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A NOVEL METHOD FOR MEASURING ACOUSTIC RADIATION FIELDS OF SUBWOOFERS BASED ON NON-FREE-FIELD ASPHERIC MEASUREMENTS

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Due to the limitation of the cutoff frequency of the anechoic chamber, it is difficult to measure the low frequency sound field by using the classical method. This paper presents a novel method for measuring the acoustic radiation field of subwoofers. Firstly, according to the sound field separation principle, a general model of spherical harmonics based on the spherical coordinate system is proposed. After that, the coefficients in the spherical harmonic expansion are worked out by the least squares method. And then we use non-free-field aspheric measurements to rebuild the radiated sound fields of the sound source in the free field. Finally, in order to testify the proposed method, simulations of dipoles and multi-poles cases are carried out.

1. Introduction

Near-field acoustic holography (NAH)¹ uses sound pressure or sound intensity data in the near-field of sound source to measure the surface vibration and sound field distribution of the source. NAH has been researched for several decades. By using the different sound field reconstruction methods, NAH mainly includes methods based on Spatial transformation of sound fields (STSF)², based on Boundary-Element Method (BEM)³ and based on Helmholtz equation least-squares (HELS)⁴ method. In 1997, Wu *et al.*⁵ proposed HELS method to analyze the external radiation of vibrating object. After that, HELS has been widely used in various acoustic measurement techniques.

In 1980, Sound Field Separation (SFS) of spherical coordinate system⁶ was first used by Weinreich⁷ for researching radiation field of sound source. By separating the inner and outer radiation term of the spherical harmonic function expansion, Weinreich rebuilt the sound field of the source in the near-field. This method can be implemented in non-free field space by measuring distributed sound pressure at double spherical surface sampling points. After the coefficients of spherical harmonic expansion are estimated using the least squares (LS) method from sound pressure or sound intensity measurement data, the radiation field of sound source inside the measurement surface can be estimated. This method has been used in recent years to restore radiation acoustic field of subwoofers by measurement in non-free field, obtained a good measurement results⁸⁻¹⁰.

In this paper, we derive a general spherical harmonic expansion expression, which shows that measurement need not be restricted in the surface of a spherical. If we get enough measurement in

non-free field to restore the spherical harmonic expansion expression, we can rebuild the radiated sound field of sound source in free field by using sound field separation method.

This paper is organized as follows: The second part of this paper describes the theoretical basis of the proposed method, the third part deduces the estimation of coefficient in spherical harmonic expansion by aspheric measurement, the fourth part shows the simulation results to evaluate the performance of the proposed method, and the final part is the conclusion.

2. Sound Field Separation of spherical coordinate system

2.1 Mathematical model of Sound field in spherical coordinates

The solution of wave equation in spherical coordinates¹ is

$$p(r, \theta, \phi, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn} h_n^{(1)}(kr) + D_{mn} h_n^{(2)}(kr)) Y_n^m(\theta, \phi), \quad (1)$$

where $h_n^{(1)}$ and $h_n^{(2)}$ are spherical Hankel functions, which represent an outgoing wave and an incoming wave separately. The relationships between Hankel functions and Bessel functions are

$$h_n^{(1)}(x) \equiv j_n(x) + iy_n(x) \quad (2)$$

and

$$h_n^{(2)}(x) \equiv j_n(x) - iy_n(x), \quad (3)$$

where $j_n(x)$ and $y_n(x)$ are spherical Bessel functions.

Equation (1) is the spherical harmonic expansion, where Y_n^m called spherical harmonic defined by

$$Y_n^m(\theta, \phi) \equiv \sqrt{\frac{(2n+1)}{4\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos \theta) e^{im\phi}, \quad (4)$$

where P_n^m is Legendre functions.

2.2 Sound field separation in spherical coordinates

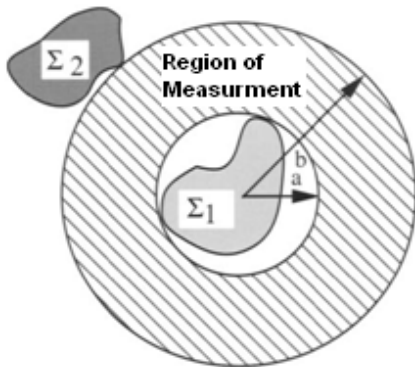


Figure 1. Mixture sources.

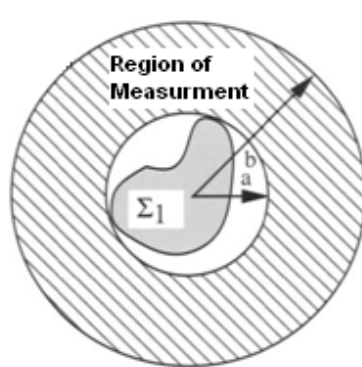


Figure 2. Inner sources.

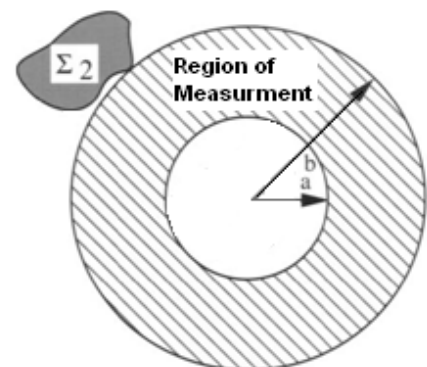


Figure 3. Outer sources.

Figures 1-3 show the sound fields generated by mixture sources, inside sources and outside sources respectively. The sphere of radius a is the smallest sphere tangent to the outermost extremity of the source and does not cut through the source at any point. The sphere of radius b is the biggest sphere tangent to the innermost extremity of the source and does not cut through the source at

any point. Therefore the whole space in spherical coordinates is divided into 3 parts, which are the region of measurement where the radius $a < r < b$, the region of inside sources where $r < a$ and the region of outside sources where $r > b$. The target of SFS in spherical coordinates is to get the radiation of Fig. 2, by measure the mixture field in Fig. 1. That is to restore radiation sound field in free field by measure mixture sound field in non-free field.

We can get the pressure field in Figs. 1-3 from Eq. (1), which is given by

$$p_a(r, \theta, \phi, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn}^{\text{MIX}} h_n^{(1)}(kr) + D_{mn}^{\text{MIX}} h_n^{(2)}(kr)) Y_n^m(\theta, \phi), \quad (5)$$

$$p_b(r, \theta, \phi, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn}^{\text{RAD}} h_n^{(1)}(kr) + D_{mn}^{\text{RAD}} h_n^{(2)}(kr)) Y_n^m(\theta, \phi) \quad (6)$$

and

$$p_c(r, \theta, \phi, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn}^{\text{INN}} h_n^{(1)}(kr) + D_{mn}^{\text{INN}} h_n^{(2)}(kr)) Y_n^m(\theta, \phi). \quad (7)$$

And we can get the relationship between the coefficients C_{mn}^{MIX} , C_{mn}^{RAD} , C_{mn}^{INN} , D_{mn}^{MIX} , D_{mn}^{RAD} and D_{mn}^{INN} in equations above from the physical meaning of spherical harmonic expression.

Firstly, in Fig. 3, there is no source inside the spherical surface at $r = a$, that is to say, the outgoing wave is equal to the incoming wave on the surface of the measurement region. So we have

$$C_{mn}^{\text{RAD}} = D_{mn}^{\text{RAD}}. \quad (8)$$

Here, we use the physical meaning of Hankel function $h_n^{(1)}$ and $h_n^{(2)}$ which represent an outgoing wave and an incoming wave respectively.

Secondly, in Fig. 2, all the sources are contained within the measurement region and there is no source outside the surface, which means it is in the free field. So we can get

$$D_{mn}^{\text{INN}} \rightarrow 0. \quad (9)$$

Finally, in Fig. 1, where in the mixture fields, according to the linear superposition principle of the spherical harmonic basis function, we can get

$$C_{mn}^{\text{MIX}} = C_{mn}^{\text{RAD}} + C_{mn}^{\text{INN}} \quad (10)$$

and

$$D_{mn}^{\text{MIX}} = D_{mn}^{\text{RAD}} + D_{mn}^{\text{INN}}. \quad (11)$$

Derived from Eqs. (8-11), the relationship between the coefficients is

$$C_{mn}^{\text{INN}} = C_{mn}^{\text{MIX}} - C_{mn}^{\text{RAD}} = C_{mn}^{\text{MIX}} - D_{mn}^{\text{RAD}} = C_{mn}^{\text{MIX}} - D_{mn}^{\text{MIX}}. \quad (12)$$

According the equation (12), p_b can be expressed as

$$\begin{aligned} p_b(r, \theta, \phi, \omega) &= \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn}^{\text{INN}} h_n^{(1)}(kr)) Y_n^m(\theta, \phi) \\ &= \sum_{n=0}^{\infty} \sum_{m=-n}^n ((C_{mn}^{\text{MIX}} - D_{mn}^{\text{MIX}}) h_n^{(1)}(kr)) Y_n^m(\theta, \phi). \end{aligned} \quad (13)$$

If we get coefficients C_{mn}^{MIX} , D_{mn}^{MIX} in mixture fields in Fig. 1, we can obtain the radiation sound fields by the inner sources, which is no relationship with outer sources.

In a real measurement, if the internal sound source is a low frequency speaker in a room, while the walls of the room for the reflected wave radiation incident as external sound sources, we can use this non-anechoic chamber measurement method to get radiated sound field of the sub-woofer in free field.

3. Aspheric measurement based on the least squares method

In order to get the spherical harmonic coefficients C_{mn}^{MIX} and D_{mn}^{MIX} , Manuel⁸ used sound pressure and sound intensity measurement in single uniform spherical, Garcia¹⁰ used sound pressure measurement in double uniform spherical respectively. However, spherical measurements are not necessary.

Assuming that we implement M measurements at M places in the region of measurement to get the sound pressure at each point, for the N order spherical harmonic expression, the number of unknown parameters is $2(N+1)^2$. Obviously $M > 2(N+1)^2$ is necessary to estimate all unknown parameters. We can rewrite Eq. (1) as

$$\begin{aligned} p_i(r, \theta, \phi, \omega) &= \sum_{n=0}^{\infty} \sum_{m=-n}^n (C_{mn} h_n^{(1)}(kr) + D_{mn} h_n^{(2)}(kr)) Y_n^m(\theta, \phi) \\ &= [c_{00} h_0^{(1)}(kr_i) + d_{00} h_0^{(2)}(kr_i)] Y_0^0(\theta_i, \phi_i) \\ &\quad + [c_{-11} h_1^{(1)}(kr_i) + d_{-11} h_1^{(2)}(kr_i)] Y_1^{-1}(\theta_i, \phi_i) \\ &\quad + [c_{01} h_1^{(1)}(kr_i) + d_{01} h_1^{(2)}(kr_i)] Y_1^0(\theta_i, \phi_i) \\ &\quad + [c_{11} h_1^{(1)}(kr_i) + d_{11} h_1^{(2)}(kr_i)] Y_1^1(\theta_i, \phi_i) + \dots \\ &= Y_i \cdot CD \quad i = [1, 2, \dots, M]. \end{aligned} \quad (14)$$

Or, we can rewrite it in vector form

$$P = Y \times CD, \quad (15)$$

where $P = [p_1 \ p_2 \ \dots \ p_M]'$ is the sound pressure vector measured at each sample point in space, $Y = [Y_1 \ Y_2 \ \dots \ Y_M]'$ is spherical harmonic function vector, determined by the angular position (θ_i, ϕ_i) of each sample points, and $CD = [c_{00} \ d_{00} \ c_{-11} \ d_{-11} \ \dots]'$ is consisted by coefficients C_{mn}^{MIX} and D_{mn}^{MIX} . So there are

$$Y_i = [h_0^{(1)}(kr_i) Y_0^0(\theta_i, \phi_i) \ h_0^{(2)}(kr_i) Y_0^0(\theta_i, \phi_i) \ h_1^{(1)}(kr_i) Y_1^{-1}(\theta_i, \phi_i) \ h_1^{(2)}(kr_i) Y_1^{-1}(\theta_i, \phi_i) \ \dots] \quad (16)$$

and

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} = \begin{bmatrix} h_0^{(1)}(kr_1) Y_0^0(\theta_1, \phi_1) & h_0^{(2)}(kr_1) Y_0^0(\theta_1, \phi_1) & \dots & h_{(N+1)^2}^{(2)}(kr_1) Y_0^0(\theta_1, \phi_1) \\ h_0^{(1)}(kr_2) Y_0^0(\theta_2, \phi_2) & h_0^{(2)}(kr_2) Y_0^0(\theta_2, \phi_2) & \dots & h_{(N+1)^2}^{(2)}(kr_2) Y_0^0(\theta_2, \phi_2) \\ \vdots & \vdots & \ddots & \vdots \\ h_0^{(1)}(kr_M) Y_0^0(\theta_M, \phi_M) & h_0^{(2)}(kr_M) Y_0^0(\theta_M, \phi_M) & \dots & h_{(N+1)^2}^{(2)}(kr_M) Y_0^0(\theta_M, \phi_M) \end{bmatrix}_{M \times 2(N+1)^2} \times \begin{bmatrix} c_{00} \\ d_{00} \\ c_{-11} \\ d_{-11} \\ \vdots \end{bmatrix}_{2(N+1)^2 \times 1}. \quad (17)$$

In order to estimate \widehat{CD} of vector CD , we use LS method

$$\widehat{CD} = \min_{\widehat{CD}} \sum_{i=0}^M \left(p_i - Y_i \cdot \widehat{CD} \right)^2. \quad (18)$$

Obviously, $\text{Rank}(Y) \geq 2(N+1)^2$ is needed to make an estimation, and there is no other restrictions on positions of sample point $(r_i, \theta_i, \phi_i), r=1, 2, \dots, M$. It means that when requirements of $a < r_i < b$ and $\text{Rank}(Y) \geq 2(N+1)^2$ are satisfied it's not necessary to choose sample points on the surface of a sphere.

We define the measurement array, which is only determined by sample positions $(r_i, \theta_i, \phi_i), r=1, 2, \dots, M$.

$$Y = \begin{bmatrix} h_0^{(1)}(kr_1)Y_0^0(\theta_1, \phi_1) & h_0^{(2)}(kr_1)Y_0^0(\theta_1, \phi_1) & h_1^{(1)}(kr_1)Y_1^{-1}(\theta_1, \phi_1) & h_1^{(2)}(kr_1)Y_1^{-1}(\theta_1, \phi_1) & \dots \\ h_0^{(1)}(kr_2)Y_0^0(\theta_2, \phi_2) & h_0^{(2)}(kr_2)Y_0^0(\theta_2, \phi_2) & h_1^{(1)}(kr_2)Y_1^{-1}(\theta_2, \phi_2) & h_1^{(2)}(kr_2)Y_1^{-1}(\theta_2, \phi_2) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_0^{(1)}(kr_M)Y_0^0(\theta_M, \phi_M) & h_0^{(2)}(kr_M)Y_0^0(\theta_M, \phi_M) & h_1^{(1)}(kr_M)Y_1^{-1}(\theta_M, \phi_M) & h_1^{(2)}(kr_M)Y_1^{-1}(\theta_M, \phi_M) & \dots \end{bmatrix}_{M \times 2(N+1)^2}. \quad (19)$$

Now we can simplify the measurement program and improve accuracy of the measurement results by optimizing the position of measuring points. However, optimization of the measurement point position relates to the volume of source, frequency of interest, function order of spherical harmonic, prior knowledge of inner and outer sources and many other factors. That will be part of the follow-up research.

4. Simulation and discussion

4.1 Radiation sound field of Dipoles

In order to evaluate the performance of the proposed method, we simulate sound field measurement of dipoles in the free field. We use a microphone array of hexahedral structure, as showed in Fig. 4. At each sample point, we add non-coherent white noises into sound signal, and set the SNR as 30dB. We use Eq. (18) to estimation the spherical harmonic coefficients of mixture sound fields, and use Eq. (13) to restore the sound fields of dipoles in free field. The results show in Figs. 4-7.

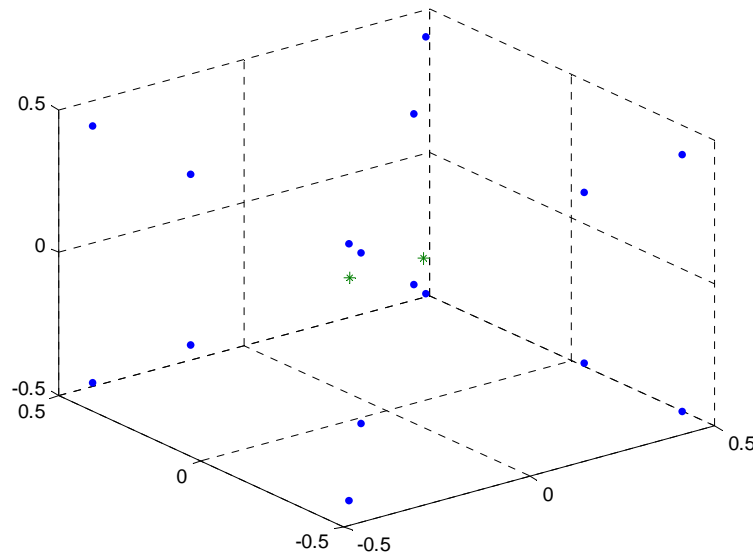


Figure 4. The positions of dipoles (*) and sample points (•)

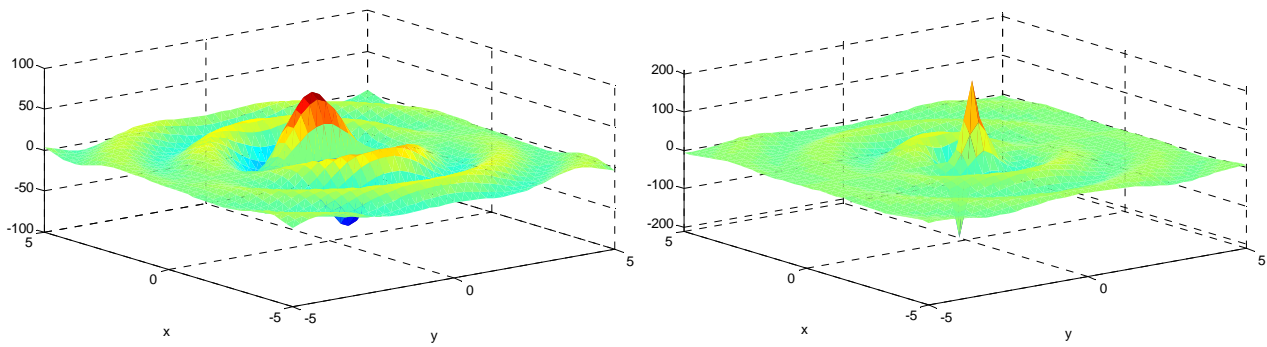


Figure 5.The radiation sound field of dipoles (Left is real part, Right is imaginary part)

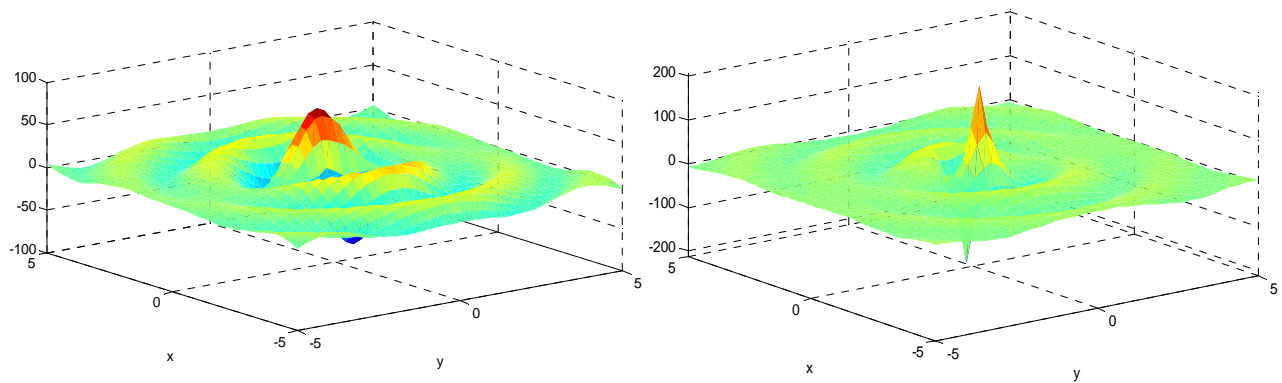


Figure 6.The estimated radiation sound field of dipoles (Left is real part, Right is imaginary part)

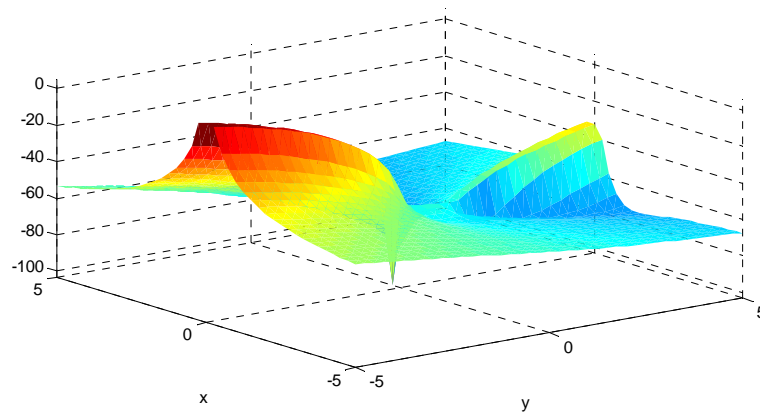


Figure 7.The error of radiation sound field estimation (dB)

Simulation results show that the estimation error is below -20dB on more than 90% positions. So the measurement method described above is effective.

4.2 Radiation sound field of random multi-poles

Without loss of generality, we use 14 random multi-poles distributed inside the sphere of radius $r = 0.4\text{m}$ and 8 random interference poles outside the sphere of radius $r = 1.5\text{m}$. The amplitudes of all the 22 poles are set randomly. The SNR of the measurement is 30dB. We choose two kinds of sample positions in space showed in Fig. 8 (a) (b) respectively.

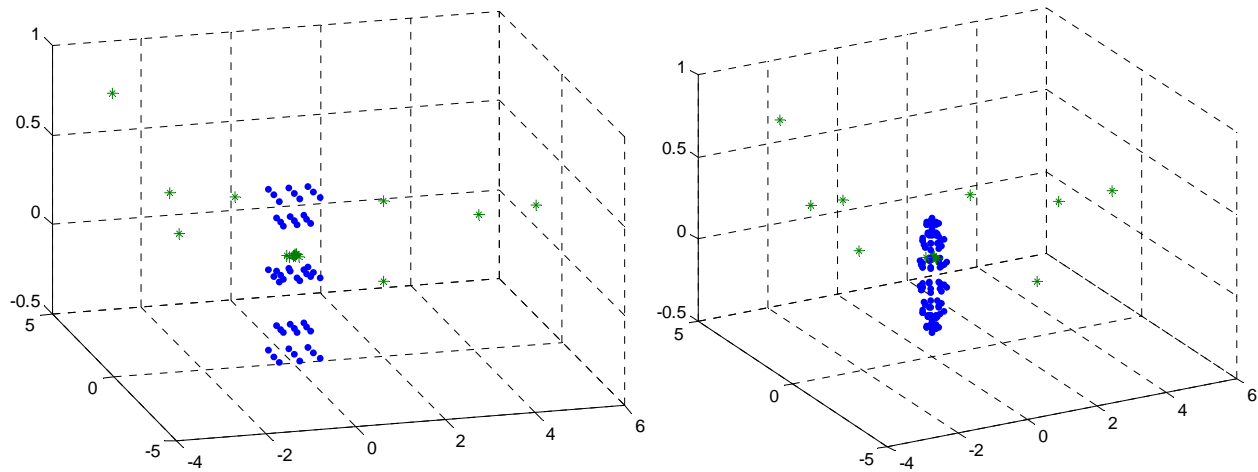


Figure 8. The positions of dipoles (*) and sample points (•)

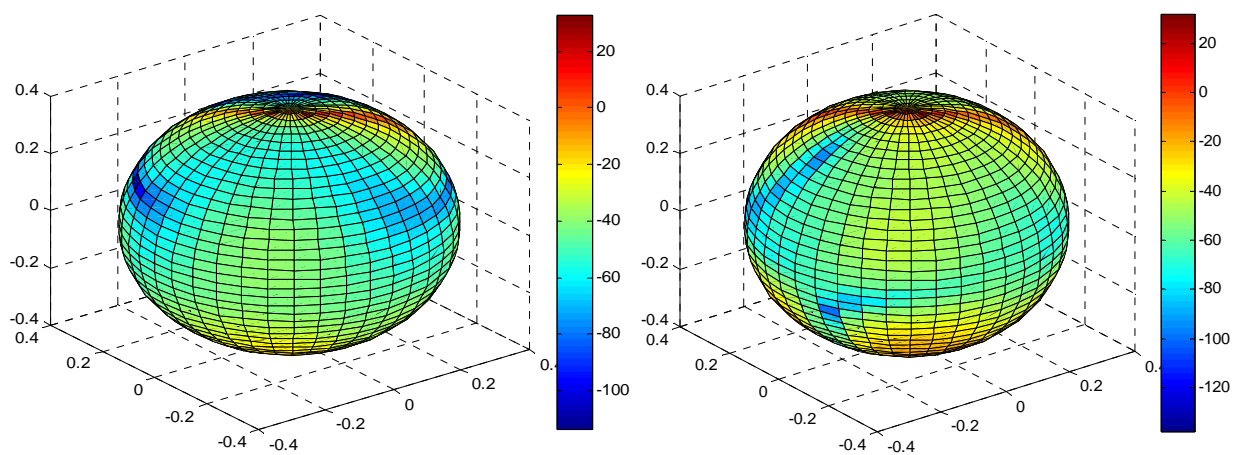


Figure 9. The error of radiation sound field estimation (dB)

The simulation results in Fig. 9 show that, the different sample positions (position of microphones in array) will affect the accuracy of the measurement. If the measurement points are appropriately selected, it is possible to reduce the complexity of the microphones array by reducing the number of measurement points. These results also show that the measurement method described in Section 3 is effective.

The simulations in Section 4 demonstrate the effectiveness of the proposed method, which use measurements in non-free field to estimate acoustic radiation in free field.

5. Conclusion

This paper proposed a method for restoring acoustic radiation field in free field by measuring in non-free field. LS method is used to estimate the coefficients of spherical harmonic function for mixture field. And then SFS method is used to restore the spherical harmonic function for the radiation field, in which only inner sources exist. Simulation results of the dipoles and multi-poles cases shows that the performance of proposed method is good. We also find that the placement of sample points in space may affect the accuracy of the measurement. Furthermore if the measurement point is appropriately selected, it is possible to reduce the complexity of the actual measurement system by reducing the number of measurement points, and improves the measurement accuracy, which is significance in practical application. Further research will focus on the optimization of sample positions for a determined sound source.

Acknowledgement

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