r, as defined in Fig. 8.20, but for more extreme bends a careful analysis is required. The problem has been considered by Nederveen (1969) and by Keefe and Benade (1983), both of whom come to similar conclusions. If we define a parameter $B = r/R$ to measure the severity of the bend, then the analysis of Keefe and Benade shows that the wave velocity in the duct is increased above the normal sound velocity c by an amount δv and the characteristic impedance of the duct is decreased from its normal value reasonable compass for playing and transport. This bending is significant in instruments such as the bassoon and saxophone, and extreme in the case of brass instruments. It is therefore important to know what effect, if any, it has on the acoustic properties of the horn.

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$$\frac{\delta v}{c} = -\frac{\delta Z}{Z_0} = \left(\frac{2I}{\pi B}\right)^{1/2} - 1, \tag{8.73}$$

where

$$I = \int_0^{\pi/2} \cos\theta \ln\left(\frac{1 + B\cos\theta}{1 - B\cos\theta}\right) d\theta. \tag{8.74}$$

Figure 8.20 shows this relation in graphical form for values of the bend

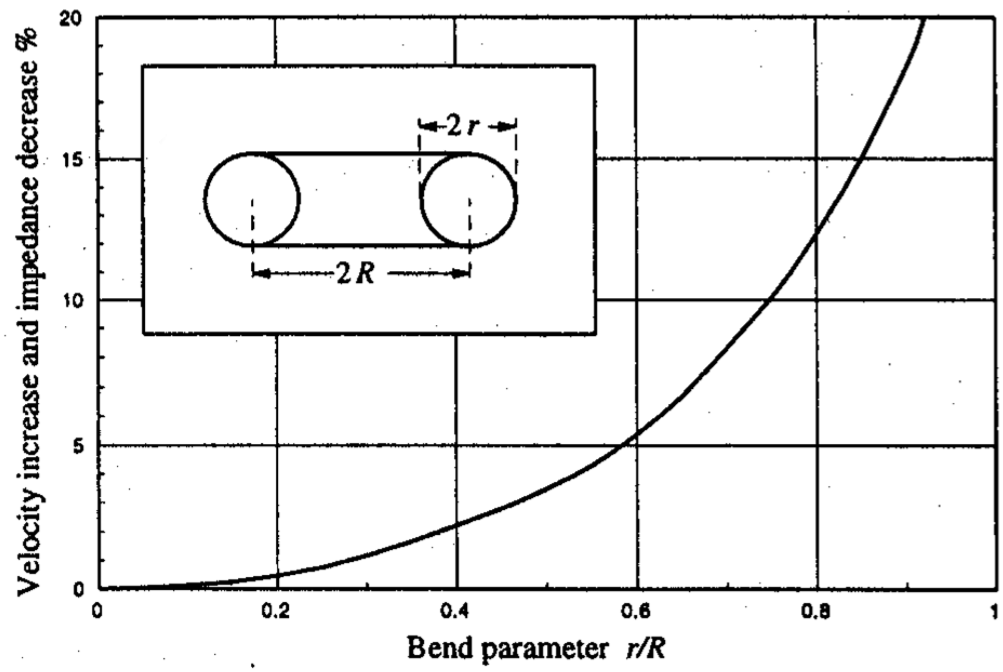

FIGURE 8.20. Increase δv in wave velocity and decrease $-\delta Z$ in duct impedance in a curved circular pipe, as a function of curvature parameter $B = r/R$, calculated from the theories of Nederveen (1969) and of Keefe and Benade (1983). Experimental values are significantly less than those calculated and are not the same for the two quantities.

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Figure 8.20 shows this relation in graphical form for values of the bend parameter between zero, a straight tube, and almost 1, which is the geometrical limit. The practical limit for bent tubing in a brass instrument is about $R \approx 0.8$.

The analysis leading to these equations, or the similar equations of Nederveen, is not rigorous, so that it is clearly necessary to check the conclusions against experiment. This was done by Keefe and Benade (1983) for tubing with about the maximum curvature found in brass instruments ($B = 0.728$). For this tubing the theory predicts $\delta v/c = -\delta Z/Z_0 = 8.9\%$. The measured values were significantly smaller than this, however, with $\delta v/c \approx 4.7\%$ and $\delta Z/Z_0 \approx -6.3\%$. The discrepancy between these two values, and between both of them and the theory, is puzzling. Keefe and Benade ascribe it to the neglect of viscosity in the calculation, but it is not clear that there is any shear except at the walls, so it may be some other aspect of the approximations made in the derivation that is to blame. Despite this, the theory does give at least a semiquantitative account of the effect of bending.

From this discussion it is clear that bending a tube shortens its acoustic length and lowers its impedance, and that the corrections may be large enough to require consideration in applications such as design of the valve tubing in trumpets and the larger bends in tubas and saxophones. As pointed out by Nederveen, insertion of a curved section into a cylindrical bore creates least acoustic missmatch if the cross-section of the curved part is reduced slightly from that of the bore, to match the impedances. In the case of a conical bore, the length of the curved section should also be increased slightly, with corresponding decrease in cone angle, to allow for the increased phase velocity.