INPUT and OUTPUT IMPEDANCE

I. PURPOSE

To familiarize with the concept of input and output impedance of a circuit. Evaluate the benefits of using of transistors to design "stiffer" electronic circuits.

II. THEORETICAL CONSIDERATIONS

II.A The concept of circuit loading

Fig 1 shows a voltage source, whose nominal output is V_{in}. When connecting this source to an external circuit, a user would expect the voltage across the terminals a and b to be V_{in}. But, because any real voltage source has an intrinsic internal resistance (r_{source}), by attaching an external load resistance R_L , the voltage across the terminals a and b will b eless than V_{in}. The difference between V_{ab} and V_{in} is more dramatic when R_L is less than or even comparable to the internal resistance r_{source}; this is illustrated through expression (1) and the corresponding graph in Fig 1.

$$V_{out} = V_{in} \frac{1}{1+(r_{source}/R_{Load})} \tag{1}$$

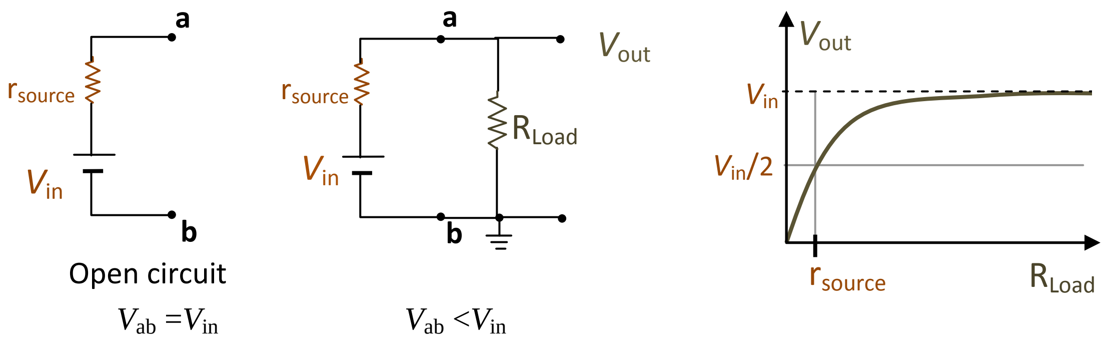


Fig. 1 "Circuit loading" refers to the undesirable reduction of the open-circuit voltage V_{ab} by the load.

Solution to avoid "loading" the circuit: Use $R_{Load} >> r_{source}$

(Rule of thumb: To use $R_{Load} > 10\ r_{source}$)

Connecting circuits one after another

In electronic circuits, stages are connected one after another.

i) Sometimes it is OK to load the circuit, as far as we know how much the loaded is, and particularly if Z_{in} is going to be constant.

ii) Of course, it is always better to have a "stiff source" ($Z_{out} << Z_{in}$), so that signal levels do not change when a load is connected.

iii) However, there are situations in which it is rather required to have $Z_{out} = Z_{in}$. That is the case in radiofrequency circuits to avoid signal reflections.

So, be aware to respond accordingly depending on the situation.

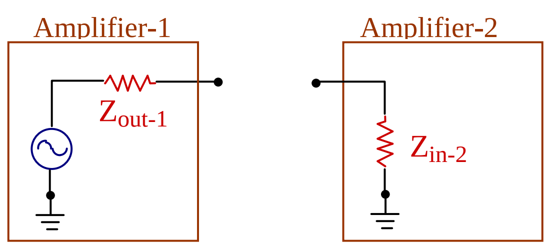


Fig. 2 Amplifiers are typically characterized by their effective output and input impedances. This is particularly important for analysis when cascading them one after another.

II.B The emitter follower circuit

We will use the effect of using a transistor to decrease the effective value of Z_{out} and/or increase the value of Z_{in}. The emitter-follower circuit (see Fig.3) will be used as a test example throughout this lab session. It is called an *emitter follower* because the output terminal (the emitter) follows the input (the base) except by a diode drop voltage:

$$V_E \approx V_B - 0.6V \tag{2}$$

- The output voltage V_{out} (in this case coinciding with V_E) is a replica of the input voltage, except for the diode voltage 0.6 V to 0.7 V.
- ***V_{in}* must stay at 0.6 V or more**, otherwise the transistor will be off and the output will stay at ground.
  By connecting the emitter resistor R_E to a negative voltage supply, one can allow negative input voltages as well. Keep this in mind for your experimental section.

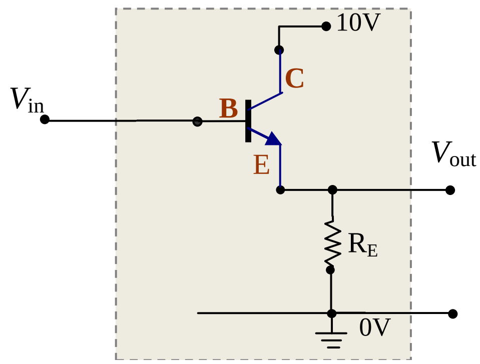
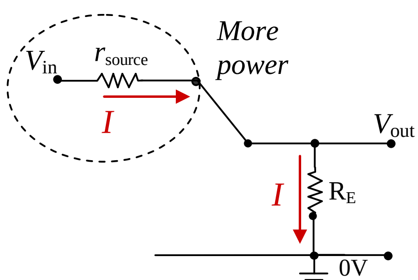


Fig. 3 Emitter follower circuit.

At first glance this circuit may appear useless, since it gives just a replica of V_{in}. until one realizes the following (see Fig. 4A),

- If a given current were needed to pass across the resistance R_E, the circuit on the right requires less power from the signal source than would be the case if the signal source were to drive R_E directly.
- That is, the current follower has a current gain, even though it does not have a voltage gain. It has a power gain. Voltage gain isn't everything ![i]



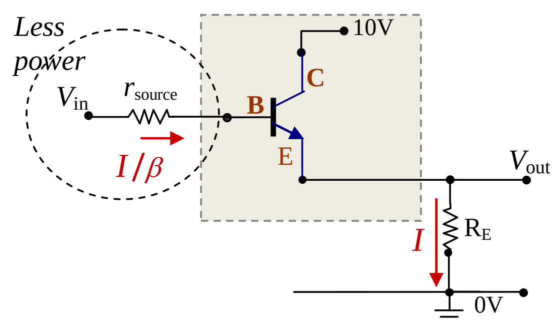
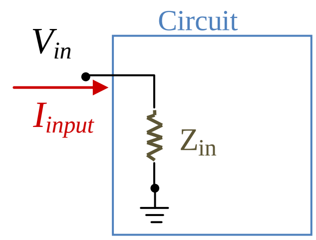


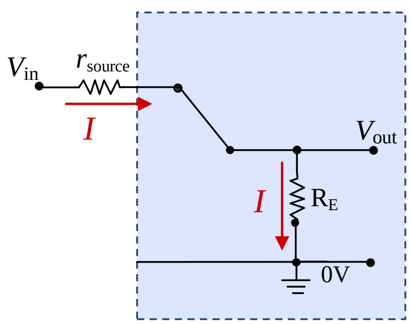
Fig. 4A For the circuit on the left, the source needs to provide a power equal to *I Vin*. For the circuit on the right, the source needs to provide a factor β less to drive the same current on the load resistor.

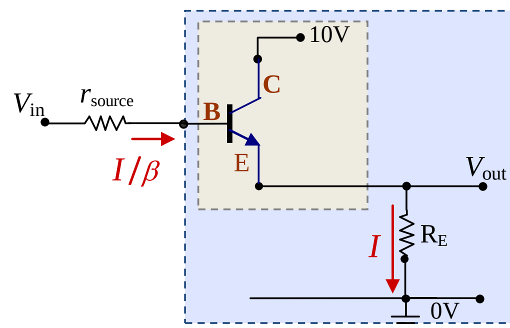
Note: In the circuit above we are considering R_E as the load resistance. In practice an additional R_L load resistance is connected (typically high). But its connection will be in parallel to R_E, in which case RE is the dominant resistance (since RL would be high).

The circuit in Fig 4A is re-draw in Fig. 4B just to highlight more clearly that the effective input impedance of circuit with the emitter follower is greater than the one without it.

Z_{in} = input voltage / (input current)







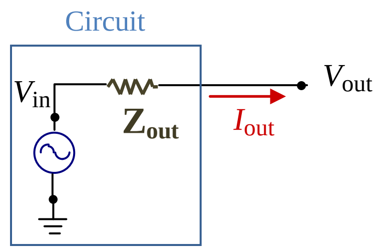
Z_{in} = input voltage / (input current)
= (V_{in} / I)

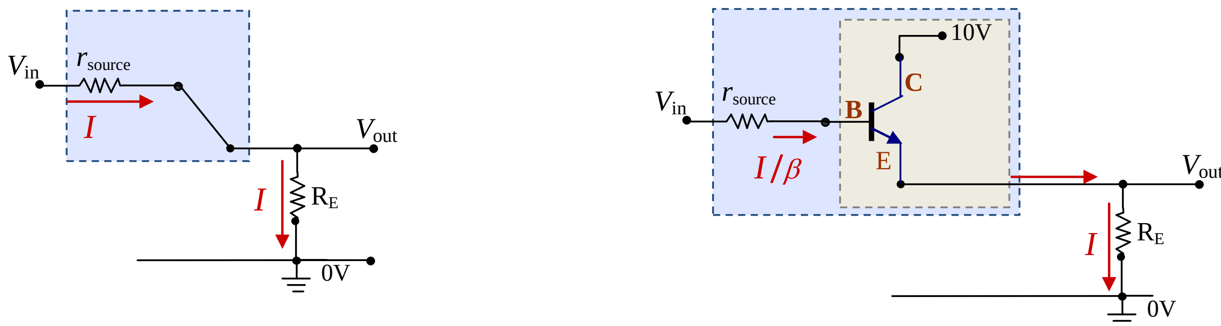
Z_{in} = input voltage / (input current)
= V_{in} / (I/β) = β (V_{in} / I)

Fig. 4B Comparison between input impedances. The emitter-follower circuit has an input impedance β times greater.

The circuit in Fig 4A is also re-draw in Fig. 4C just to highlight more clearly that the effective output impedance of circuit with the emitter follower is smaller than the one without it.

Z_{out} = [V_{in} - V_{out}] / (output current)





Z_{out} = (V_{in} - V_{out}) / (ouput current)
= ($I\, r_{source}$ / I) =
= r_{source}

Z_{out} = [V_{in} - V_{out}] / (output current)
= [(I/β) r_{source}] / I)
= r_{source} / β

Fig. 4C Comparison between the output impedances. Notice, the effective output impedance of the emitter-follower is a factor β smaller than the circuit on the left. [In the calculations we have overlooked the V_{BE} of 0.6-0.7 volt (typically one is interested in the variations of the changes of the voltages and currents in the circuit rather than the steady values.]

From here you may want to jump to the experimental Section III, which is devoted to the experimental measurement of the input and output impedance of an emitter-follower circuit. Make sure the transistor is working in the active region. After that you can come back to this section.

Between this line and Section III (Experimental section) there are more precise analytical calculations of the input and output impedance. These expression will make more useful after you have implemented your emitter follower circuit.

II.B.1 Calculation of the effective (load) input impedance Z_{in} in the emitter-follower circuit

In what follows we will use lower cases to signify small signal (incremental quantities. Typically one is interested in the variations of the changes of the voltages and currents in the circuit rather than the steady values. Also, " the distinction between the current gain h_{FE} and small-signal current gain h_{fe} isn't always made clear and the term beta (β) is used for both; that is $h_{FE} = h_{fe} = \beta$." Horowitz and Hills book.

In the circuit shown in Fig.5 below, we are looking for an expression for

$$z_{in} \equiv \text{input voltage / (input current)} = \Delta v_B / \Delta i_B$$

in terms of load impedance R_E. Let's work out expressions for Δv_B and Δi_B

- First, an expression for Δi_B is obtained from the charge conservation, $I_E = I_B + I_C$. For the case of ac-input signals this implies,

  $$\Delta i_E = \Delta i_B + \Delta i_C$$

  We know that $I_C = \beta I_B$, or $\Delta i_C = \beta \Delta i_B$; hence

  $$\Delta i_E = (\beta+1)\Delta i_B$$

  or,

  $$\Delta i_B = \Delta i_E / (\beta+1) \qquad (3)$$

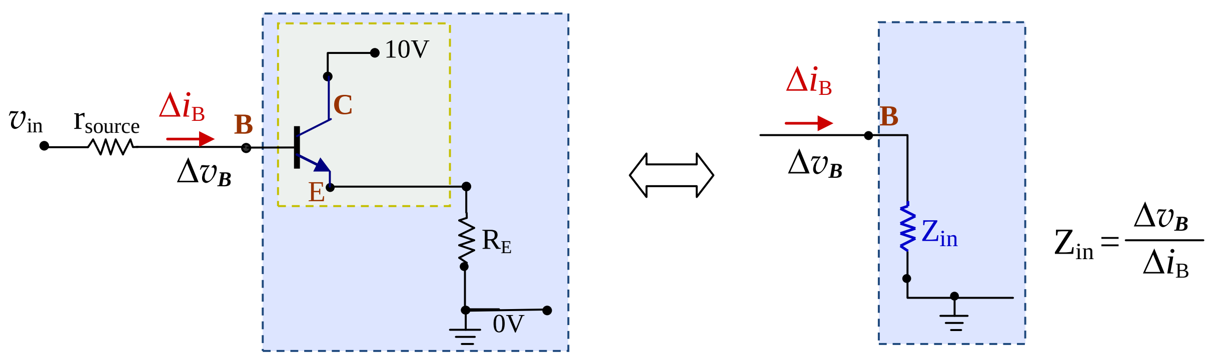


Fig.5 Evaluation of the effective (load) input impedance Z_{in} of the emitter-follower circuit.

- Next, we want an expression for Δv_B that relates to Δi_E and the load impedance R_E.

  For the case of ac-signals, expression (2) gives $\Delta v_E = \Delta v_B$

  $$\Delta v_E = R_E \, \Delta i_E$$

  $$\Delta v_B = R_E \, \Delta i_E \qquad (4)$$

Equating (3) and (4) one obtains,

$$Z_{in} \equiv \frac{\Delta v_B}{\Delta i_B} = (\beta + 1) R_E \tag{5}$$

Since β is typically of the order of 100, then Z_{in} is ~ 100 times greater than R_E. For R_E = 0.5 kΩ, Z_{in} ~ 50 kΩ

[Notice the mathematical derivation above is independent of the particular accessory to the circuit (see Fig. 5) that may be used to experimentally measure Δi_B and Δv_B. It only requires that the transistor is properly biased (i.e. working in the active region, so one can justify the use of the expression $I_C = \beta I_B$.]

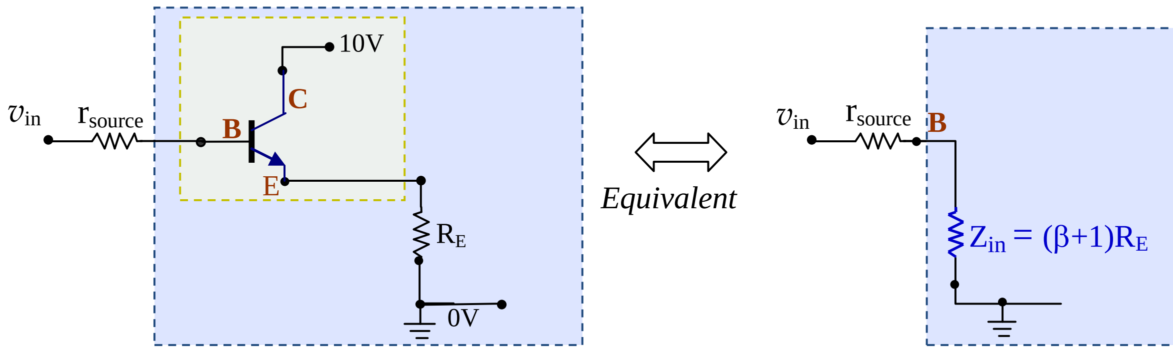


Fig.6 HIGHER (LOAD) INPUT IMPEDANCE. The presence of the transistor has the net effect to increasing the (load) impedance R_E by a factor of $\beta+1$. "*The emitter-follower has high input impedance*".

II.B.2 Calculation of the effective (source) output impedance Z_{out} in the emitter-follower circuit

We will use upper case for DC voltages (like V_{BB} and I_B) and the lower case used for AC small-signals (like v_{in}, Δi_B, and v_{out}).

In the circuit shown in Fig.7 below, we are looking for an expression for

$$Z_{out} \equiv [\, v_{in} - v_{out} \,] \,/\, (\text{output current})$$
$$= (v_{in} - \Delta v_B) / \Delta i_E$$

in terms of source impedance r_{source}.

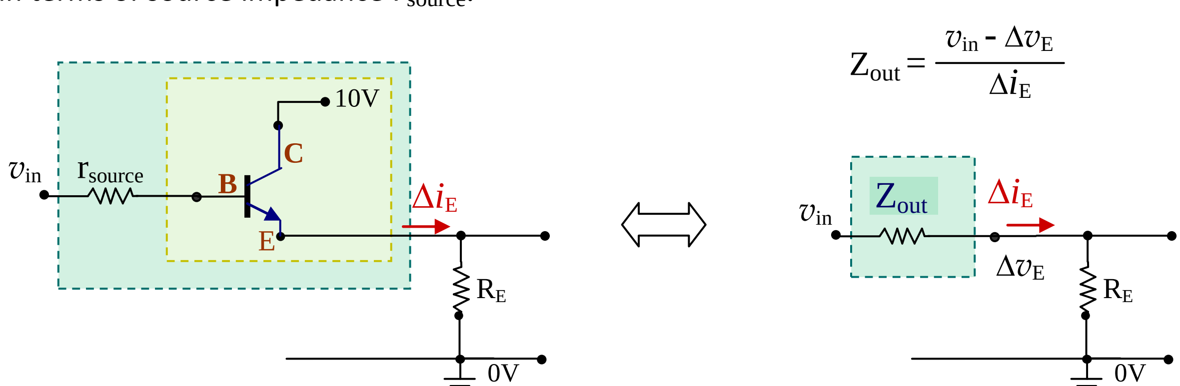


Fig.7 Evaluation of the effective (source) output impedance Z_{out} of the emitter-follower circuit.

In the circuit on the left side of Fig.7, applying expression (2) for the case of ac-voltages gives,

$$\Delta v_E = \Delta v_B \,.$$

Accordingly,

$$v_{in} = r_{source}\, \Delta i_B \quad + \quad R_E\, \Delta i_E$$

Using $\Delta i_E = (\beta + 1)\Delta i_B$

$$v_{in} = r_{source}\, \Delta i_E\, /(\beta + 1) \quad + \quad R_E\, \Delta i_E$$

$$v_{in} = [\ r_{source} / (\beta + 1) \quad + \quad R_E\ ]\, \Delta i_E \qquad (6)$$

In the circuit on the right side of Fig.7,

$$v_{in} = (Z_{out} + R_E)\, \Delta i_E \qquad (7)$$

From (6) and (7),

$$Z_{out} = r_{source} /(\beta + 1) \qquad (8)$$

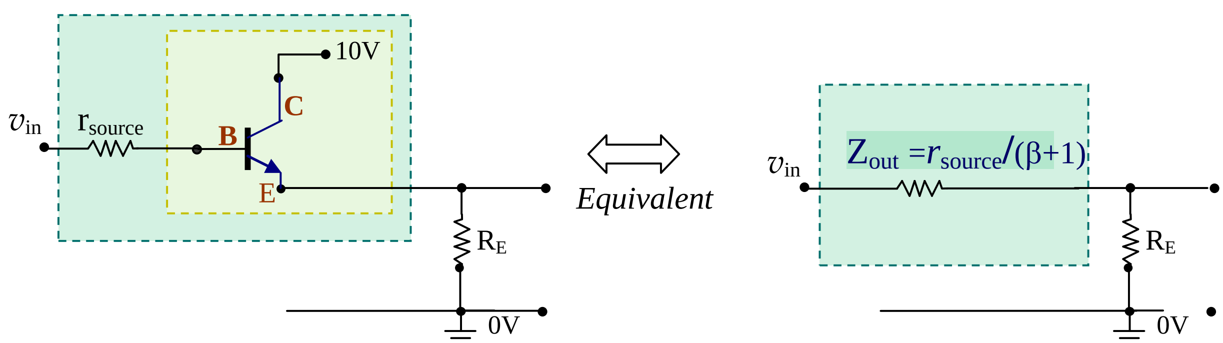


Fig.8 LOWER (SOURCE) OUTPUT IMPEDANCE. The presence of the transistor has the net effect to reducing the (source) output impedance r_{source} by a factor of $(\beta+1)$. "*The emitter-follower lowers the (source) output impedance*".

II.B.3 Alternative derivation of the output and input impedances.

We have obtained in two separate sections, II.B.1 and II.B.2, explicit expressions (5) and (8) for the corresponding values of Z_{out} and Z_{in} of a emitter-follower circuit. Both expressions can in fact be obtained from a single more compact expression that compares the input and output voltages, as shown below.

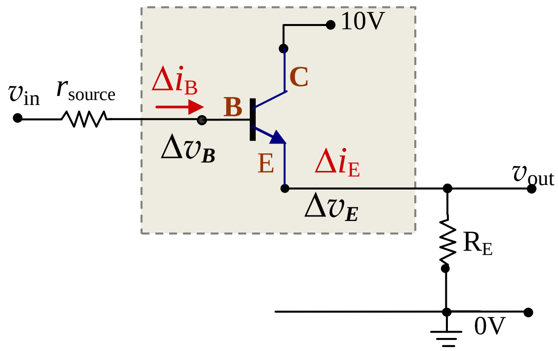


Fig.9 Emitter-follower circuit.

Notice in Fig.9,

$$v_{in} - r_{source}\,\Delta i_B - \Delta v_E = 0$$

Since $\Delta v_E = v_{out}$,

$$v_{in} - v_{out} = r_{source}\,\Delta i_B \tag{9}$$

To make R_E intervene in expression (9), we use

$$v_{out} = R_E\,\Delta i_E$$

Since $\Delta i_E = (\beta+1)\Delta i_B$

$$v_{out} = R_E\,(\beta+1)\Delta i_B$$

$$\Delta i_B = v_{out} \;/\; R_E\,(\beta+1) \tag{10}$$

Replacing (10) in (9),

$$v_{in} - v_{out} = r_{source}\,v_{out} \;/\; R_E\,(\beta+1)$$

$$v_{in} = v_{out}\,(1 + \frac{r_{source}}{(\beta+1)R_E})$$

$$v_{out} = \frac{1}{1+\frac{r_{source}}{R_E}\frac{1}{(\beta+1)}}\,v_{in} \qquad \text{(For circuit in Fig.9 that, uses a transistor)} \tag{11}$$

Compare this last expression for Δv_{out} with the case in which the transistor circuit were not used (Fig.1):

$$v_{out} = v_{in}\,\frac{1}{1 + \frac{r_{source}}{R_E}} \qquad \text{(For circuit in Fig.1, no transistor used)} \tag{12}$$

Notice in (11), one obtains the same result whether

- Considering an effective (source) output impedance $r_{source}/(\beta+1)$ and load impedance R_E, or
- Considering a (source) output impedance r_{source} and an effective input impedance $R_E(\beta+1)$.

III. EXPERIMENTAL CONSIDERATIONS

III.1 Biasing the emitter follower

It is necessary to bias the emitter-follower so that collector current flows while the input signal v_{in} changes. A voltage divider is the simplest way, which is shown in Fig. 10.

R_1 and R_2 are chosen to put the base halfway between the ground and V_{CC} when there is not input signal. Hence R_1 and R_2 are approximately equal.

TASK. Design and build the following emitter-follower circuit

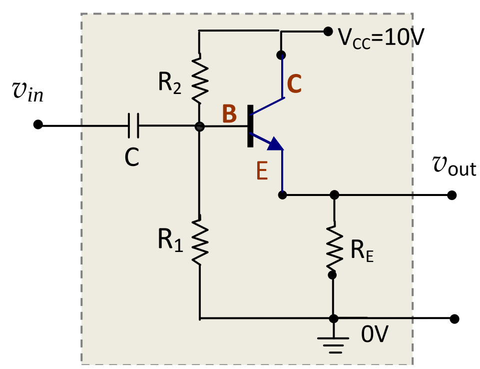


Fig. 10 Emitter follower circuit. Notice, we are interested in using small signal ac-voltages at the input

Notice that the stiffer emitter-follower does not invalidate the calculation performed above with the simpler emitter-follower circuit. The support for argument is shown in Fig.11. The voltage divider is a DC bias that provides a DC base current that bias properly the transistor to work in the active region. The connection of another AC-source (v_{in}) produces an additional AC base current. (See also Appendix-1 below for Thevenin equivalent circuit analysis).

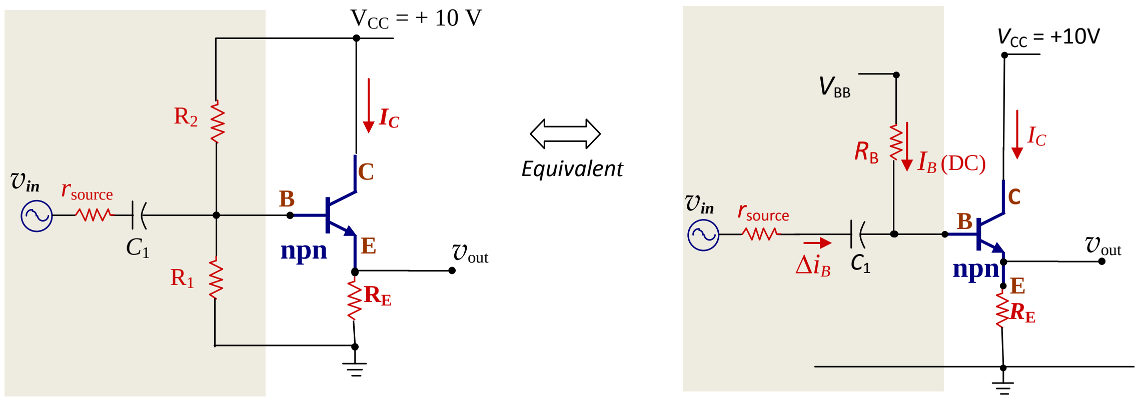


Fig. 11 Thevening equivalent circuit. **Left:** Emitter-follower circuit. **Right:** Equivalent circuit. The drawing helps to differentiate the additional (AC) base current injected by the signal generator (v_{in}) from the DC base current established by the bias circuit. Notice the upper case used for DC voltages (like V_{BB} and I_B) and the lower case used for AC small-signals (like v_{in}, Δi_B, and v_{out}).

Thevening analysis indicate the value for R_B is,

$$R_B = (R_1 // R_2) \qquad \text{and } V_{BB} = 5V \tag{13}$$

Notice, if we want the base to be about half way between the the ground and V_{CC}, that is around 5 volts. Since V_{BB} is already 5 volts a criteria to select R_1 and R_2 is that the drop of voltage across ($R_1//R_2$) to be much smaller than $R_E I_E$,

$$(R_1 // R_2)\, I_B << R_E\, I_E$$

$$(R_1 // R_2)\, I_B << \beta\, R_E\, I_B$$

$$(R_1 // R_2) << \beta\, R_E \tag{14}$$

How to choose R_E?

If we decided to work with a collector current of 1 mA, and we want the output to swing around the middle between the ground and V_{CC}, then a 5 kΩ resistor would do the job.

Method of analysis

Underlying our method of analysis is to consider the emitter follower as a black box, which to the effect of measuring its effective load it will be considering having input impedance Z_{in}, and to the effect of driving a subsequent circuit stage it will be considered having output impedance Z_{out}. Our objective in this lab is to **calculate** and **measure** these two impedances (of the same emitter follower circuit.) See figures 5 and 7 above.

Experimental procedure

- First, ensure the transistor in the emitter followers is working in the active region.

  In this case, do not hook yet any ac input signal yet. This step is just to make sure the transistor is working in the active region under DC bias voltage.

Measure V_{EB}

Measure the base current, and verify if the value agrees with the Thevenin analysis.

- Next, apply an ac input voltage v_{in}.

  Use a coupling capacitor (try C_1= 0.1 μF, for example).
- For a given input frequency (1 kHz, for example), increase gradually the amplitude and check the max amplitude the circuit tolerates before the output signal gets distorted.
- Establish the range of frequency at which the circuit works as an emitter follower.

  Find out what happens when the frequency is lowered. Check if there is distortion when too low frequencies are used.

Repeat the procedure above by experimenting a smaller capacitance (10 nF).

III.1B Input impedance of the emitter-follower circuit

- Measure the input impedance of the circuit

The diagram on the left in Fig. 12 shows in a very straightforward manner that the input impedance can be determined experimentally by measuring the base-voltage Δv_B and the input base-current Δi_B. We want to measure $\Delta v_B / \Delta i_B$.

The diagrams on the right in Fig. 12 show two optional ways of implementing the measurement. For Δv_{in} use an ac-voltage of amplitude ~ 20 mV. (Notice, in the first option an additional "small" resistance R_x has been introduced for the purpose of measuring Δi_B.)

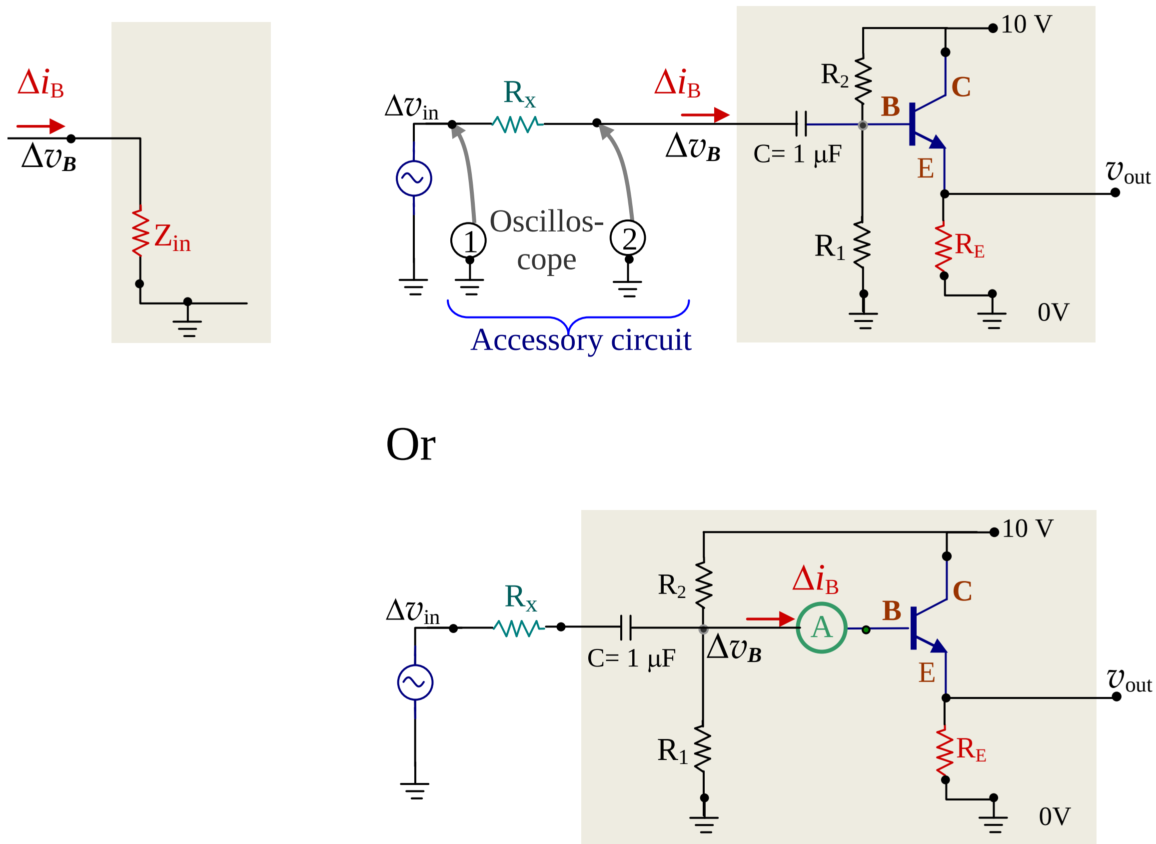


Fig. 12 Left: Black box diagram representing the emitter-follower. Measuring Δv_B /Δi_B gives the value of Z_{in}. **Right:** Optional experimental implementations for measuring Δv_B and Δi_B. In the first option, the measurement of Δi_B is implemented by using a small resistance R_x and setting the oscilloscope inputs to ac-mode; invert channel 2 and measure $v_1–v_2$ in order to obtain the input current Δi_B. In the second option, an ammeter is inserted to measure directly Δi_B. Verify also that Δv_{out} follows Δv_B (i.e. ensure the transistor is working in the active region.)

TASKS: Verify also that Δv_{out} follows Δv_B (i.e. ensure the transistor is working in the active region.)

Verify if the predicted value given in (5) is close to the experimental value you measured in the section above.

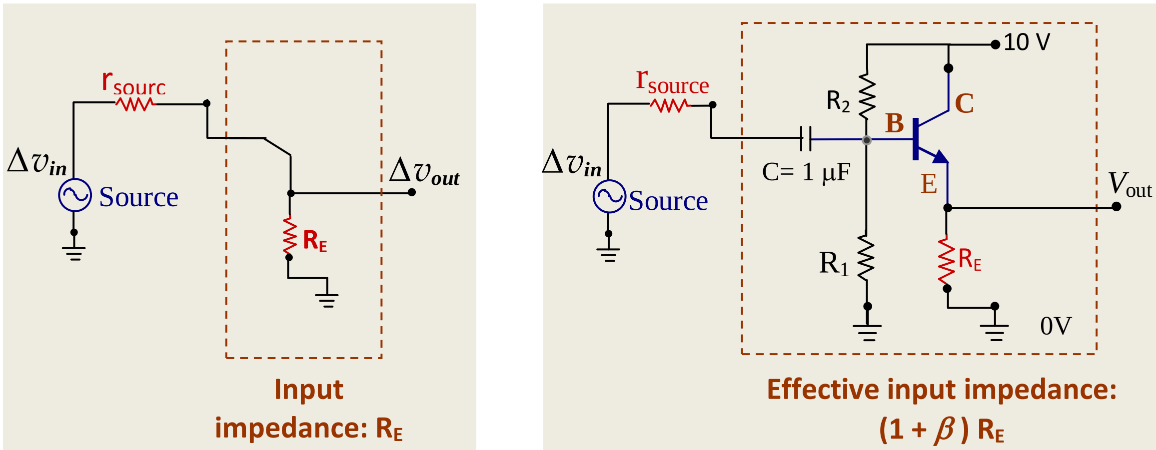


Fig. 13 Schematic comparison of the effective input impedance caused by R_E, depending on whether or not a transistor is used. The higher impedance of the latter is an advantage feature.

III.1C Output impedance of the emitter-follower circuit

Similarly to the procedure for finding the input impedance,

Method-1 Using a variable resistance R_{LOAD}.

We will consider the emitter follower circuit (Fig. 14, left diagram) as a black box (right diagram) for the purpose of finding its equivalent output impedance.

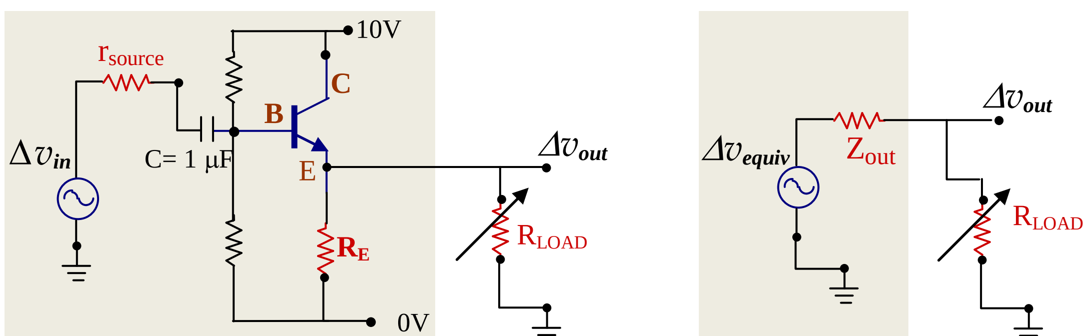


Fig. 14 The follower emitter circuit (left) and its equivalent circuit (right)

In this setup, the output voltage across R_{LOAD} is given by,

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv} \qquad (13)$$

By choosing two different values for R_{LOAD}, R_1 and R_2, one obtains,

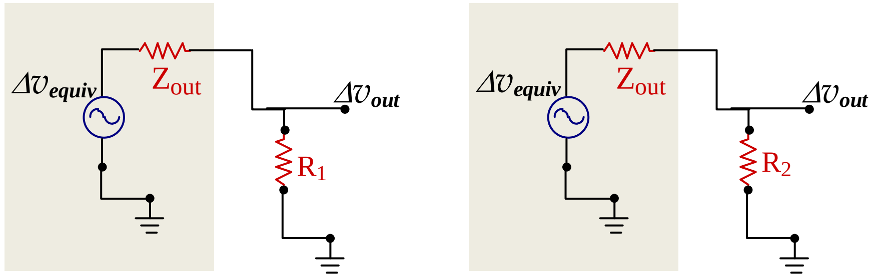


Fig. 15 The equivalent follower emitter circuit hooked to two different output loads.

$$\Delta v_{out;1} = \frac{R_1}{Z_{out} + R_1}\Delta v_{equiv} \quad \text{and} \quad \Delta v_{out;2} = \frac{R_2}{Z_{out} + R_2}\Delta v_{equiv}$$

Solving for Z_{out} (see proof at the end of these notes, appendix 2),

$$Z_{out} = R_2 \frac{\left[\dfrac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1\right]}{\left[1 - \dfrac{R_2}{R_1}\dfrac{\Delta v_{out;1}}{\Delta v_{out;2}}\right]} \tag{14}$$

- As we expect Z_{out} to be very low ($\leq 100\ \Omega$) compared to the input impedance, you may have to use very low values for R_1 and R_2 (potentially 30 Ohms).
- Alternatively, in the description above, R_1 can be the input impedance of the oscilloscope, and R_2 a low value resistance R_2.

  i) Connect the output to the oscilloscope (assumed here that the impedance of the oscilloscope is infinite; $R_{LOAD} = R_1 = \infty$). This allows measuring the amplitude of Δv_{equiv}.

$$\Delta v_{out,1} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}}\Delta v_{equiv} = \frac{R_1}{Z_{out} + R_1}\Delta v_{equiv} \xrightarrow[R_1 \to \infty]{} \Delta v_{equiv}$$

  That is,

$$\Delta v_{out,1} = \Delta v_{equiv}$$ (*measured by simply connecting the output of the transistor to the oscilloscope*.)

  ii) Use an arbitrary external resistance R_2 (typically low values work better, like 30 Ohms for example) as R_{LOAD}, and measure the corresponding $\Delta v_{out.2}$.

$$\Delta v_{out,2} = \frac{R_2}{Z_{out} + R_2}\Delta v_{equiv}$$

$$(Z_{out} + R_2) = \frac{R_2 \Delta v_{equiv}}{\Delta v_{out,2}}$$

$$Z_{out} = R_2\left[\frac{\Delta v_{equiv}}{\Delta v_{out,2}} - 1\right] = R_2\left[\frac{\Delta v_{out,1}}{\Delta v_{out,2}} - 1\right]$$

Note about checking the results:

Even if you get smaller values for Z_{out}, how to know if the results make sense? That is, how do we know the output resistance without and with the transistor has decrease by a factor of β? In fact, the latter may not have been fulfilled (check your results once you measure independently the output impedance of your signal generator, as requested in the next section of this lab).

One alternative way to verify our results is to do the following:

- Insert a resistance of 1k ohm between the ac-voltage source and your circuit. That is, we are adding a 1k ohm to the output impedance.
  Repeat the procedure to measure Z_{out}.
  Check if your new results has increased the value for Z_{out} by $1k\Omega/\beta$.

Method-2

The shortcoming of method-1 is that it requires the testing R_{LOAD} (Fig. 14) to be too small (which may disturb the working point of the transistor).

Another way is to use the circuit of Fig. 13 and measure directly,

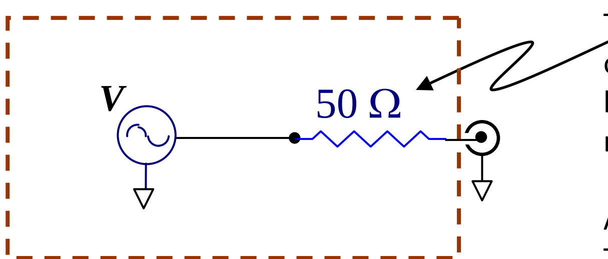
$$Z_{out} \equiv [\Delta v_{in} - \Delta v_{out}] / (\text{output current})$$
$$= (\Delta v_{in} - \Delta v_{out}) / \Delta i_E$$

This will require to measure $v_{in} - v_{out}$, which will be very small. One way to attempt its measurement is to use the oscilloscope to monitor v_{in} (in channel-1) and v_{out} (in channel-2), invert the second signal, and then use the scope in summing mode. For Δi_E you can use $(\Delta v_{out}) / R_E$

In both methods, be aware of the role played by the capacitor impedance. Choose the proper frequency such that *a*) the capacitor impedance is minimum, but at the same time *b*) the used frequency is within the frequency-bandwidth response of the transistor.

III.2. Matching Impedance

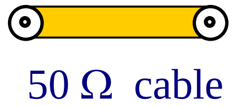
50 Ω Output Impedance



This is a real resistor, physical placed at the output (as literally drawn in the figure.) A high quality resistor, with the lowest reactance possible, is preferred.

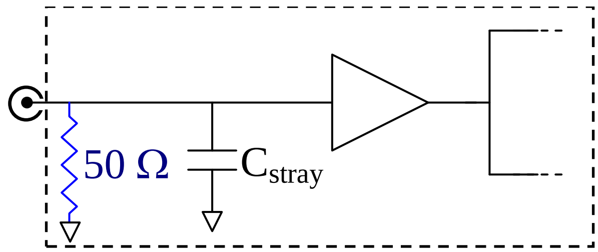
Alternatively, you can think this is the Thevenin's equivalent circuit.

50 Ω Impedance Cable



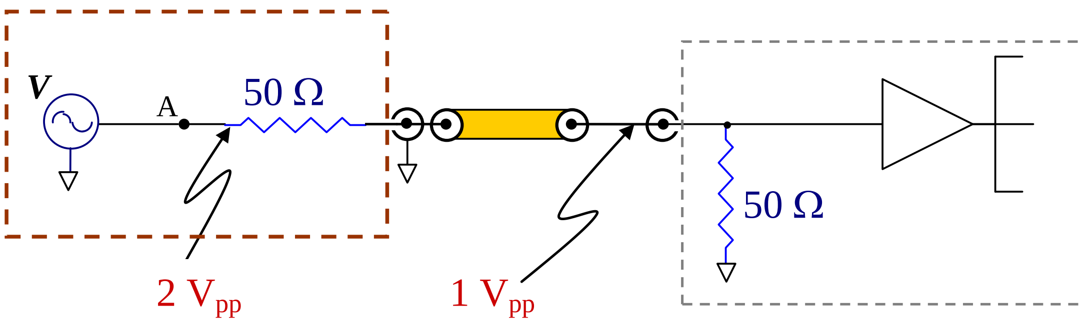
This is a 50 Ω complex impedance cable

50 Ω Input Impedance



MEASUREMENTS

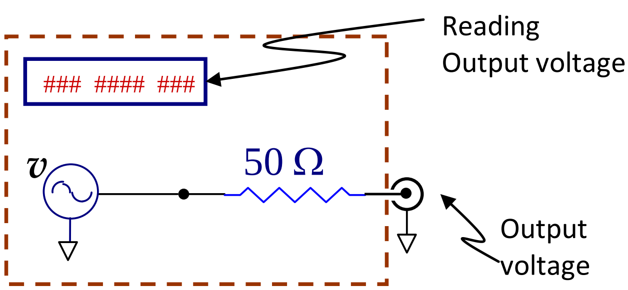
Some signal generators are specified to have 50 Ω output impedance AND expect to be connected to 50Ω input impedance devices. ONLY if the latter requirement is satisfied, the reading of the output voltage will coincide with the actual output voltage.



The input and output impedance create a voltage divider, hence the voltage readings (2 V_{pp} and 1V_{pp}) shown in the figure.

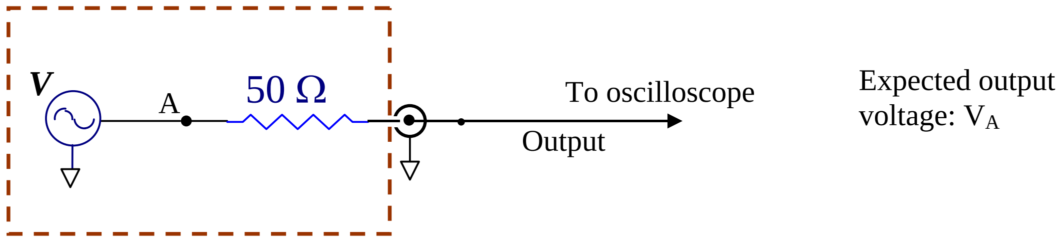
Notice in the case above that if, for example the output voltage were 1 Vpp (1 volt peak-to-peak) the voltage at A would be 2 Vpp

HOW TO TEST IF AN EQUIPMENT HAS A 50Ω OUTPUT IMPEDANCE OR NOT?



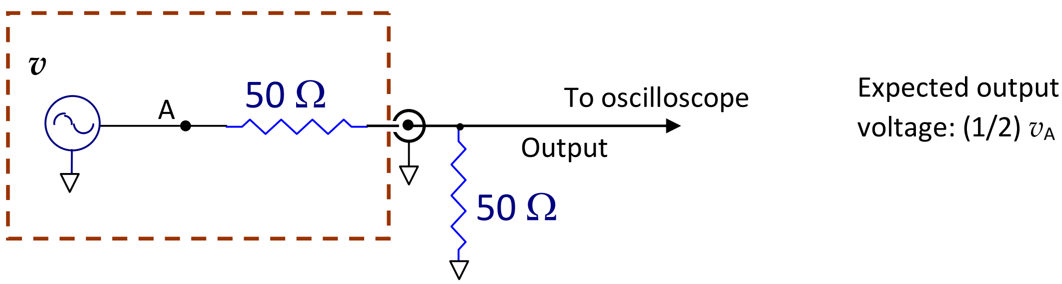
Task: Verify whether or not your signal generator has a 50Ω output impedance.

i)



ii)

Now, connect a 50 Ω resistor as shown in the figure below. If the voltage drops to 50%, then the output impedance is 50Ω.



Appendix-: Thevenin equivalent circuit analysis

The example presented here is for a transistor amplifier circuit. But the essence of the Thevening analysis is valid also for a emitter-follower circuit.

We can use the Thevenin's theorem to show the equivalence between the circuits in Fig. 9 and Fig. 11.

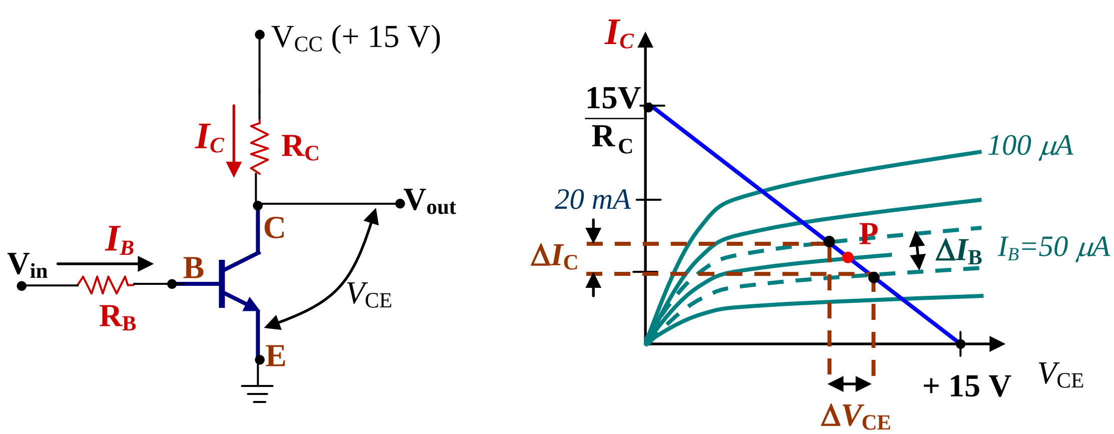


Fig. 9

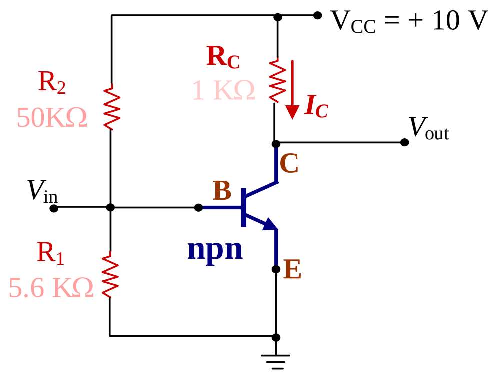


Fig. 11 DC bias circuit (accomplished by the resistance R_1 and R_2).

This is made more evident by re-drawing Fig. 11 as shown in Fig. 12 below.

Through the Thevenin theorem one can claim that both circuits, the ones at the left and right sides of Fig.12 are equivalent. Analyzing the shaded area one obtains:

- The Thevenin voltage V_{BB} is the open-circuit voltage (voltage across XY in the circuit when no external load is applied) $V_{BB} = \frac{V_{CC}}{R_1 + R_2} R_1$. ($V_{BB}$ = (10V/55 kΩ)(5 kΩ) = 1V.)
- R_B designates the Thevenin equivalent series resistance. The short circuit currents (i.e. the currents when X and Y are shorted) are V_{CC}/R_2 and V_{BB}/R_B respectively. Since the circuits are equivalent these two current must be equal. Hence, $R_B = \frac{R_2}{V_{CC}} V_{BB}$; using the value for V_{BB} obtained above, results $R_B = \frac{R_2 R_1}{R_1 + R_2}$ (R_B = (5 kΩ x 5 kΩ) / (55 kΩ) = 5 kΩ).

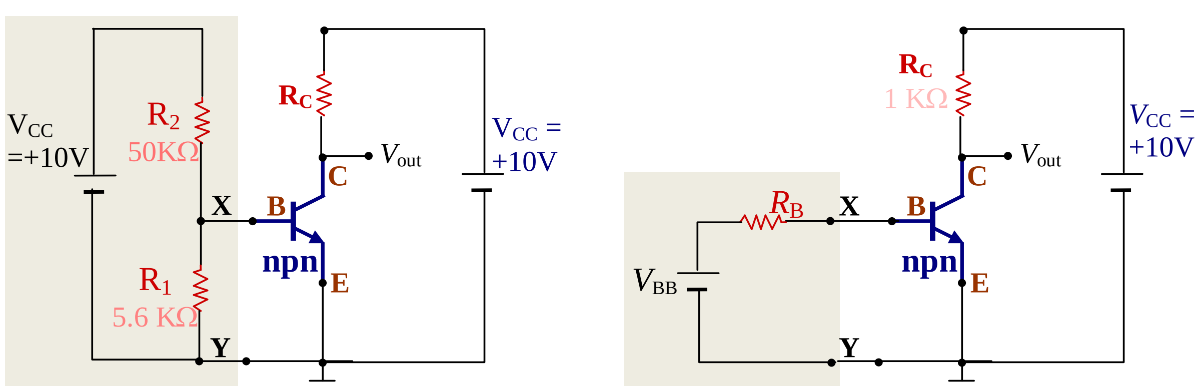


Fig. 12 DC bias circuit and its Thevenin equivalent. The latter helps to calculate the different parameters associated to the intended operating point of the transistor (using the analysis described in the previous section) based on the values of R_1 and R_2.

Appendix-2

Proof of expression (14):

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv}$$

When using $R_{LOAD} = R_1$ and $R_{LOAD} = R_2$ respectively, one obtains,

$$\Delta v_{out;1} = \frac{R_1}{Z_{out} + R_1} \Delta v_{equiv} \quad \text{and} \quad \Delta v_{out;2} = \frac{R_2}{Z_{out} + R_2} \Delta v_{equiv}$$

$$\frac{\Delta v_{out;1}}{\Delta v_{out;2}} = \frac{R_1}{R_2} \frac{(Z_{out} + R_2)}{(Z_{out} + R_1)}$$

$$(Z_{out} + R_1) \frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} = (Z_{out} + R_2)$$

$$Z_{out} \left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right] + R_1 \frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} = R_2$$

$$Z_{out} \left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right] = R_2 \left[1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}} \right]$$

$$Z_{out} = R_2 \frac{\left[1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}} \right]}{\left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right]}$$

---

[i] Horowitz and Hill, "The Art of Electronics