Smith then analyzes the case wherein the horn throat is an annular ring at the air chamber as shown in Figure 8. Again the mode shapes inside the air chamber are found to be complex functions of Bessel functions. It is further found that the first null can be suppressed by properly choosing the radius of the horn throat annulus. To suppress the *jth* node it is necessary to choose r_1 such that $k_j r_1$ is a root of J_0. The parameter k_j has already been chosen such that $k_j a$ is a root of J_1.

The radius of each of m annuli is chosen to suppress each of the first m modes that exist in the air chamber by finding the first m roots to the J_0 function. Table 1 lists the first five roots of the J_0 and J_1 functions.

Table 1

Roots of the Functions $J_0(m)$ & $J_1(m)$

m	$J_0(m)$	$J_1(m)$	k_j
1	2.41	3.83	$3.83/a$
2	5.52	7.02	$7.02/a$
3	8.65	10.17	$10.17/a$
4	11.79	13.32	$13.3/a$
5	14.93	16.47	$16.47/a$

The procedure, then, is to choose a number m corresponding to the number of concentric rings desired. If this number is five then the *mth* root of J_1 is 16.470. The radius of each of the rings is obtained by dividing each of the first five roots of J_0 by this fifth root of J_1 and multiplying by the radius of the air chamber. Note: If the air chamber is not flat, i.e. spherical, then the radius of the chamber is the distance from the center to the outside edge measured along the curve defining the chamber, i.e. arc length.

Bessel Function Roots (Zeros)

m	JØ(m)	J1(m)
1	2.4048255577	3.8317059702
2	5.5200781103	7.0155866698
3	8.6537279129	10.1734681351
4	11.7915344391	13.3236919363
5	14.9309177086	16.4706300509

Note: Use of these values, instead of those provided in the paper, will improve the accuracy of the calculations, particularly those related to determination of slit area.
WHG (13-Oct-2ØØ8)

For the five ring case under consideration

$$a_1 = \frac{2.405a}{16.47} = 0.146a$$

$$a_2 = \frac{5.520a}{16.47} = 0.335a$$

$$a_3 = 0.525a$$

$$a_4 = 0.716a$$

$$a_5 = 0.907a$$

Up to this point the ratio of horn throat area to the diaphragm area has been arbitrary. Indeed, if additional concentric annuli are added their total area is still the choice of the designer. However, the area allotted to each additional annulus now becomes defined by the solution to the problem. To determine the width of each of these annular slots we must solve m^{-1} simultaneous equations and one simple equation to find the areas of each of the slots. Smith's equation 25 defines the simultaneous equations and for the case of the five rings they are written as:

$$
\begin{array}{ccccc}
S_1 J_0(k_1 a_1) & \cdot & \cdot & \cdot & S_5 J_0(k_1 a_5) = 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
S_1 J_0(k_4 a_1) & \cdot & \cdot & \cdot & S_5 J_0(k_4 a_5) = 0
\end{array}
\tag{3}
$$

The fifth required equation is obtained when the designer chooses the total area of the rings S_s

$$S_1 + S_2 + S_3 + S_4 + S_5 = S_s \tag{4}$$

As before the values of k_i are defined by the roots of J_1 and are listed in the last column of Table 1. The five unknowns in the four equation matrix may be reduced to four by dividing each of the S_j values by S_1 to produce the four unknowns $s_j = S_j/S_1$

The matrix then becomes:

$$
\begin{aligned}
0.629\, s_2 + 0.217\, s_3 - 0.161\, s_4 - 0.377\, s_5 + 0.923 &= 0 \\
0.029\, s_2 - 0.398\, s_3 - 0.170\, s_4 + 0.236\, s_5 + 0.754 &= 0 \\
-\,0.366\, s_2 - 0.062\, s_3 + 0.269\, s_4 - 0.143\, s_5 + 0.520 &= 0 \\
-\,0.328\, s_2 + 0.300\, s_3 - 0.200\, s_4 + 0.066\, s_5 + 0.256 &= 0
\end{aligned}
\tag{3a}
$$

Solving for s_j gives

$$s_2 = 2.142$$

$$s_3 = 3.463$$

$$s_4 = 4.917$$

$$s_5 = 5.929$$

If $S_s = \frac{S_D}{10}$ (5)

$$= \frac{\pi a^2}{10}$$

then

$$S_1\ (1 + 2.142 + 3.463 + 4.917 + 5.929) = \frac{\pi a^2}{10}$$

$$S_1 = 0.018a^2$$

$$S_2 = 0.0386a^2$$

$$S_3 = 0.0623a^2$$

$$S_4 = 0.0885a^2$$

$$S_5 = 0.1067a^2$$

If Δa_j is taken as 1/2 the width of the *jth* slot (Figure 8) then it can be shown that:

$$W_j = 2\Delta a_j = s_j/2\pi a_j \tag{6}$$

giving $W_1 = 0.0196a$

$W_2 = 0.0183a$

$W_3 = 0.0189a$

$W_4 = 0.0197a$

$W_5 = 0.0187a$

This completes the review of Smith's article. The transitions between these annular rings and the throat of the horn are left to the designer. The main requirement is for a smooth transition. An exponential area function in this transition is exotic, but will frequently be found to be too short to justify its economic impact.

EXPERIMENTAL DESIGN

JBL has designed and constructed a new phasing plug for the model 2440 and 375 compression drivers. These have a 0.1 metre voice coil diameter. Figure 9 is a photograph of this four ring plug. Since the velocity of sound is sensitive to the spacing between the plug and the diaphragm, the suppression of a given mode is also sensitive to this spacing.

Figure 10 shows the frequency response curve of a typical 2440 compression driver on a 0.0254 metre diameter terminated tube using the previous phasing plug design as a solid curve. Predicted chamber resonances are at 3,520, 6,740, 10,360 and 14,460 Hz for this transducer. The existing phasing plug does a good job of suppressing the first resonance, but the 6,740 Hz resonance shows through at about 7,600 Hz. The third resonance, predicted for 10,360 Hz appears to occur at 9,600 Hz and is almost coincident with the voice coil decoupling frequency just above 10 kHz. The 14.5 kHz resonance is not obvious. The null appearing at 17 kHz is a cross mode of the loading tube and could fill an entire paper by itself.

Figure 10 also shows the response of the same diaphragm mounted on the new Smith plug as a dashed curve. The normal spacing between the diaphragm and the plug has been slightly increased to illustrate, for this report, the existance of the predicted resonances. Chamber resonances are subtly seen at 4,100, 7,300 and 9,500 Hz.

Mechanical resonance of the diaphragm is visible at 8,400 Hz and the decoupling frequency is on schedule at 10 kHz. Considerable hash is seen around 14 kHz and the predicted dip may or may not be there. The tube cross mode is still at 17 kHz.

A final dotted response curve is seen in Figure 10 using an experimental voice coil and diaphragm which will be the subject of another paper in this session. The three diaphragms of Figure 10 are driven at different input voltages.

Since the Smith design is characterized by a finite spacing between the edge of the chamber and the largest ring, the magnetic field strength is enhanced by as much as 15 - 20% over the case wherein the pole piece is brought to an edge to accommodate the phasing plug.

CONCLUSION

Recent inovations in phasing plug design have added spice to the life of the transducer designer. It still remains to be shown, however, that any one design is superior to any others. A design optimization procedure for the concentric ring type has been reviewed. Such procedures for optimization of the salt shaker and the radial slot are eagerly awaited.